Proportional Reasoning: Focus on Sense-Making

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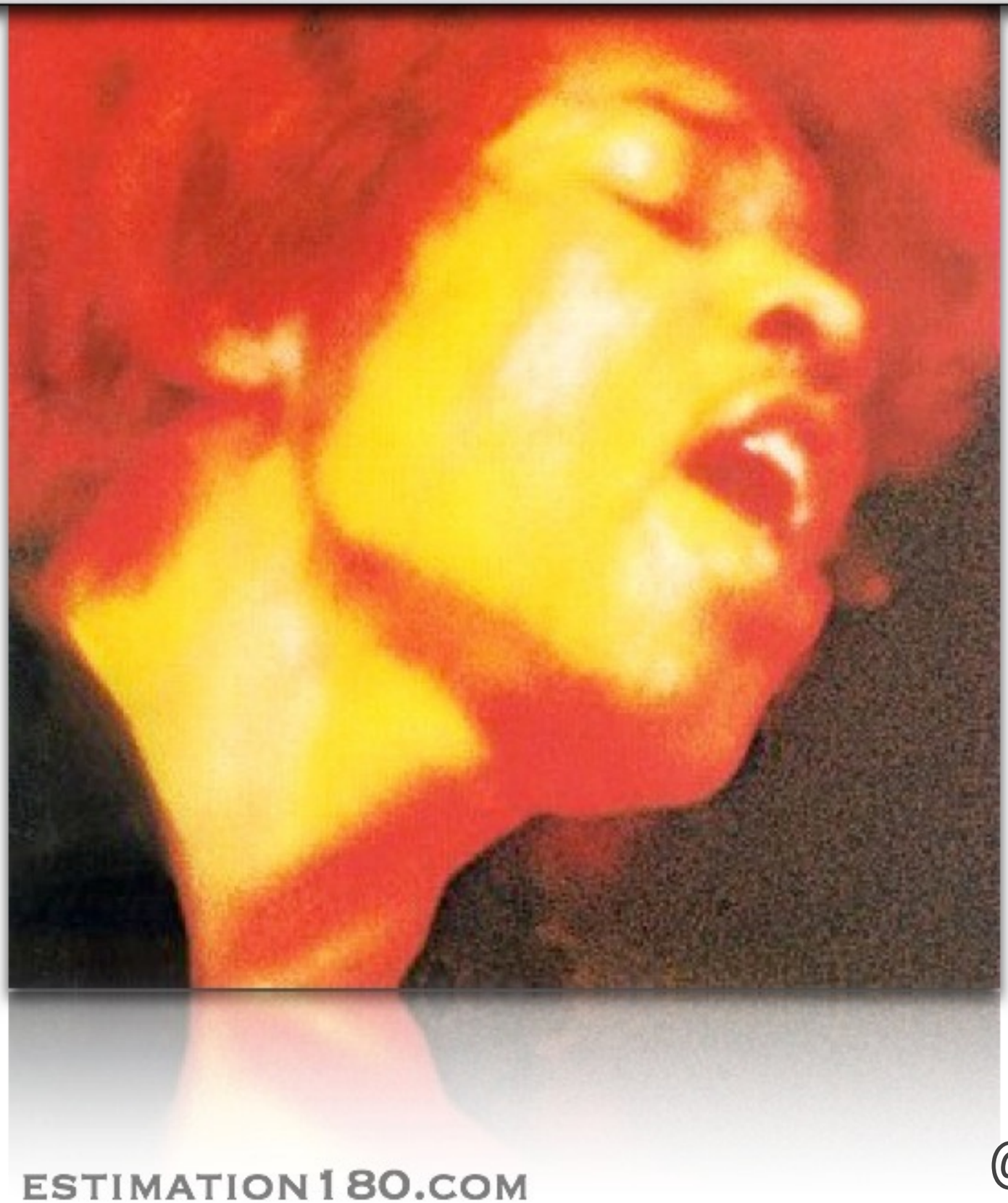
All Along The Watchtower
The Jimi Hendrix Experience — Electric Ladyland
2:40



