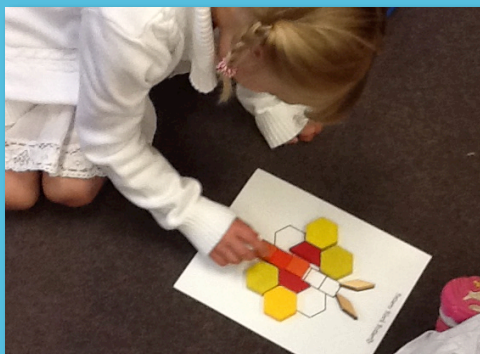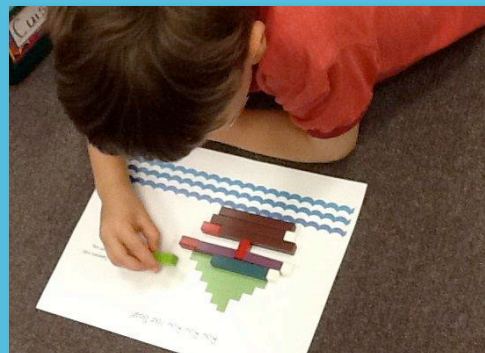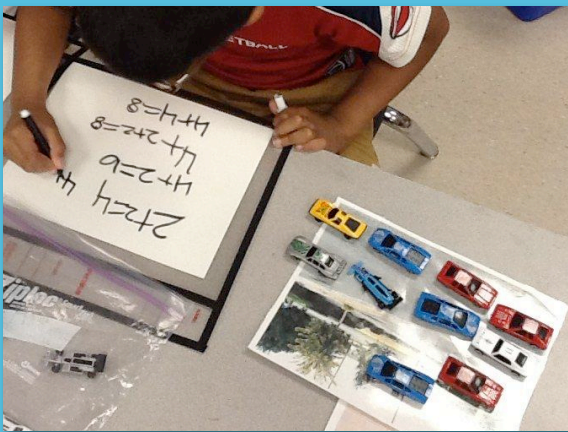# Daily Math Investigations

## Meaningful Math Routines

**ALTERNATIVES TO CALENDAR DESIGNED TO KEEP YOUR STUDENTS ENGAGED, THINKING AND REASONING MATHEMATICALLY!**

SANDRA BALL

CAROLE FULLERTON

# What is the Purpose of Calendar?

**Calendar – Past and Present**
For early primary educators, daily calendar has been a staple in instructional practice.  Seen as an opportunity to expose students to patterns and counting, calendar was considered an important part of daily math instruction.

What we have learned and are continuing to learn about the brain, how children acquire mathematical concepts, and developmentally appropriate practice has lead us to re-examine this traditional approach.  Calendar, in its original form was a fairly passive experience for children.  Most young learners watched as one of their peers completed a pattern, listened as others counted, or chanted along with the group.  While we recognize the value of daily exposure to mathematical ideas, the passive nature of this imagining of calendar often did not meet the needs of many of our learners.

**Moving Forward**
In this resource, we would like to present a more active, participatory version of "calendar" – a daily opportunity for students to truly engage with meaningful math concepts, to play with materials, to process, think, and problem-solve. The tasks, questions and problems we have included in this resource are intended to inspire thoughtful math investigations into number, shape, measurement and pattern. We call these experiences "**Daily Math Investigations**".

Daily Math Investigations allow students to explore math concepts in real and embedded ways.  They promote connection-making and meaning-making between concepts.  In our literacy practice, we know that balance is important.  Likewise, it's important to consider balance in our math programs, including hands-on explorations, independent practice, open-ended tasks, direct instruction and opportunities to share and compare what students are learning.

**Re-imagining: Daily Math Investigations**
Daily Math investigations are an opportunity for students to think and play with mathematical ideas.  Teachers present tasks and pose questions that are intended to promote curiosity about numeracy concepts. In opening up the kinds of questions we ask, we include more students in the learning of math, and help to address the range of learners in our classrooms.  A combination of entry tasks and rich routines allow for balance between whole group, small group and independent learning, a chance for students to explore the math at their level.

# Daily Math Investigations

In designing entry tasks and rich routines, we considered the following:

**Timing**
• How long is too long?
• How much time is just right?

We know that young learners have a limited attention span. For the learning to happen, our whole group carpet time must be short, and spent wisely.

**Activity level of the students**
• Who's doing the talking?
• Who's doing the math?

Little children need to move, to touch, to talk in order to learn. The richness of the math is lost when it is presented orally only. Introducing materials into carpet time, providing opportunities for students to think-pair-share, and recognizing and celebrating students' "aha moments" are essential.

**Grouping**
• Whole group?
• Small group?
• Independent?

Daily Math Investigations do not need to be done all together, all the time. Consider opportunities for students to work in small groups, pairs or even alone around a particular question or task.

**Content**
• What important math idea(s) will be explored?
• How does it address the range of learning needs?

Within the curriculum, there are several skills and concepts that require time to master. It makes sense to present these ideas over time and with intention in thought-provoking ways through a Daily Math Investigation! As they develop as thinkers throughout the year, students will engage with the important concepts again and again seeing it through a different, more evolved lens.

**Connectedness**
• What connections are being highlighted?
• What math-to-math, math-to-self, math-to-world connections can be made?

Students learn best – and remember more – when the learning they are doing is connected.  Supporting students to think about how the mathematical idea you're exploring is like another they already know is an important foundational aspect of learning.  Connecting to their own experience is likewise critical - embedding the math in something relevant to your students is highly motivating!

**Engagement**
• How will I know if my students are learning?
• What kinds of questions keep students thinking?

In the past, students' experience of calendar was at best, minimally engaging.  Being the "VIP" happened far to seldom for real learning to happen!  As we evolve in our practice, it's important to consider the degree of engagement at this rich learning time. The engaging part of the task must be the math - the complexity of the question, the curiosity it inspires and the conversations that emerge.

**Assessment**
• What can I learn about my students?
• What can I watch and listen for?

Daily Math Investigations are ideal opportunities for teachers to observe, listen and reflect on what their young learners know and can do.  As they wrestle with important math, students' thinking is exposed through their actions and words. Take advantage!
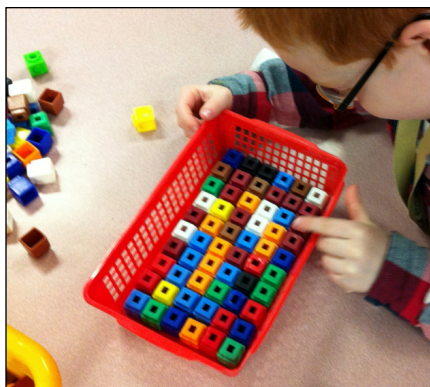
# Engaging Tasks for Daily Math Investigations

In our math instruction, thoughtful planning is essential. If we are to take advantage of the rich learning time that presents itself at the beginning of our students' day, we must be mindful and make good instructional decisions about how this valuable time is used.

> **Daily Math Investigations:**
> Get them engaged.
> Get them thinking.
> Get them reasoning.