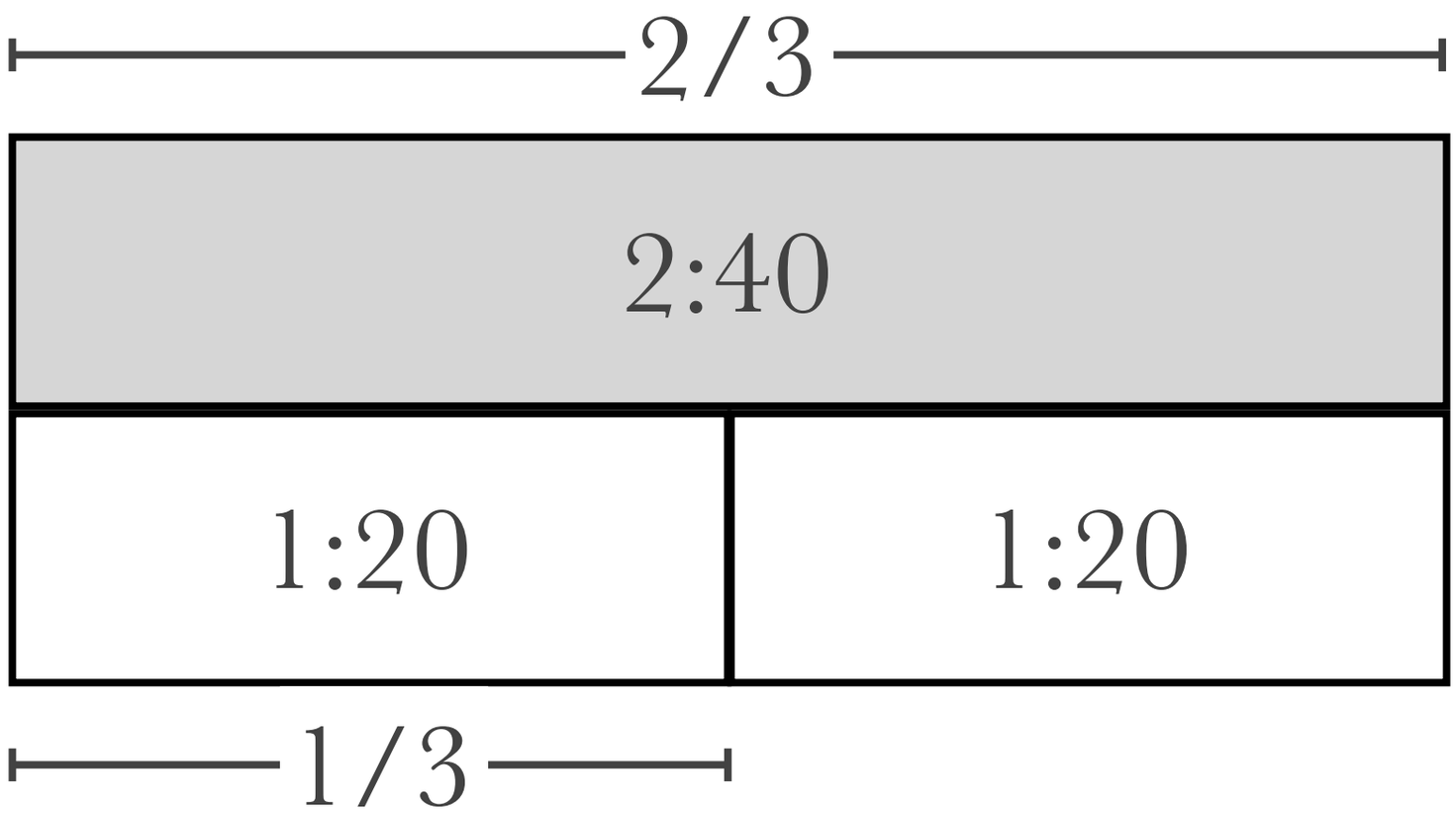
ESTIMATION180.COM

@mr_stadel

Representation: Bar Model

2:40

Representation: Bar Model



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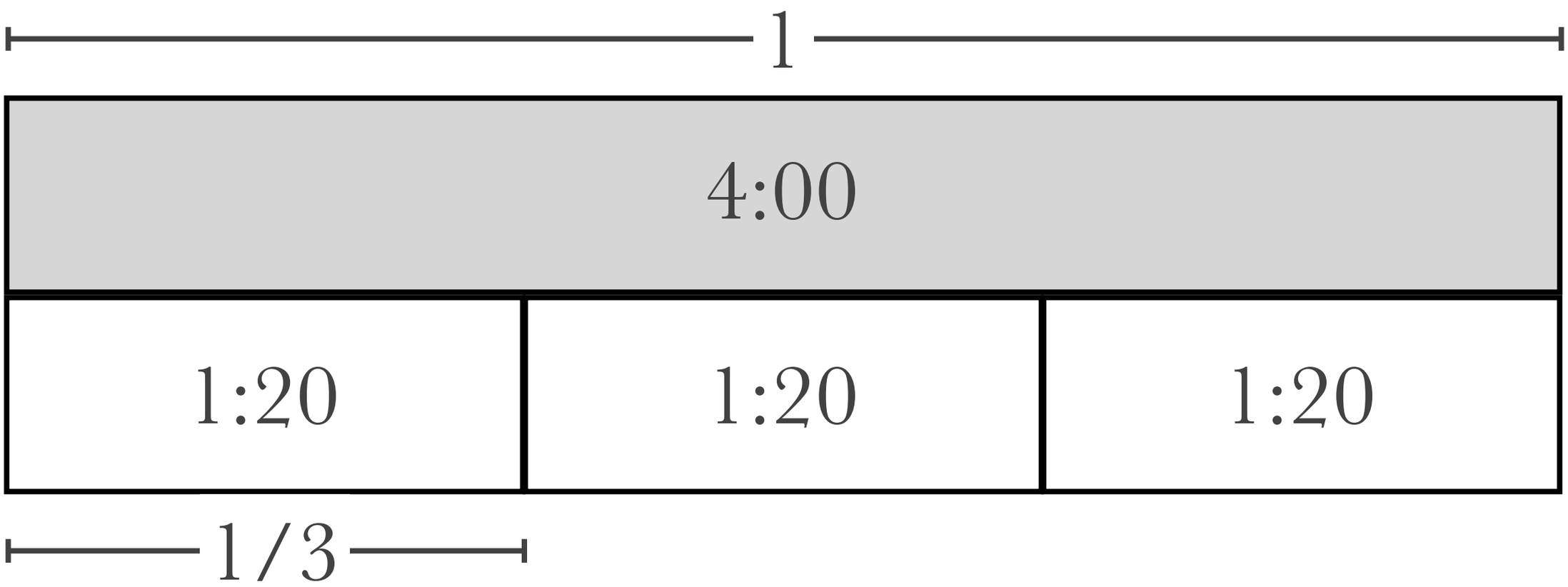
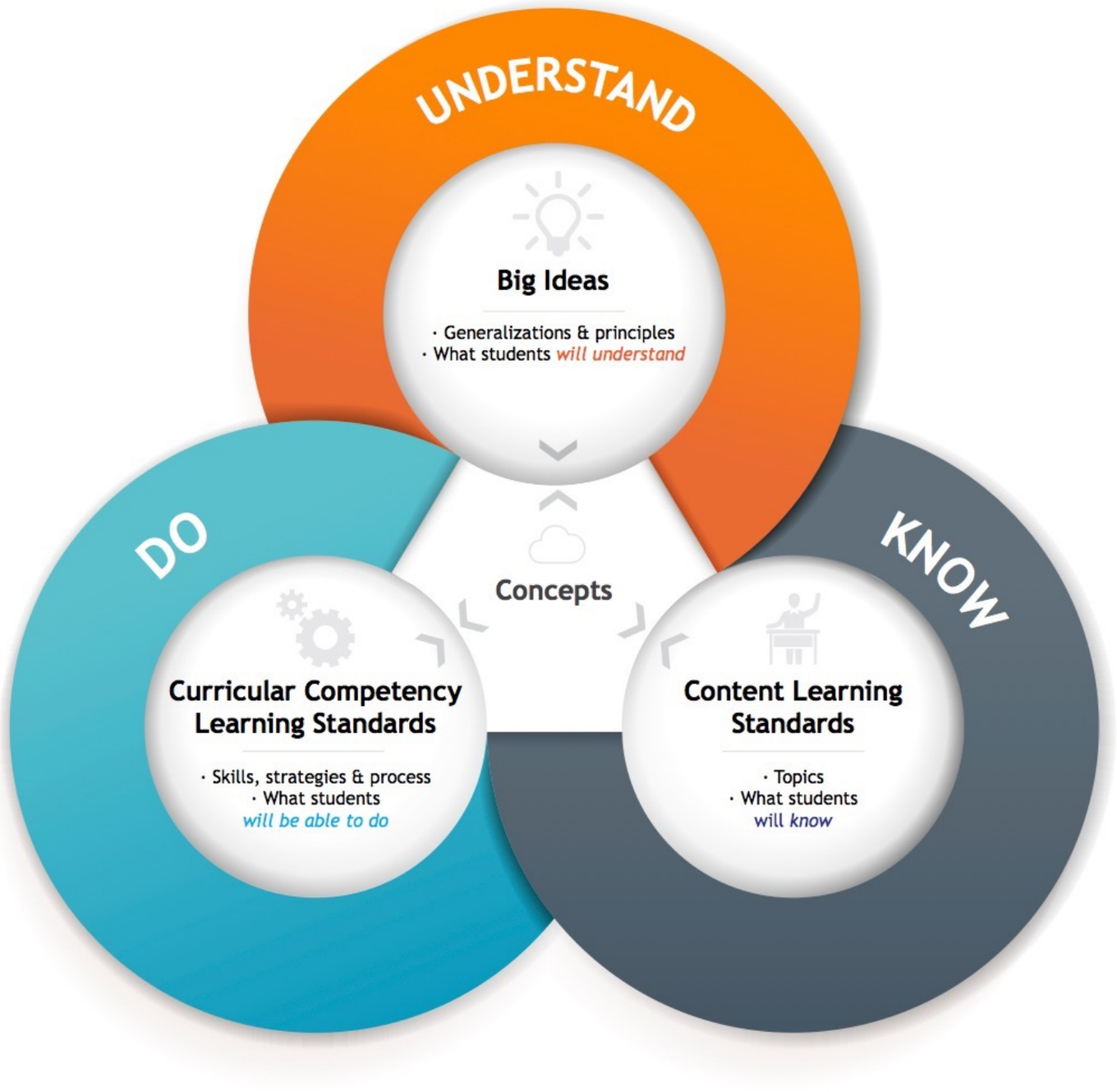
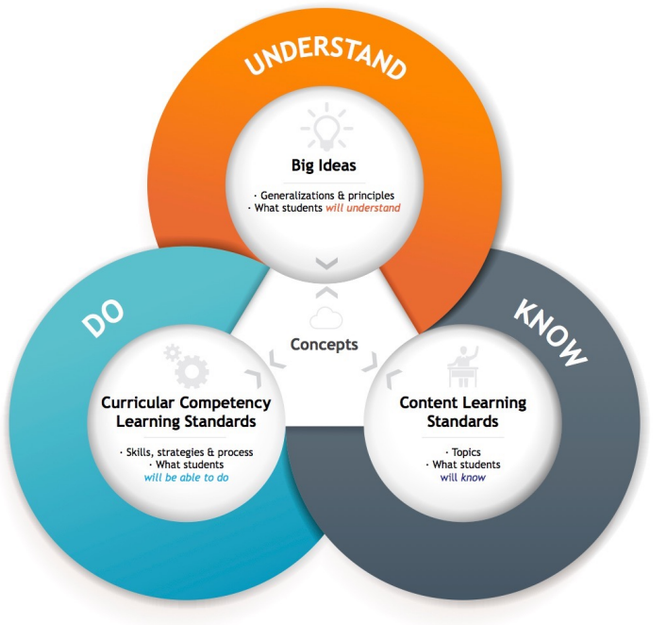



UNDERSTAND
Big Ideas
· Generalizations & principles
· What students will understand
DO
KNOW
Concepts
Curricular Competency Learning Standards
· Skills, strategies & process
· What students
will be able to do
Content Learning Standards
· Topics
· What students
will know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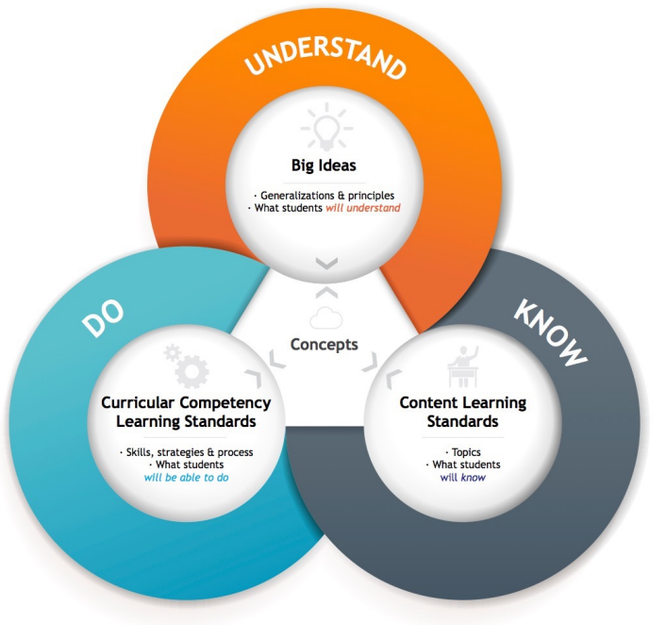
Big Idea(s)
Competencies
Content



ratios

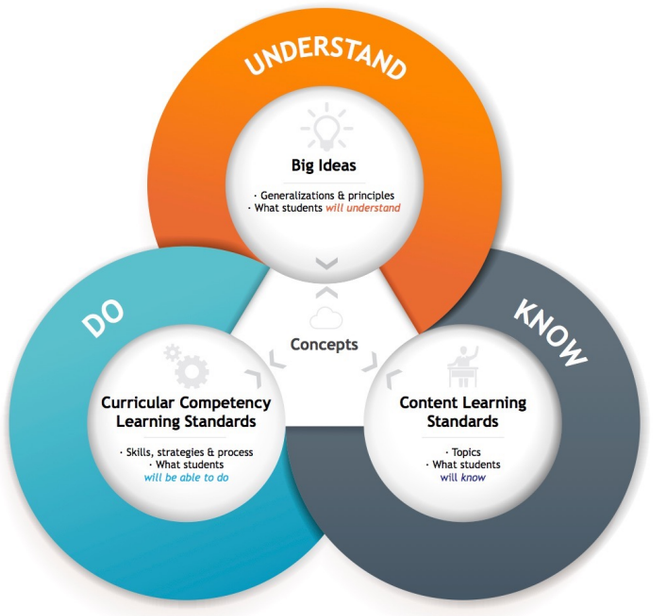
rates

proportions

unit price

percent

coupons



Big Idea(s)

Proportional reasoning helps us make sense of multiplicative relationships.

Competencies

Content

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rates

proportions

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percent

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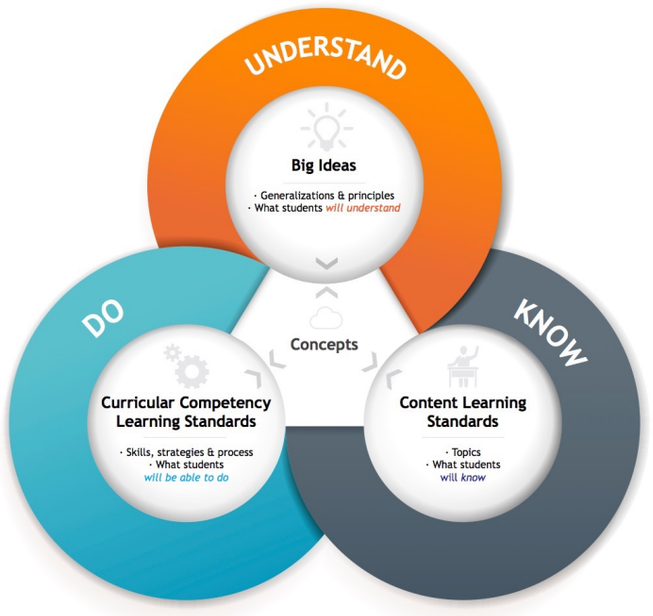


Big Idea(s)

Proportional reasoning helps us make sense of multiplicative relationships.

Competencies

Use multiple strategies to solve problems

Communicate in a variety of ways to explain and justify ideas

Content

ratios

rates

proportions

unit price

percent

coupons

Big Idea
Students will understand that • Thinking about how quantities are related using multiplication is essential for solving a wide variety of problems • Ratios, rates, and percent make comparisons easy; one term is made the same

Big Idea(s)

Proportional reasoning helps us make sense of multiplicative relationships.

Competencies

Use multiple strategies to solve problems

Communicate in a variety of ways to explain and justify ideas

Content

ratios
rates
proportions
unit price
percent
coupons

Curricular Competencies	Content
Students will be able to: • choose correct and efficient strategies • monitor progress to completion of task and make necessary adjustments along the way • propose and consider or critique alternative strategies • share mathematical *ideas*–not just *steps*!–needed to solve problems (verbal & written) • present work that is clear and easy to follow • effectively use tables, equations, etc. to support conclusions or arguments	*Students will know that:* • two equivalent ratios represent the same relationship • ratio tables list equivalent ratios in an organized way • a rate represents an infinite number of equivalent ratios • a unit rate (or price) is an equivalent rate where one term is "1" • a proportion is an expression of the equivalence of two ratios • proportion problems can be solved by looking for scale factors within or between ratios • a percent is a fanatical comparison to 100

Grade 8 Math

Curricular Competencies

Big Ideas

*Number represents, describes, and compares the quantities of ratios, rates, and percents.

*Computational fluency and flexibility extend to operations with fractions.

*Discrete linear relationships can be represented in many connected ways and used to identify and make generalizations.

*The relationship between surface area and volume of 3D objects can be used to describe, measure, and compare spatial relationships.