In order to ensure that we are making the most of these activities and that the intent of these daily experiences remains sound, we have developed criteria for engaging tasks.  The entry tasks and rich routines are characterized by the following:

1.  **Interactive**
    - All students are <u>doing</u> math
    - Good questions are asked to promote wondering

2.  **Oral language opportunities**
    - Mediated conversations exist between teacher and students
    - Frequent opportunities for students to share and describe thinking

3.  **Relevant**
    - Connected to to classroom context and students' lives

4. **Focused on important mathematics**
    - Connected to PLOs

By keeping the mathematics at the forefront and by focussing on students' active engagement with the math, we maximize the potential of these experiences to promote deep mathematical learning.  Daily Math Investigations are designed to get students engaged, thinking and reasoning.

## Traditional Calendar – What to keep and what to let go

A monthly calendar gives us interesting information.  We can use it to mark important events, like an upcoming holiday, a student's birthday or a school celebration.  Highlighting these events on a calendar and counting the days until they happen is fun for students.  That said, the abstract nature of a calendar - the repeating 7-day pattern of days in a week, the recursive aspect of the chart itself and the random number of days per month - makes it virtually undecipherable for young students.

For many of us, calendar time (and all the activities associated with it) is ingrained in our script for primary teaching.  It's important however to consider carefully the purpose of these tasks - and more importantly, their effectiveness.  Choose wisely and consider the above criteria.

• Are the pieces of your calendar time truly relevant?
• Are students talking?
• Are they engaged?
• Are they doing math?

# Entry Tasks

Entry tasks are exactly that - tasks that are prepared and waiting for students as they enter the classroom at the beginning of the day.  Knowing that our students' minds tend to be the most fresh in the morning, presenting these problems and mathematical curiosities to students as they arrive makes sense.  Likewise, they will allow teachers to welcome and talk to their students as they transition into the learning space.

Entry tasks are invitational in nature. They are intended to inspire wonder and to encourage questioning.  Students love these interesting questions and eagerly anticipate these morning tasks.  These explorations can be very motivating - even for reluctant learners.

## Set up
Imagine a single table or counter space set up where students can easily gather and access materials, or multiple tables set up with inviting materials and a clear prompt.  Some days the tasks may be to sort a set of objects, to vote in a survey, to estimate a collection, to create, extend or correct a pattern, to complete a puzzle, or to contribute to a collaborative solution.  The power of an entry task is in its immediate engagement in something thoughtful and mathematical.  Examples of what these tasks could look like are described below.

## Timing
The timing and content focus of these tasks are flexible and should be responsive to the needs - and abilities - of your learners.  Feel free to choose tasks from the list below and to adapt them as you see fit.  You might consider using one task each day for all students to explore, or you might create 5 stations and allow your students to explore different tasks each day over a week.  Don't feel as though you have to present something new every morning. Novelty is important, of course, but students may find these entry tasks engaging enough to explore for more than one day in a row.  Learning will still happen!

## Choice and learning
Children naturally choose tasks that evoke their curiosity and have learning potential - sometime several days in a row. Given a rich task and an engaging context, exploring the same materials and question over and over can continue to promote thinking for many children. Since these entry tasks are designed as an invitation to think and explore, it makes sense to let students lead the way. Trying to be too prescriptive will ruin the experience - for both you as a teacher and for your students as explorers.

# Entry Tasks

**Spatial Tasks**
- Cuisenaire Rods Puzzles
- Tangram Puzzles
- Pattern Block Puzzles
- Pentomino Puzzles

**Fix-its**
- Dot Sort
- Mitten Match
- Hundred Chart Puzzles
- Number Tiles in a Bag
- Pattern Problems
- Cuisenaire Capers

**Investigating Measurement**
- Order's Up!
  - Tray of Treasures
  - String Fling
  - Scoop It!
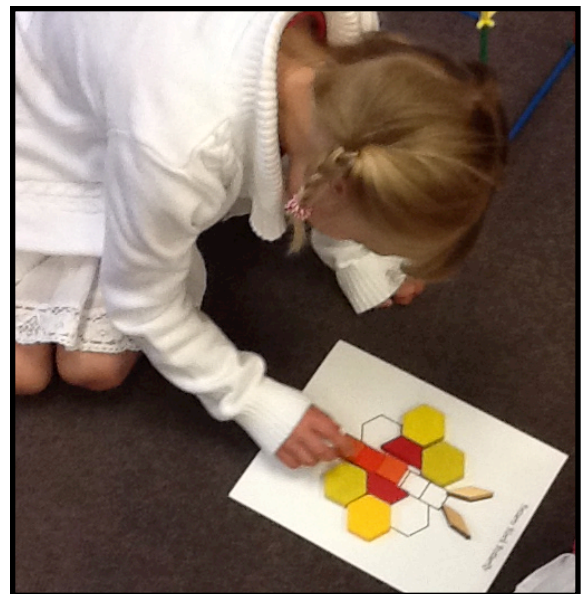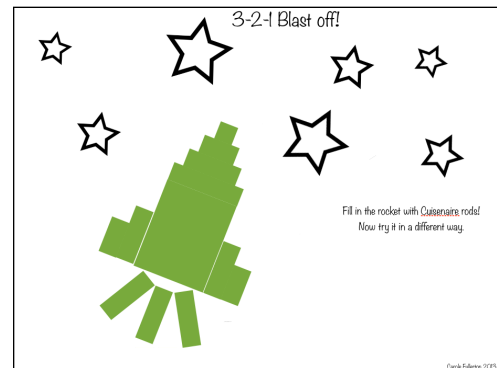
**Messing with Data**
- Yes-No!

**Dots Crazy!**
- Dots Crazy
- Dotty Scavenger Hunt

**Fluency Builders**
- Simple Games
- Domino Targets
- Domino Stackers
- Number Tile Puzzles
- Magic Wands

**Plotting Patterns**
- Pattern Police
- Pattern Creation Station

# Spatial tasks

Mathematical thinking depends on being able to visualize and manipulate images. These tasks will promote talk, reasoning and thinking in your youngest learners.

### Cuisenaire matching tasks
Use a small set of Cuisenaire rods and the attached puzzles. Have students match rods to the outlines. Encourage them to complete the puzzles in more than one way! Older students can calculate the total length of their rods to make a number-based connection.
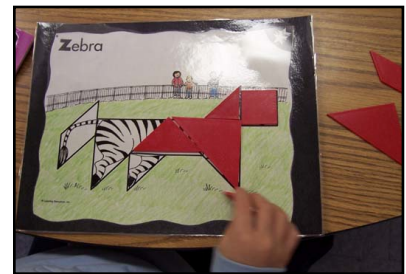
### Pattern block puzzles
Use a small set of Pattern Blocks and the attached puzzles. Have students match blocks to the outlines. Encourage them to complete the puzzles in more than one way. Highlight the relationships between the pieces!

### Tangram puzzles
Use a complete set of 7 tangram pieces and the attached puzzles. Have students match pieces to the outlines. Encourage them to complete the puzzles in more than one way! Give students the language of flipping and turning to describe the translations they are using.
Consider reading "Grandfather Tang's Story" as an introduction to these puzzles.

### Pentomino puzzles
Use a complete set of 12 pentomino shapes and the attached puzzles. Have students match pieces to the outlines. Encourage them to complete the puzzles in more than one way!
Encourage older students to create puzzles for friends.

## Fix-its

These are puzzles – mathematical mysteries – to be solved.  Each one promotes reasoning through an important concept. Students can work on these problems alone or with a partner, showing their work to a teacher upon completion.  Afterwards, the mystery is reset to allow the next group of children to solve it.  Over time, students might like to create their own "Fix-Its" for friends. The following are brief descriptions of possible Fix-It tasks.  Be creative and make or adapt your own tasks.

### Dot Sort!
Use a collection of 6-8 dot cards.  Have students put them in order from smallest to largest or the reverse.

### Mitten Match
Have students match numeral cards to pictures of sets.  Use seasonal images to keep it interesting!

### Hundred Chart Puzzles (Grade 2)
Cut up a hundreds chart along the grid lines into 5-6 interesting shaped pieces.  Have students reassemble the pieces to make a complete chart.  Photocopy hundreds chart puzzles onto colored paper of various colours to keep the pieces distinct.

### Number Tiles In A Bag (Grade 2)
Use ceramic tiles (or colour tiles) for these puzzles.  Write numerals on each tile to create mixed sets of connected numerals and place them in a bag.  Students dump out their tiles and arrange them as if there were in a hundreds chart.  For example, in the picture at right, the numbers 5, 6, 7, 16, 25, 26, 27, 37, 46, 47, 56 and 66 have been rearranged to show one more, one less, ten more, ten less relationships.

| 5 | 6 | 7 |
|---|----|---|
|   | 16 |   |
| 25| 26 | 27|
|   |    | 37|
|   | 46 | 47|
|   | 56 |   |
|   | 66 |   |

### Pattern Problems
Create 4 repeats of a simple pattern using materials.  Make an intentional error in on of the pattern cores.  Have students propose a solution to the problem.
In order to allow multiple students to work on the same pattern problem, consider creating the pattern on a strip of paper. Students build, then fix to problematic patterns. Over time, students will enjoy making flawed patterns for their peers!

## Cuisenaire Capers
Pose questions like the following of your students.
- *I ran out of orange (blue, black) rods. How can I make one?*
- *Use a rod length of exactly 20 or 50 or 100. Make a design or a tower. Make it in another way. Trade in some pieces and make it again.*
- *Make a tower, then take it apart.  Trade your pieces in for orange rods.  How many oranges did you use? How many extras?*

## Investigating Measurement

Each of these "Order's Up" tasks is open-ended and full of possibility.  The intention of these tasks is to promote wonder and exploration of important measurement concepts.  As an Entry Task, these investigations get students thinking about measuring and comparing different attributes of objects in their world.  Debrief what students have discovered in your Rich Routine time.  They will no doubt be excited to share!

## Order's Up!

### Tray of Treasures
Place a set of objects on a tray.  Have students put them in order according to an attribute of their own.  Explore and discuss what attribute they used (length, width, mass, etc).

### Scoop It!
Provide a set of small containers and unifix cubes.  Have students fill the jars and then count their cubes to see which one hold the most.  Older students should record the total number and make statements of comparison (This one held three more than that one...).  On another day, switch the unifix cubes for puff balls, erasers, stacking chips or foam counters and have students explore again.
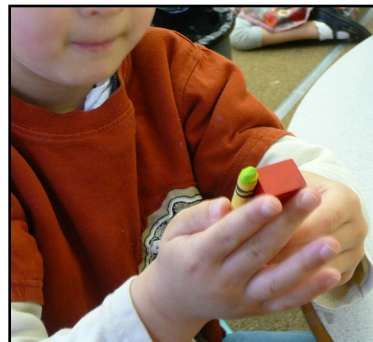
### String Fling!
Cut a set of 8-10 strings of different lengths.  Have students place them in order from shortest to longest, or the reverse.

# Messing with Data

Data investigations provide us with rich, meaningful opportunities to count and compare.  Posing a dialing question of your students and then discussing the results is a great way to introduce the language of more than, less than, and the same as within a context.  Likewise, as we explore different ways to represent and describe what we have learned from these surveys.

### Yes-No

Pose questions with 2 possible answers (yes / no, or a preference between 2 options). Have students record their answers in a variety of ways. Consider:

- Unifix cube stacks - one for yes, one for no.
- Stamp your response in a yes-no chart with a bingo dabber
- Place a counter in an egg carton, where each column is labeled yes-no
- Place a cube in a ten frame (one for yes one for no)
- Place your photograph in the yes or no circle
- Post a sticky note in the yes or no column
- Put a unifix cube in the 'yes tray" or the "no tray" of a balance scale.
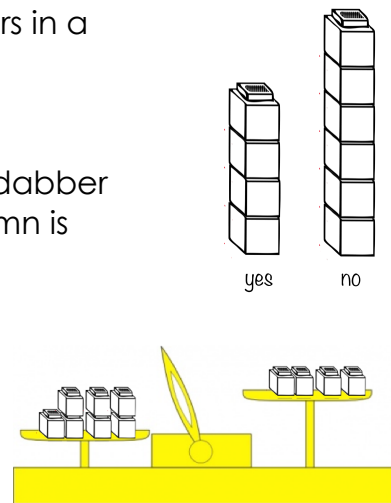
Collate, count and examine the data during your Rich Routine time.  Students are very motivated to discuss what is important to them!

### Dot's Crazy!

Post a number of the day and have your students represent that number in as many ways as they can using puff balls, counters, shells or other small manipulatives.  They should record their representations of the number of the day using bingo dabbers.

Consider using these representations for games and other tasks like the Dotty Scavenger Hunt, below.

**Dotty Scavenger Hunt**
Post signs around the classroom for numerals 5, 6, 7, and 8. Distribute a dot card to each child.  Students read their card, sort out how many, then at a signal, find their matching numeral in the classroom.  All of those with "5" compare and say how their numbers are alike and how they are different.


### Fluency Builders

Students need and benefit daily practice in order to develop fluency with the facts.  Be aware though, that students need to understand what the operations mean and have strategies for finding sums and differences before they practice them in isolation.  The following tasks promote thinking rather than memorization - a developmentally sound approach to mastery of the facts in early primary.

**Simple Partner Games**
There is a wide array of quality games for practicing and mastering number relationships.  Consider your favorites, from Concentration, Go Fish, and Snap to games from BEAM, and those drawn from other core resources.  Be sure that students know how to play these games before setting them out. Even a set of dice, a stack of counters and a spinner can be used by students to play games of their own creation.

## Domino Stackers

Students collect and create piles of dominoes with each of the following sums: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.  They create stacks of dominoes with the same sum and explore how many of each there are.
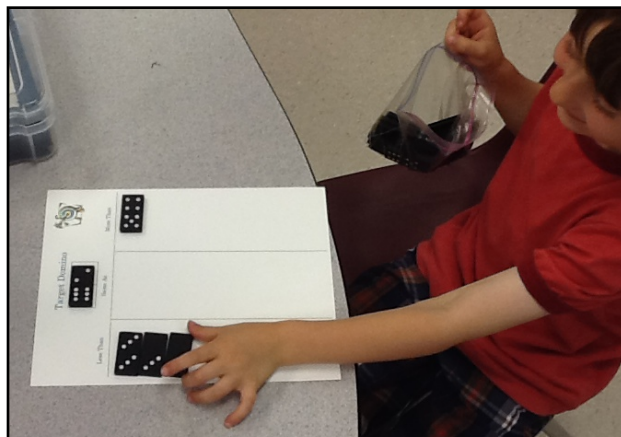
## Domino Targets

Have students draw one domino from a pile and find the sum of the dots on it.