*Analyzing data by determining averages is one way to make sense of large data sets and enables us to compare and interpret.

			Reasoning and Analyzing						Understanding and Solving				Communicating and Representing				Connecting and Reflecting			
			Use logic and patterns to solve puzzles and play games	Use reasoning and logic to explore, analyze, and apply mathematical ideas	Estimate reasonably	Demonstrate and apply mental math strategies	Use tools or technology to explore & create patterns & relationships, & test conjectures	Model mathematics in contextualized experiences	Apply multiple strategies to solve problems in both abstract and contextualized situations	Develop, demonstrate, and apply mathematical understanding through play, inquiry, and problem solving	Visualize to explore mathematical concepts	Engage in problem-solving experiences that are connected to place, story, cultural practices, and perspectives relevant to local First Peoples communities, the local community, & other cultures	Use mathematical vocabulary & language to contribute to mathematical discussions	Explain and justify mathematical ideas and decisions	Communicate mathematical thinking in many ways	Represent mathematical ideas in concrete, pictorial, and symbolic forms	Reflect on mathematical thinking	Connect mathematical concepts to each other and to other areas and personal interests	Use mathematical arguments to support personal choices	Incorporate First Peoples worldviews and perspectives to make connections to mathematical concepts
			A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
Content	perfect squares and cubes	1																		
	square and cube roots	2																		
	percents less than 1 and greater than 100 (decimal & fractional %)	3																		
	numerical proportional reasoning (rates, ratio, proportions, & %)	4																		
	operations with fractions (addition, subtraction, multiplication, division, & order of operations)	5																		
	discrete linear relations (extended to larger numbers, limited to integers)	6																		
	expressions- writing and evaluating using substitution	7																		
	two-step equations with integer coefficients, constants, and solutions	8																		
	surface area and volume of regular solids, including triangular and other right prisms and cylinders	9																		
	Pythagorean theorem	10																		
	construction, views, and nets of 3D objects	11																		
	central tendency	12																		
	theoretical probability with two independent events	13																		
	financial literacy — best buys	14																		

Chris

$75

Jeff

$60

Marc

$45

Peter Liljedahl

BUY TWO PAIRS, GET ONE PAIR FREE!

3rd pair must be of equal or lesser value

Chris

?

Jeff

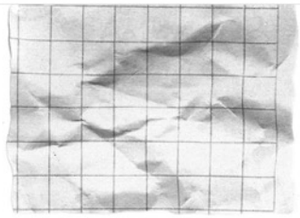
?

Marc

?

Chris Jeff Marc

$45 $45 $45



Sharing Pairs

C J M

Three friends, Chris, Jeff, and Marc, go shopping for shoes. The store is having a *buy two pairs, get one pair free* sale.

Chris opts for a pair of high tops for $75, Jeff picks out a pair of low tops for $60, and Marc settles on a pair of slip-ons for $45.

The cashier rings them up; the bill is $135.

How much should each friend pay? Try to find the fairest way possible. Justify your reasoning.

Original
C = $75
J = 60
M = 45
Total = 180

With the sale the total would be 135. Since there is 3 friends you would divide the total by 3 (to get 45). If you left it there they would all be paying the same amount. That wouldn't be fair because to Mark he would be paying $45.00 either way and Chris would be getting a huge discount. If you take away $15.00 from each of the original prices you get a more fairer way of dividing the money. Chris would still be paying more because his shoes cost more and Mark would be paying less because his shoes cost less. All of them would get a $15.00 discount and would be fair

135 / 3 = 45

C	J	M
75	60	45
-15	-15	-15
60	45	30

60
45
+30
135

h/t Carley Brockway

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$60

Jeff

$45

Marc

$30

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Chris
$75
-$15
$60

Jeff
$60
-$15
$45

Marc
$45
-$15
$30

This is the fairest way because in the beginning all their shoes prices were a $15 difference. so basically they subtract $15 from the orginal price, chris would then pay $60, Jeff would pay $45 and Marc would pay $30. It is fair because chris' shoes are the most expensive so he should pay more than Jeff and Marc. since Marc's shoes were the least, thats why he pay's the least amount. Also, in the beginning all the prices had a difference of $15 and with the prices $60, $45, $30, there is a difference of $15. I think this is the fairest way to split up the money.

Chris

$56.25

Jeff

$45.00

Marc

$33.75

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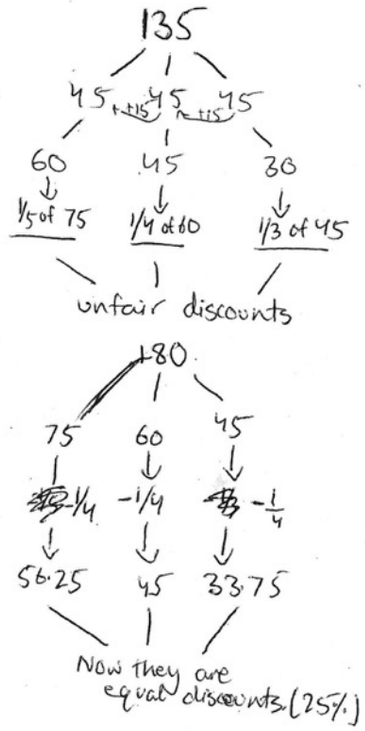
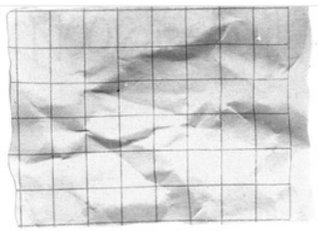


March. 27th, 2015



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Ex.1 $\frac{\$135}{3} = \$45$ per person

Ex.2 CHRIS 60 + JEFF 45 + MARC 30 = $135 $15 difference

Ex.3 55 (73.3%) + 45 (75%) + 35 (77.7%) = $135 $10 difference

FINAL ANSWER: Ex.4 56.45 (75.27%) + 45 (75%) + 33.55 (74.56%) = $135 ← 75% paid per person from the original price

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FINAL ANSWER: Ex.4 56.45 (75.27%) + 45 (75%) + 33.55 (74.56%) = $135 ← 75% paid per person from the original price