They should sort their remaining tiles into 3 piles:

• More than the target
• Less than the target
• The same as the target

Older children should also record an equation to match the total and place it in their chart.

## Magic Wands

Have students create a stack of unifix cubes, all in the same colour, to a maximum of 10 cubes. This is their Magic Wand! Students break their Magic Wand into 2 parts and count to see how many are in each hand. Older students should record the length of the 2 parts by drawing and writing an equation or number story.
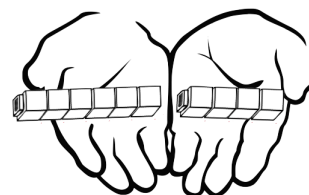
## Number Tile Puzzles

Students use one set of number tiles with the numerals 0-9 written on each one.  They use their tiles to complete the missing parts on the templates. Introduce more complex tasks over time, and encourage students to explain and share their strategies!  Consider putting the templates into clear plastic sleeves and have students write and record their answers in dry erase pen.

Use all of your 0-9 tiles!

$\square$ + 2 = 9

2 + $\square$ = 8

$\square$ + 5 = 9

$\square$ + $\square$ = 7

6 + $\square$ = 6

$\square$ + $\square$ = 4

$$\begin{array}{r} 5 \\ + \ 3 \\ \hline \square \end{array}$$

$$\begin{array}{r} 8 \\ + \ 1 \\ \hline \square \end{array}$$
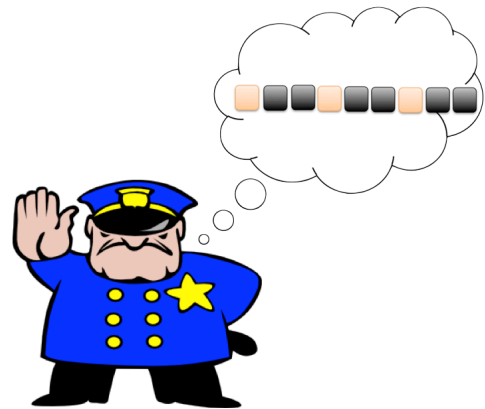
Puzzle **A**

## Plotting Patterns

### Pattern Police
*Stop right there!*
*Make my pattern to gain entry!*

To play this entry game, the teacher records
and posts a pattern in numbers, images,
letters, or in another interesting form. Then
students must make 3 repeats of that same
pattern with unifix cubes and leave their
pattern on the entry table.

Debrief the collection of "same pattern different ways" on the
carpet during whole class Rich Routine time.

### Pattern Creation Station
On entry, students find a set of pattern materials and a set of
instructions.  The daily task is to create either a growing pattern, a
shrinking pattern, a circular pattern or a number pattern to
match it.
Again, debrief the collection of "same pattern different ways" on
the carpet during whole class Rich Routine time.
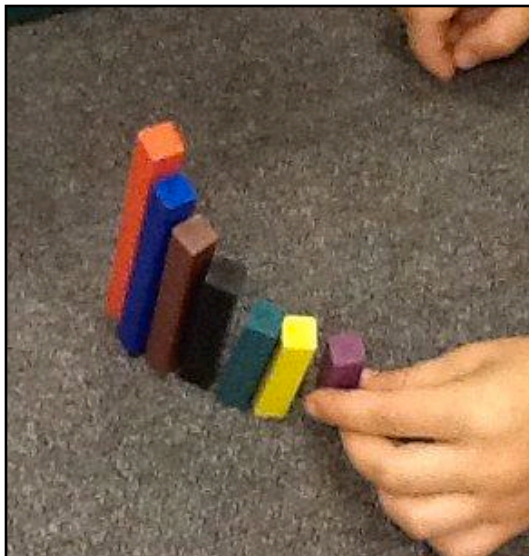
# Rich Routines

### Rich Routines - What are they?
Rich routines are explored in more structured, less fluid ways.  These questions - open-ended and interesting - are more often posed during a carpet time. Unlike a traditional calendar time, rich routines include far more opportunities for talk and modeling.  Students engage with them in active ways and in a variety of groupings. These questions may be a springboard to another more involved investigation, but the focus is on connectedness and content.  The power in rich routines is in their potential to revisit important content over time.

### Routine ≠ Rote
Do not confuse "routine" with "rote" - routinized structures are predictable, but remain engaging owing to their ongoing, increasing complexity.  The questions may, over time, be the same, but the the intent is to deepen knowledge and understanding as the content becomes more complex.  Mastery of an idea comes through multiple experiences and the chance to connect each new experience to prior knowledge. Rich routines provide the structure for making these connections. Examples, prompts and frames are described below.  They are loosely separated into content areas - number, measurement, data, etc.

### Be Selective!
Consider the suggested tasks on the following pages as a choose your own adventure. Taking into consideration the needs and interests of your students, pick carefully from the tasks that follow.  You might like to work through 3-4 of them a day, for several weeks, and then swap out one or 2 of the tasks to keep it fresh.  Revisiting tasks later in the year is likewise a good idea! Students in the spring of the year are much different thinkers than in the fall!

# Rich Routines

## All About Number
• Thinking Frames

## Building Number and Measurement Concepts
• Cuisenaire investigations:
  • Addition
  • Subtraction
  • Extending the Task
  • Developing Referents

## Exploring the Open Number Line
• Grade K/1 Tasks
• Grade 2 Tasks
• Grade 3 Tasks

## Exploring the 100's Chart
• Building a 100's Chart
• Fill-It
• Fix-It

## Numbers of the Day
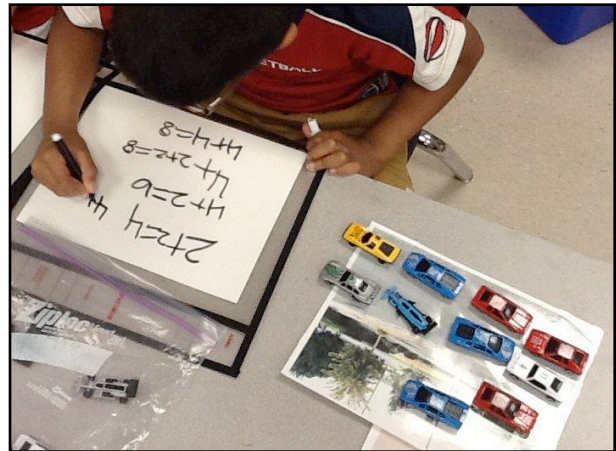• Math Picture of the Day
• Equation of the Day
• Referent of the Day

## Counting Collections

## Messing with Data

## Sort It Out!

## Teeter Totter Tales
• Kindergarten
• Early Grade 1
• Late Grade 1
• Grade 2

## All About Number

For this rich routine, select a number within your students' range, and model with the whole group how to complete a set of prompts.

Consider creating pocket chart cards or magnetized cards with the following sentence frames.

For example:

17 **is more than** 12.
17 **is less than** 18.
17 **is the same as** 10 and 7.

17 **is too many** hats for my head.
17 **is too few** cookies.
17 **is just enough** friends.

Do a think aloud as you record your numbers to support students in understand why you picked them.  Invite some ideas from your students for a second number and record them.  Then have students work collectively - in the whole group or in small groups - on numbers at their respective levels.  That is, you might create a group of students and have them work on statements for the number 11, and have another group working on the number 20.

Consider your students and the number and kind of sentence frames you use each day.  Don't use them all at one go!

Line masters for these thinking prompts are included in this resource for your use.  Have students place them in dry erase sleeves or laminate them. While students are gathered on the carpet, have them record their own ideas for the number of the day and share them by holding them up.  This is an excellent strategy of engaging multiple learners at once.  Each child will get to create - and share - their thinking with the group.

Over time, these frames could become part of students' Entry Task work.  Assign a number for the day or have older students roll a double die and create their own from the digits rolled.

## Thinking About Number – Frames

_____ is more than _____.
_____ is less than _____.
_____ is the same as _____.

_____ is too many _____.
_____ is too few _____.
_____ is just enough _____.

_____ is about _____.
_____ is close to _____ but far from _____.

_____ people could _____.
_____ unifix could _____.
_____ paperclips would _____.
_____ drops of water could _____.
_____ jellybeans could _____.

_____ seconds could _____.
_____ is too short for _____.
_____ is too long for _____.
_____ is just right for _____.

$_____ is just enough for _____.
$_____ is not enough for _____.

$_____ is more than _____.
$_____ is less than _____.
$_____ is the same as _____.

---

**Thinking About Number**

$_____ is more than _____.

$_____ is less than _____.

$_____ is the same as _____.

---

## Building Number and Measurement Concepts

Primary students explore and engage with number in sets or groups. When we build 13 for example, we might use 13 blocks, or a full ten frame and a 3, or we might use a ten rod and three units. In each case, the quantity is represented as a set.

Cuisenaire rods, however provide a linear model for representing number. When using Cuisenaire rods, we line the pieces up end-to-end, creating a longer rod of a certain length. In using an orange Cuisenaire rod as our referent (point of comparison) we help students to visualize length and over time, make connections both to measurement and to a number line.

### Cuisenaire Rods - modeling addition

To help students make connections between the relative lengths of the rods, build each day, using the giant Cuisenaire rods. Post an orange rod of length 10 on the board. On day 1, line up a white rod at the far left end of the orange rod, as shown below.

Ask part-whole questions about the 2 rods. For example:

- *How many more whites would we need to to get to the end of the orange?*
- *If we added another white each day, how many days will it be until it is full?*
- *What single rod do we need to fill the rest in right now?*
- *What other rods might work?*
- *What colour do you think we will use tomorrow?*

Model solutions to the problem, using the large rods. The following shows that on day 1, the missing part to get to the end of the orange (10) is a blue (9). Make connections to the number names as well as the colours while you explore these materials!

**1 + 9 = 10**

On day 2, add a second white rod, then exchange the 2 white rods for a red, confirming that they are the same length.

Depending on your grade level, (ie grades 1 and 2) consider recording an equation to match the action (1 + 1 = 2). Continue in this way on day three, adding a white rod to the end of the red and exchanging the red and the white for a light green rod of length three.

Continue to ask part-whole questions as you progress.  When you reach day ten, celebrate and start again!

## Cuisenaire Rods - Modeling Subtraction

Depending on the age and stage of your students, consider using the Cuisenaire rods in the reverse way to model subtraction and/or counting backwards. Starting at 10 (orange) remove one white block each day, counting backward until you reach 0. Record the matching subtraction equation each day.  Note that in order to remove a white rod from an orange one, first you have to exchange the orange for a blue and a white (ie 9 + 1 = 10) before you can subtract it (10 – 1 = 9).

## Cuisenaire Rods - Extending the Task for Older Grades

*Exploring number to 20*
Use 2 orange rods laid end-to-end to create a single length of 20. Explore numbers greater than ten (ie the teens).  This is an ideal task for grade 1 and 2 students.

*Skip Counting*
Create a number line with 2 orange rods end-to-end.
Replicate the process on the board as above, only use red rods instead of white ones. Record the skip counting pattern in 2's, making a list of the numbers built. Explore 4's, 5's and even 3's in this way.

**2,   4,   6,   8,   10,   12!**

## Cuisenaire Rods - Developing Referents
Another approach to this task is to focus on the relative lengths of the rods. This will support students in their ability to estimate and measure. Your conversations should include and highlight phrases like *close to, far from, about the same as, about halfway to, more than half*, etc. These concepts will take a long time to develop. They are the perfect topic for revisiting in a structured routine!

For example,
*What do you notice about the rod of the day?*

(A purple, or a rod of length 4)

You might hear:

• *It's longer than light green by 1 white.*

4 = 3 + 1

• *It's as long as 2 reds.*

4 = 2 + 2

• *It's shorter than a yellow.*

4 < 5

• *It's going to take a dark green to get to 10!*

If desired, continue to prompt with questions like these:

- *Is it closer to one? Closer to ten?*
- *Is it a lot more than one?  A lot less than ten?*
- *Is it more than half way to ten?  Less than half way to ten?*

Over time, your students will naturally use a referent of "orange" or ten-ness in their description of the other rods.  What's exciting about Cuisenaire rods is that they introduce standard units of measure (centimeters) in a visual and tactile way.  The white rod measures exactly 1 cm in length, while the orange rod is 10 cm long. Mastering these referents will be of great support as students begin to consolidate their learning in more formal ways.

## Exploring the Open Number Line

An open number line is a line segment with endpoints, but no hash marks to show divisions.  It's a powerful tool for thinking about the relative size of numbers and is conceptually connected to the use of Cuisenaire rods.  Open number lines ask students to make approximations about the placement of a quantity, to compare and order.

As mentioned earlier, most of primary students' experiences of modeling and representing number is limited to sets and groups of objects.  If we really want to compare these objects we need to line them up to see their relative size.



| 5 | 6 | 7 | 7 | 8 | 9 |
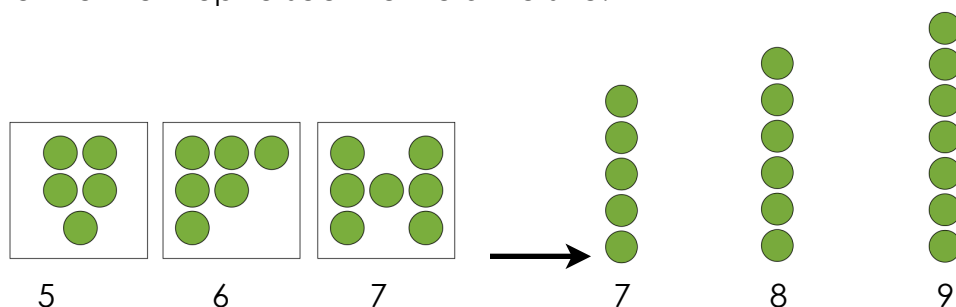
Open number lines help students to reason about the relative size of numbers. The relationships that exist between quantities - that 7 is more than 5, that it's almost 10 - are mapped out when a child estimates where to place a 7 on that same number line.  These referents (points of comparison) are a critical aspect of number sense, and foundational to flexible application of the operations.

Students will begin to develop these linear referents in the previous activities with Cuisenaire rods.

Most importantly, though, an open number line makes it impossible to count by ones.  Students who rely solely on counting by ones (on their fingers or on a number line / ruler) never learn to apply more efficient strategies like doubles, compensation, and using known facts.