Chris

$150

Jeff

$90

Marc

$60

Sharing Gasoline Costs

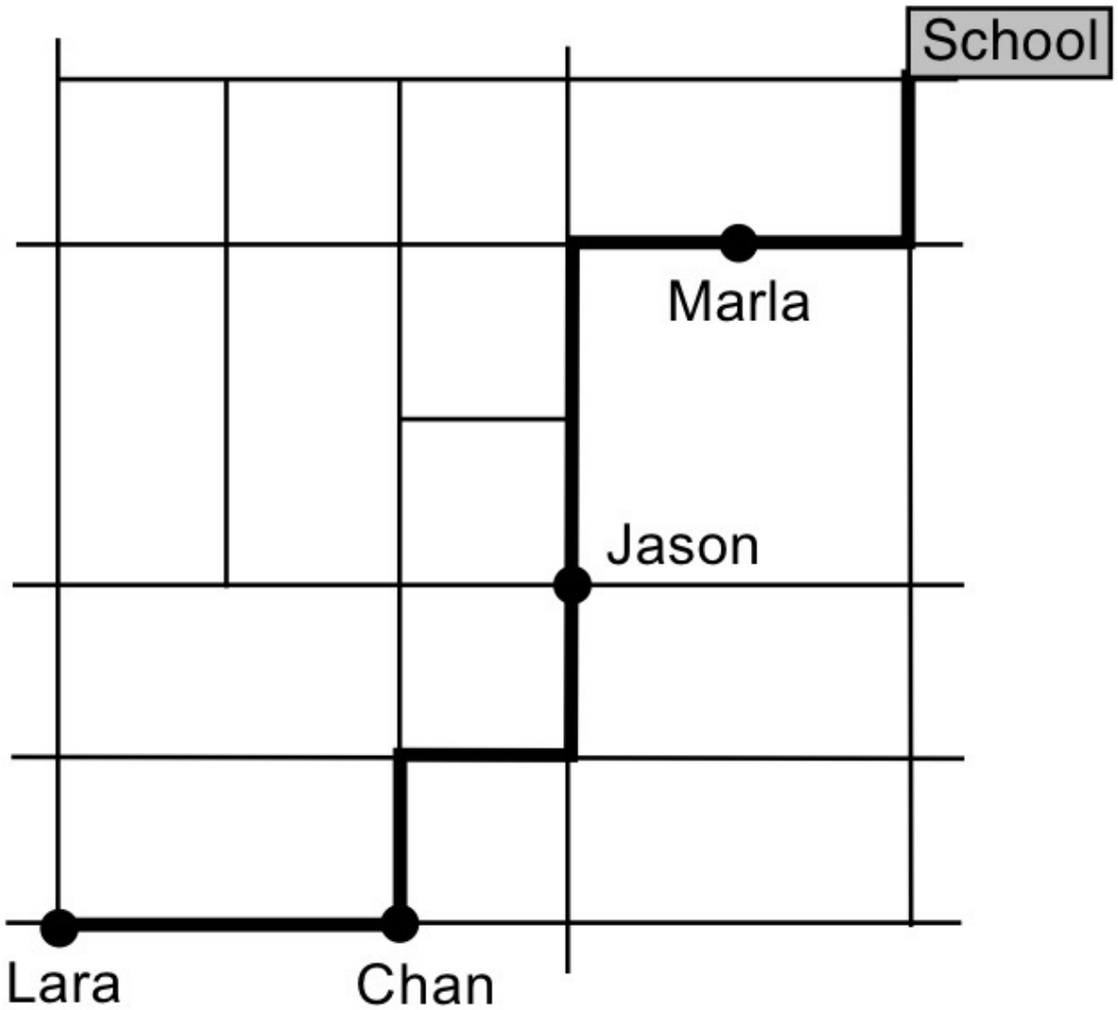
Each day Lara's mom drives her to school.
On the way, she picks up three of Lara's friends, Chan, Jason and Marla.

Each afternoon, she returns by the same route and drops them off at their homes.

This map is drawn to scale.

It shows where each person lives and the route taken by Lara's mom.

At the end of a term, the four students agree to pay $300 in total towards the cost of the gasoline.



How much should each person pay?
Try to find the fairest possible method. Show all your work.

Rod Ratios

What is the ratio of this pair of Cuisenaire Rods?

How do you know?

How many pairs can you find with the same ratio?

Proportional Problem Types:

(1) *missing-value*, and

(2) *comparison*

Split Time

@ddmeyer

Split Time

What's the first question that comes to your mind?

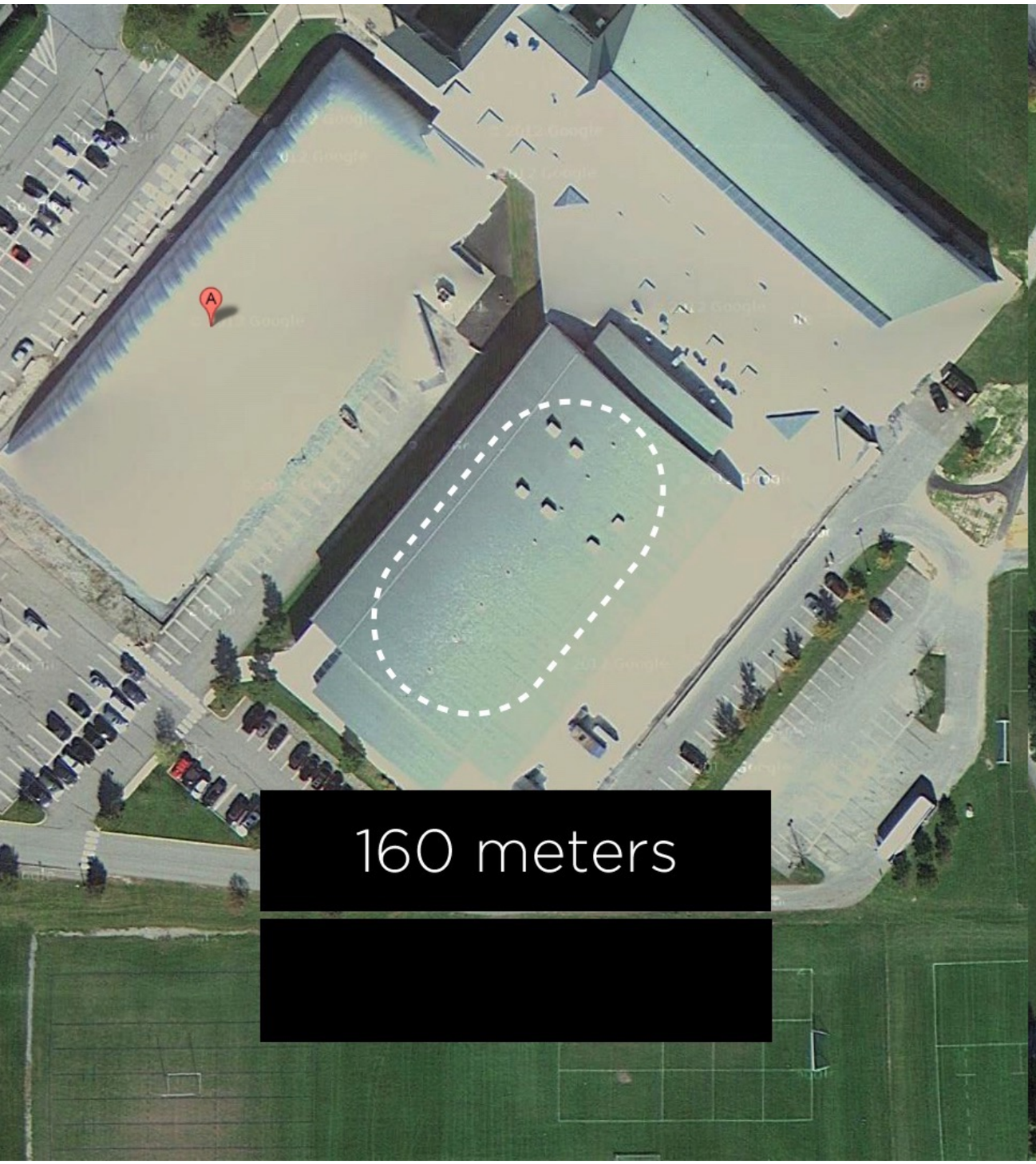
What's a guess that's too *low*?

What's a guess that's too *high*?