## Exploring the Open Number Line

**Grade K/1 Tasks**
Draw an open number line with endpoints of 0 and 10 for kindergarten and K/1.  Mark a spot on the number line with a star and pose these and other engaging questions:
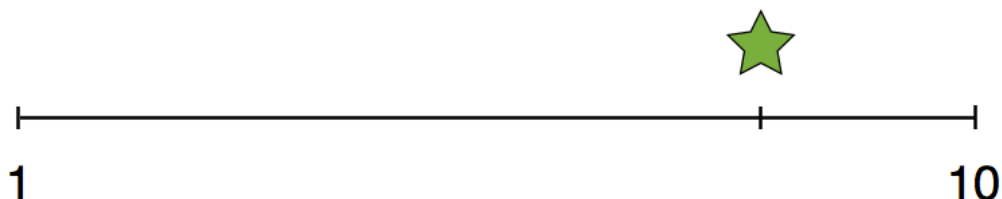


- What number could the star be?
- Where does 6 go?
- I am a number that's less than 9 but more than 5.
- What could I be?

Be sure and make connections to the Cuisenaire rod tasks as you work through these ideas.  It will help students immeasurably!

**Grade 1 Tasks**
Draw an open number line with endpoints of 0 & 20 for grade 1.
Pose these and other questions:
- Mark a spot on the number line.
- What could I be?
- Where does 14 go?
- I am a number that's less than 13 but more than 9.  What could I be?

**Grade 2 Tasks**
Draw an open number line with endpoints of 0 & 100 for grade 2.
Pose these questions:
- Mark a spot on the number line.
- What could I be?
- Where does 65 go?
- I am a number that's less than 90 but more than 55.  What could I be?

## Exploring the 100's Chart (Grade 2)

Students in grades 2 and up benefit from seeing how a hundreds chart is constructed.  It gives them time to learn the patterns as the chart is built. Consider the following tasks to make sense of this powerful tool.

### Fill It! – Day 1

Use a large blank 10 by 10 grid, laminated so that you can write in the squares. Consider also a digital version, using a document camera or a SmartBoard. There are commercially available pocket charts for this purpose as well.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|----|
| 11 | 12 | 13 | 14 | 15 | | | | | |
| | | | | 25 | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
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| | | | | | | | | | |

To begin, start at upper left hand corner and record the numerals from 1 to 10 in the top line.  Ask students what they think goes in the next square – that is, the first square of the second row. Confirm that it is 11, and continue filling in squares until you reach 15.

Stop and ask:
*What number comes next?  How do you know?*

Point to the square below the 15.
Ask:
*What number goes here?  How do you know?*

Enter the number 25, and leave it until the next day.

### Fill It! – Day 2

On another day, continue to record numbers in the 100's chart, starting where you left off, at 16.  When you reach 25, celebrate the fact that it's already there, and in the correct spot.  Continue filling in the chart to 30 and stop.  Point to the space where 41 would go.
Ask:
*What number goes here?  How do you know?*

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|----|
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| | | | | | | | | | |
| 41 | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |

Be sure students are talking about their strategies.
They could count by ones to find the missing number, or they could use the existing patterns to predict what number goes in the "41" spot.

## Fill It! – Day 3

Continue in this same way, each day writing 10-15 numbers and then making a prediction about a missing numeral down the line.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  | ? |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |

## Fill It! - Daily Conversations

Focus on the patterns within the 100's chart. Notice and talk about how the tens are the same in each row. Talk about how the ones increase each square. Observe that the ones digit are the same in each column.

As students develop understanding of these patterns, they will likewise be developing concepts of place value in a visual way. The 100's chart is a complex tool that deserves daily conversation and demystifying. We cannot assume that students "get it" without the chance to construct and deconstruct the chart over time.

Don't forget to count beyond 100 when you reach the end of the chart. These important patterns do continue! Bridging a century (going up and over 100, 200, 300...) is an essential understanding.

## Hundreds Chart Fix-Its

Once the hundreds chart has been built and well understood in terms of it parts, present a daily challenge to students.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 |  | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 |  | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

Consider:
- Remove one or 2 numbers from the chart. Have students tell which numbers are missing and how they know.
- Remove a set of intersecting numbers from the chart and have students use logic to sort out which ones have gone missing.
- Switch 2 or 3 numbers in the chart, creating an error in the pattern. Ask students to find and correct the error, and explain how they know.

Remember that the conversation about how students solve these problems is more valuable than the correct answer.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 |  | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 |  |  |  | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 |  | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

## Math Picture of the Day

### Math Actions
Show an image of removal or joining. Use a story book page, or something like the image below, from Splash! by Ann Jonas.
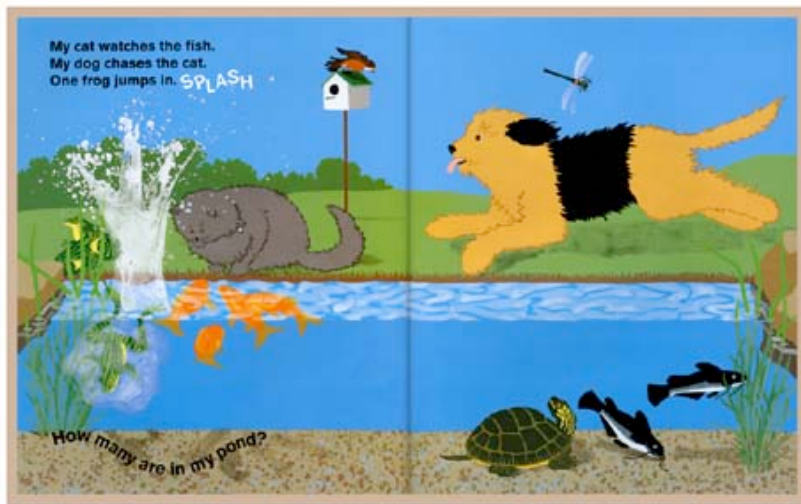


Have students tell a math story to match, describing the action in the image.  Have them say whether the creatures are coming or going.  For each story, the teacher should make a simple drawing with arrows to show the action in a clear way.  For older students, consider also recording an equation.

Extend the learning to remove or add ones, many ones, tens, or even multiple tens.

### Grouping and Counting
Use an image from a book that features an interesting arrangement of a number of objects to count. Present the image to your students, and ask:
*How would you count these _____?*

Accept all suggestions. Illustrate the groupings made by students by circling on the image itself.  Consider scanned copies of these images, laminated or protected in clear plastic sleeves, or project the images using a document camera or SmartBoard software.  Focus on the many logical ways to group and count.

When the group sizes are the same, model skip counting to find the total.

For example, consider this image drawn from the book <u>Calvin Can't Fly</u> by Jennifer Berne.
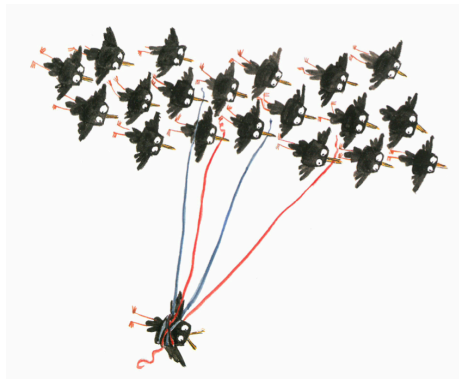


We can circle and count 5's, or even 3's in this image.

These classic stories have many great images worth exploring:
<u>The House of Twelve Bunnies</u>  by Caroline Stills
<u>Let's Count to 100</u> Masayuki Sebe
<u>365 Penguins</u> Jean Luc Fromental
<u>Pattern Fish</u> by Trudy Harris
<u>Too Many Frogs</u> by Ann and John Hassett

## Numbers of the day

**Equation of the day**… (Grade 1 and up)
Provides a simple addition or subtraction equation each day.
Have students tell a story to match it.   For example:

Teacher records the following equation:  **6 + 2**

A student volunteers:

*There were 6 shells on the beach.*
*Two more washed up.*
*How many shells are there now?*

For added fun, use the Location Spinner to
provide a setting for the problem.

For example, if a student sees the equation 8 – 3
and spins the location "bakery", she might offer
the following problem:

*"There were 8 cookies in the bakery.*
*3 were eaten.*
*How many cookies are left?"*
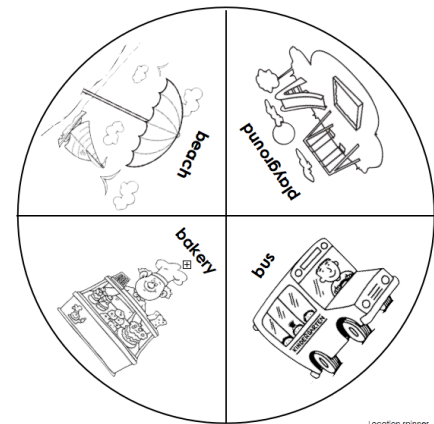


Location spinner
Carole Fullerton 2013

**Referent of the day**
In order to practice using referents or units of comparison,
engage students in these daily tasks. Use a spinner to sort out
what you will use as a referent each day, and what you will find.

Spinners are included to compare size and/or mass of different
classroom objects, body parts and sets of counters.

It is important to match a SIZE spinner with another SIZE spinner,
and a MASS spinner to its partner MASS spinner.  See the footers
on each of the spinners to be sure!

For example, the following pair of MASS spinners tell us to find things in the classroom that are **heavier than** a **glue stick**.



Mass spinner
Carole Fullerton 2013

Classroom objects spinner - mass
Carole Fullerton 2013
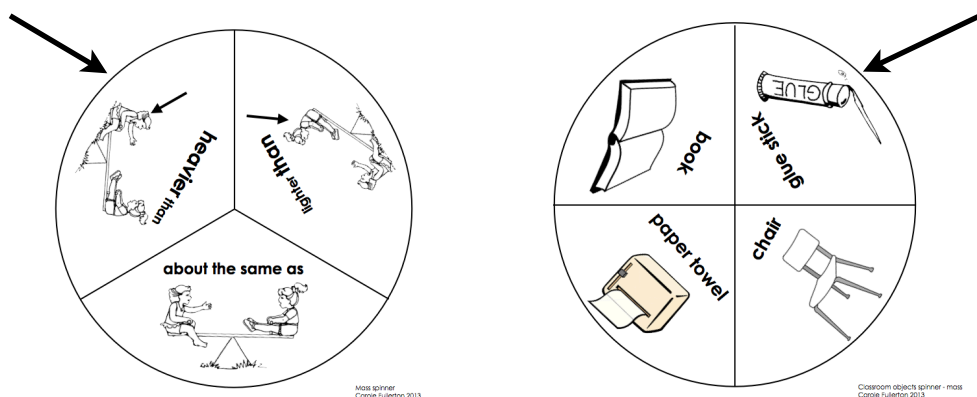
Students should find just one object that is heavier than a glue stick and return to the carpet with it. Invite them to grab a glue stick in one hand to compare its mass to the object they find. Debrief and describe the set of collected objects, and compare them to the referent of the day – in the above case, a glue stick.

## Measure of the day

When we explore the length of ten objects, we can use that referent to make predictions about the length of multiples of ten. For example, when we understand that 10 unifix cubes is about as long as one hand, then 20 unifix cubes is as long as 2 hands, etc.  Exploring these ideas in sequence, and by making connections to previous explorations is a meaningful daily routine.

It's important to use the same materials over the course of these investigations (multiples of 10 unifix, multiples of 10 pennies, multiples of 10 plastic cups, etc) so that students are comparing like objects each time. Don't be afraid to explore others as well - just be sure you have lots of them!

Celebrate the connections students are able to make between 10 and 20 objects, between 10 and 30 objects, etc.  Important place value ideas will roll out!

**Measure of the day prompts**
Try some of these "measure of the day" sequences, then come up with your own!

**How long is…?**
How long is 10 unifix?
How long is 20 unifix?
How long is 30 unifix?
How long is 40 unifix?
How long is 50 unifix?

**How long is…?**
How long is 10 pennies?
How long is 20 pennies?
How long is 30 pennies?
How long is 40 pennies?
How long is 50 pennies?

**How long is…?**
How long is 10 plastic cups?
How long is 20 plastic cups?
How long is 30 plastic cups?
How long is 40 plastic cups?
How long is 50 plastic cups?

**How long is...?**
How long are 10 new pencils?
How long are 20 new pencils?
How long are 30 new pencils?
How long are 40 new pencils?
How long are 50 new pencils?

## Counting Collections

Build opportunities into your daily routine to estimate and then count collections of different sizes. Start out with small collections (5-10 objects) and then increase them gradually.  Changing the size of the objects to be counted will provide novelty for students, and force them to think hard about their estimates.  Consider using: unifix, counters, small beads, marbles, cotton balls, pennies, pencils, or fun counters in the shape of animals. Spread the objects out on a flat surface and cover them with a large sheet of paper.  Show the children the collection briefly and accept responses. Alternately, you might consider putting objects in a clear container or jar.  It's much harder to estimate in this way!

Challenge students to explain how they arrived at their estimation each time.

Did they:
• Compare to a referent (to 5, to 10, to 20)?
  • *It's a little bit more than 10.*
  • *I know 100 is this much, and that's about half....*

- Use multiple groups of a number and skip count?
  - *I knew that this was 5, so I counted 5, 10, 15 and some more, so it's about 17.*

When you count to confirm the actual amount in the collection, make groups of 2's, 5's or 10's and model how to skip count to find the total.  Count the sets multiple times to be sure you're right!

## Messing with Data

Collate, count and examine the data collected during your Entry Task time during your Rich Routine time.  Students are very motivated to discuss the results of surveys that are all about them! Ask:
- How many children _____?
- How many more like _____ than _____?
- What does this mean?
- Would grade 2's answer in the same way?
- How many children answered the survey / are present today?

Show the data in an equation like:
13 + __ = 24
to show "How many more kids like apples than bananas?"

## Sort It out!

Consider integrating a sorting task into your daily routine time.  Start with 8-10 objects that share more than just colour based attributes...  Invite students to sort the objects in one way and have their peers guess their sorting rule.  Then invite another student to sort the same collection again, using a different attribute.  Celebrate the different ways we can sort a set of objects! Explore 2-D shapes and 3-D objects as well as other interesting counters and natural objects.  As students improve in their ability to sort and name how the group is alike, sort on more than one attribute at a time!