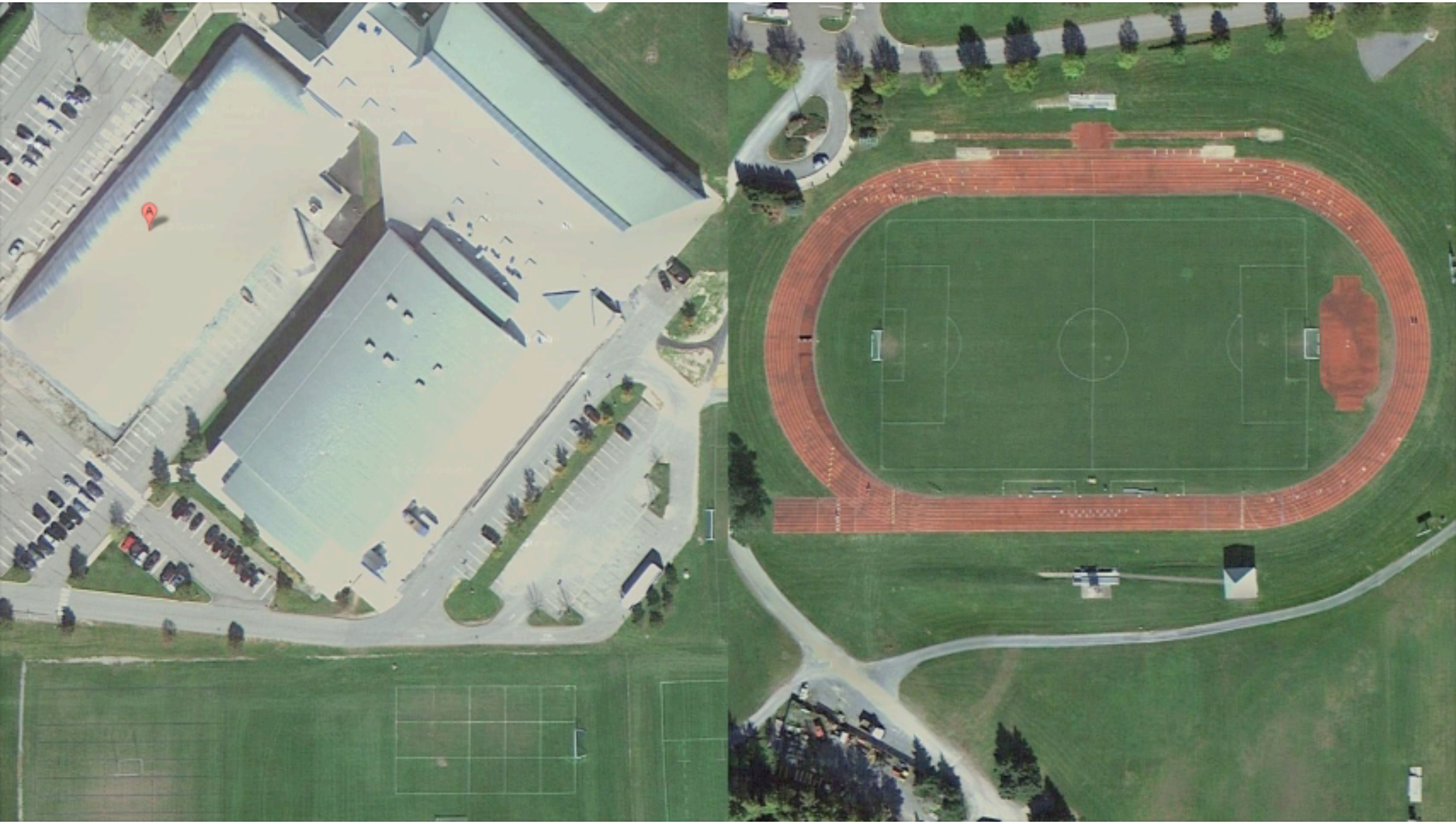
Write down your estimate.

What information would be helpful to know here?

Split Time



Split Time



Representation: Ratio Table

metres	400			
seconds	75			

Representation: Ratio Table

metres	400	40		
seconds	75	7.5		

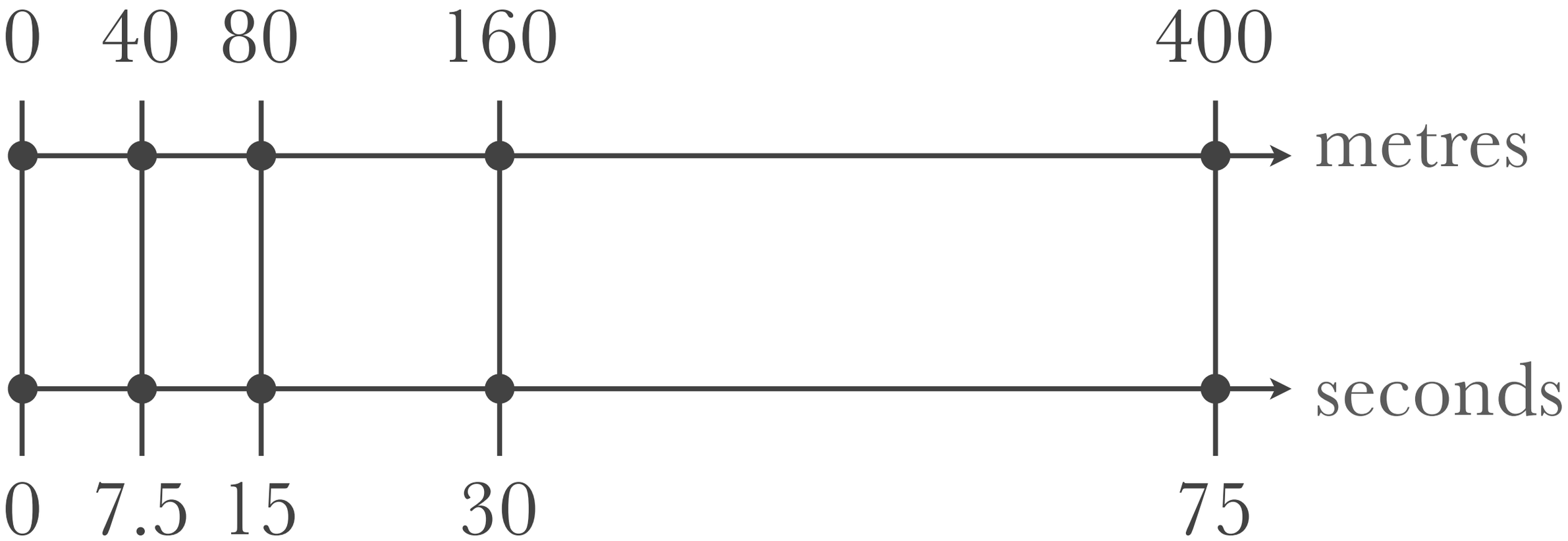
Representation: Ratio Table

metres	400	40	80	
seconds	75	7.5	15	

Representation: Ratio Table

metres	400	40	80	160
seconds	75	7.5	15	30

Representation: Double Number Line



Steph Curry 3-point Tracker: On pace for [redacted] makes in 2015-16

By Zach **Harper** | NBA writer

November 22, 2015 10:57 pm ET

We're nearly through [redacted] percent of the 2015-16 season and Golden State Warriors' superstar is still making 3-pointers at not only a historic pace but a historically ridiculous pace at that. As a refresher, Stephen Curry has three of the five most prolific 3-point

NBA VIDEO

November 23, 2015
NBA Power Rankings: 11/23 (2:28)

November 23, 2015
Brad Stevens talks Celtics
(7:21)

November 23, 2015
How long can Warriors keep win streak alive?
(2:22)

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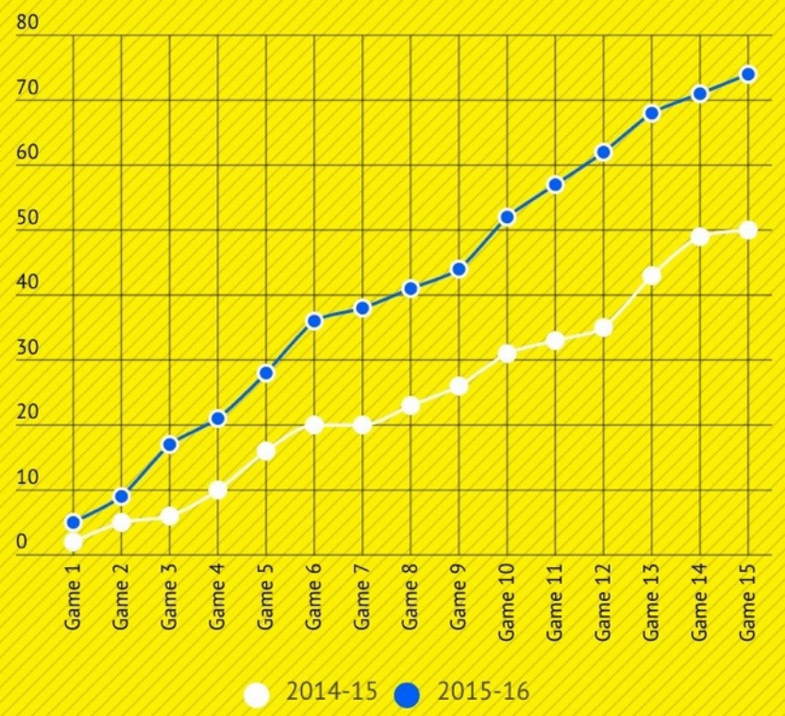
Los Angeles Clippers season performance
(1:03)

LATEST

CHASING HISTORY

3-POINT TRACKER



"I always believed 3 is better than 2."

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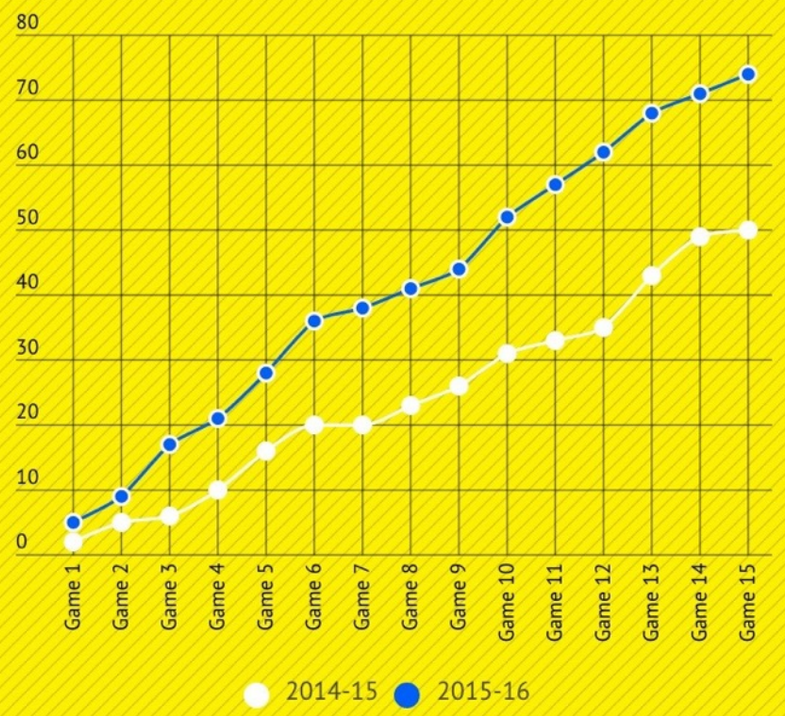

"I always believed 3 is better than 2."

Carnival Tickets

What questions do you have?

TICKET BOOTH

1 TICKET = $.50

12 TICKETS = $5.00

25 TICKETS = $10.00

50 TICKETS = $25.00

120 TICKETS = $50.00

HAVE FUN!


WESTERN FAMILY
Original Cola
12 CANS
WESTERN FAMILY
Original Cola
12 CANS
WESTERN FAMILY
Diet Cola
12 CANS
GR 14-51
14362-8 /2
62639-31194
4 54
12X355ML
+ Dep/Recycle Fee
Wf Cola
card price
3 99
each
save .55 each
14362-8
102314
in-store special
GR 14-51
14365-1 /2
62639-31193
4 54
12X355ML
+ Dep/Recycle Fee
Wf Diet Cola
card price
3 99
each
save .55 each
14365-1
102314
in-store special


PB GR 19
14 01
4176-4 /1
67000-01056
Ccola Classic 20pk
10 49
20X355ML
sale
sale
sale
Special
5 99
each
save 4.50 each
4176-4
102314
CANETTES DE 355 mL (7,1 L)
160
Coca-Cola
®/MD
20 x 355 mL CANS (7.1 L)
CANETTES DE 355 mL (7,1 L)
160
BRE
à 100 %
ODA
GEMBRE
turel à 100 %


PB GR 19
14 53
52525-3 /1
69000-00429
+ Dep/Recycle Fee
Pepsi Cola 24pk Tray
1249
24X355ML
sale
sale
card price
699
each
save 5.50 each
52525-3
102314
24
pepsi
pepsi
pepsi

What do you *notice*?

What do you *wonder*?

“Best use of textbooks ever.”

Solution

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

	Hummingbird	Butterfly
a) Estimate speed.	$\frac{800 \text{ km}}{20 \text{ h}} = 40 \text{ km/h}$	$\frac{800 \text{ km}}{40 \text{ h}} = 20 \text{ km/h}$
b) Calculate speed.	$\frac{800 \text{ km}}{18.5 \text{ h}}$ C 800 ÷ 18.5 = 43.243243 The speed is 43.24 km/h.	$\frac{800 \text{ km}}{41.6 \text{ h}}$ C 800 ÷ 41.6 = 19.230769 The speed is 19.23 km/h.

Strategies
Estimate and Check

A rate can be expressed as a fraction that includes the two different units. A rate cannot be expressed as a percent because a percent is a ratio that compares quantities expressed in the same units.

The speed of the hummingbird is 43.24 km/h and the speed of the monarch butterfly is 19.23 km/h, to the nearest hundredth. The estimates suggest that these answers are reasonable.

Show You Know

Determine the unit rate in each situation.

a) Brandon runs 150 m in 25 s.

b) Kira earns $88 for working 8 h.

c) Cat food costs $9 for five cans.

Example 2: Compare Prices Using Unit Rates

Brett went to the grocery store to buy his favourite brand of orange juice. He found the following container sizes and prices. Which container of orange juice is the best buy?

$1.69 $2.99 $5.49

unit price

- a unit rate used when shopping
- often shown per 100 g or per 100 mL
- makes it easier for shoppers to compare costs of similar items

Solution

Calculate the **unit price** of each container of orange juice and then compare.

414 mL for $1.69

$$\text{Unit price} = \frac{\text{cost}}{\text{volume}}$$
$$= \frac{\$1.69}{414\text{ mL}}$$
$$= \$0.00408/\text{mL}$$

C 1.69 ÷ 414 = 0.0040821

The unit price is $0.00408/mL or 0.408¢/mL.

$1 = 100¢
To convert dollars to cents, multiply by 100.

946 mL for $2.99

$$\text{Unit price} = \frac{\text{cost}}{\text{volume}}$$
$$= \frac{\$2.99}{946\text{ mL}}$$
$$= \$0.00316/\text{mL}$$

C 2.99 ÷ 946 = 0.0031607

The unit price is $0.00316/mL or 0.316¢/mL.

1.89 L for $5.49
To compare unit prices, the numbers must be in the same units.

1 L = 1000 mL
1.89 L = 1000 × 1.89
= 1890 mL

$$\text{Unit price} = \frac{\text{cost}}{\text{volume}}$$
$$= \frac{\$5.49}{1890\text{ mL}}$$
$$= \$0.00290/\text{mL}$$

C 5.49 ÷ 1890 = 0.0029048

The unit price is $0.00290/mL or 0.290¢/mL.

The unit price for the 1.89-L container is less than the unit prices of the other two containers. The best buy is the 1.89-L container for $5.49.

Show You Know

At Ed's Grocery, one brand of salsa is sold in the following container sizes. Which container of salsa is the best buy? Show your work.

3. a) Give two examples of rates that are common in every day life. Share your examples with a classmate.

b) What units measure each of the rates in part a)?

c) Explain why a rate cannot be expressed as a percent.

Check Your Understanding

Practise

For help with #4 to #6, refer to Example 1 on pages 56–57.

4. Determine the unit rate in each situation.

a) An orca swims 110 km in 2 h.

b) A Canada goose flies 800 km in 12.5 h.

c) Cathy plants 45 daffodils in 30 min.

5. What is the unit rate in each?

a) A blue whale eats 8 t of krill in 2 days.

b) The cruising speed of a blue whale allows it to travel 193 km in 10 h.

c) A bull moose bellows 15 times in $2\frac{1}{2}$ h.

6. Gina earns $78.00 for working 6 h. Asad makes $192.50 after working 14 h. Determine each person's unit rate of pay. Who has a greater hourly rate of pay?

For help with #7 to #9, refer to Example 2 on pages 57–58.

7. The table shows the price of different-sized packages of mixed nuts.

Nut Package	Mass	Price
1	300 g	$2.19
2	500 g	$3.09
3	700 g	$4.83

a) What is the unit price per 100 g for each package?

b) Which package is the best buy? Explain your choice.

8. Fraser is shopping for milk. It is available in three sizes.

a) What is the unit price for each carton of milk?

b) What is the unit price per 100 mL for the l-L carton?

c) Which carton of milk is the best buy? Explain why.

9. Mala is shopping for honey. Her favourite brand is available in two sizes.

a) Estimate which is the better buy. Show your thinking.

b) Determine the better buy. Show your work.

Math Challenge #25 GRAPE JUICE JUNGLE

Figure This! Math Challenges for Families

Which tastes JUICIER??

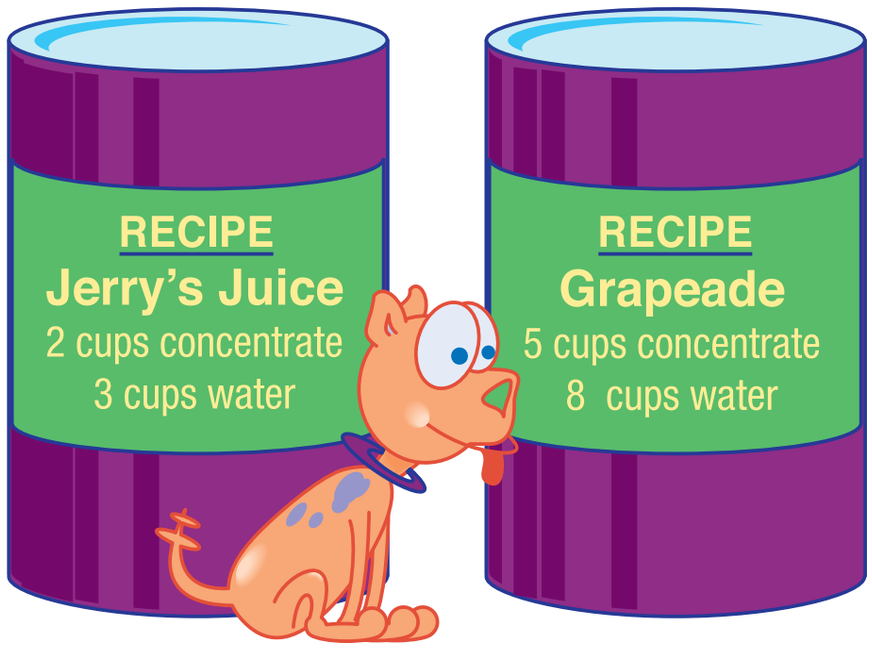

Figure This! **If all grape juice concentrates are the same strength, which recipe would you expect to have the strongest grape taste?**

Hint: For each recipe think about how much water should be used with 1 cup (c.) of concentrate, or how much concentrate should be used with 1 cup of water.

Ratios are fractions that compare two or more quantities. Shoppers use ratios to compare prices; cooks use them to adjust recipes. Architects and designers use ratios to create scale drawings.

NCTM

Which city is "selfier"?

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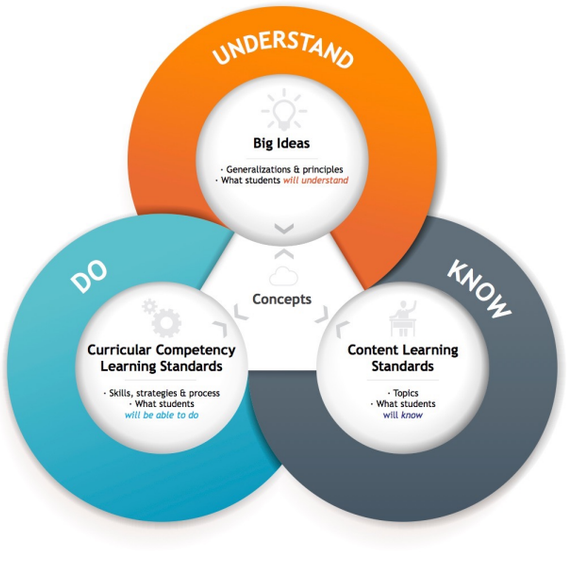
Curricular Competencies Revisited



Big Idea(s)

Proportional reasoning helps us make sense of multiplicative relationships.

Competencies

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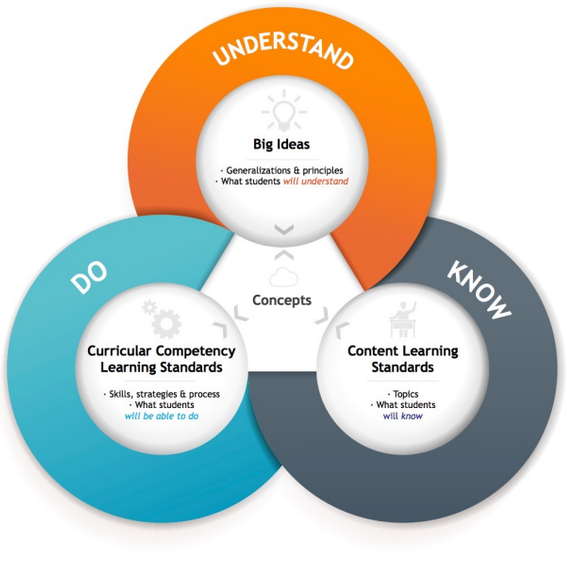
Curricular Competencies Revisited



Big Idea(s)

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Competencies

?

Content

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