# Teeter Totter Tales

The purpose of this set of daily routines is to compare quantity and to describe the sets as an equality (the same as) or as an inequality (less than, greater than, not the same as). If you used a pan balance to measure students' responses to your "Yes-No" surveys, help students to recall how that scale behaved when the load was out of balance.
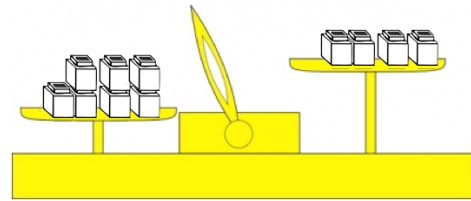
### Teeter Totter - Kindergarten
Draw 2 dot cards at random from a pile and, holding one in each hand, show the children. Ask:

*Am I balanced?*
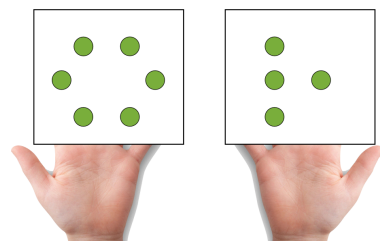
Have students show you with their bodies which way the teeter totter tips. Have them tell you why this is so. Rephrase and say:

*That's right. 6 is more than 4, so I'd tip towards the six.*

### Teeter Totter - Early Grade 1
Draw 2 ten frames at random from a pile and, holding one in each hand, show the children. Ask:
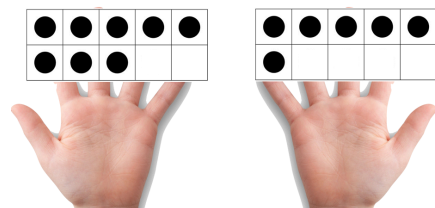
*Am I balanced?*

Have students show you with their bodies which way the teeter totter tips. Have them tell you why this is so. Rephrase and say:

*That's right. 6 is NOT the same as 8, so I'd tip towards the 8.*
**6 ≠ 8**

## Teeter Totter - Late Grade 1

Draw 2 numeral cards at random from a pile and post them on the board. Ask:

*Does the teeter totter balance?  Are these the same?*
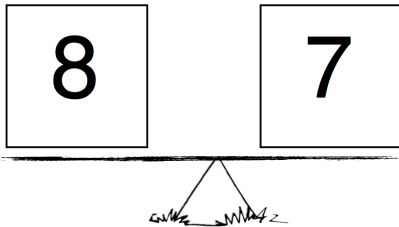


Have students show you with their bodies which way the teeter totter tips. Have them tell you why this is so.

Draw the teeter totter tipped to the appropriate side.
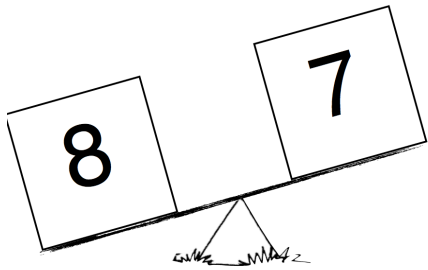


Say:
*That's right.  8 is more than 7, so I'd tip towards the 8.*
                    **8 > 7**
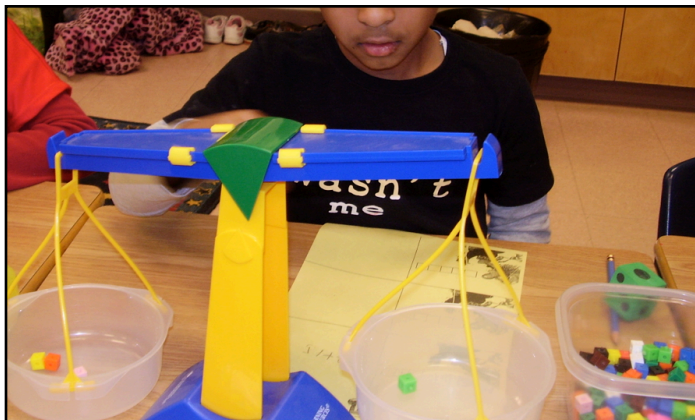
## Teeter Totter - Grade 2

Draw 2 sets of numeral or dot cards at random from a pile and post them on the board, 2 on each side of the balance.
Write an addition sign between them.

Ask:
*Does this show a set of balanced equations?*
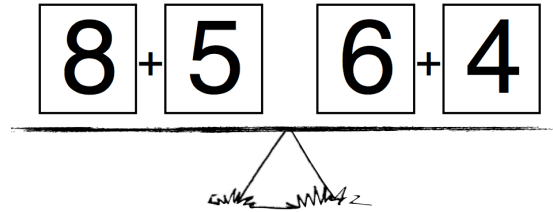


Have students show you with their bodies which way the teeter totter tips. Have them tell you why this is so.

Rephrase and say:

*That's right.  6 + 4 is NOT the same as 8 + 5.*
*This shows an inequality, since 10 is less than 13.*
*The teeter totter will tip towards the 13.*
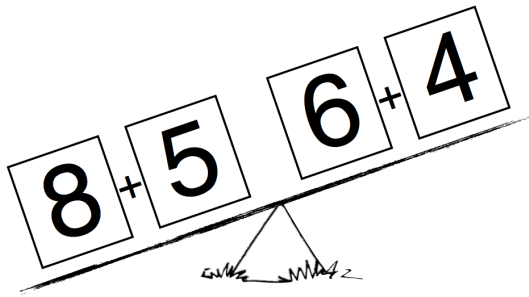


**8 + 5 ≠ 6 + 4**
**13 ≠ 10**
**13 > 10**

# What's next?

As early primary teachers, we recognize that for students to learn they must be actively engaged in thought-provoking tasks.  Our morning routines and entry tasks are an important part of our early primary math programs - and are loaded with potential for developing important conceptual understanding. Although the routines and entry tasks on the previous pages promote deep thinking about important math concepts, they should not be considered a complete math program.  Rather, these tasks are a supplement to a balanced math program in which students work alone, in groups or in partners, on open-ended, structured or directed tasks.

We hope that this resource has given you food for thought, and that, by using these Daily Math Investigations, you are able to get your students engaged, thinking, and reasoning mathematically.

~ Sandra and Carole

## Notes and other ideas

# Line Masters and Companion Resources

On the following pages, you'll find a selection of line masters for the tasks presented in this resource.

More tasks and ideas can be found at:
http://mindfull.wordpress.com

For additional professional development opportunities, contact:
Carole Fullerton at mindfull.consulting@gmail.com
Sandra Ball at ball_s@surreyschools.ca

HAPPY
BIRTHDAY
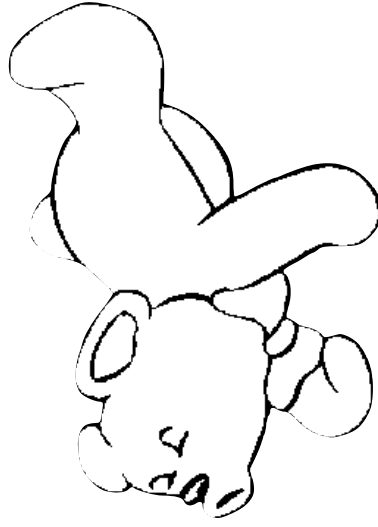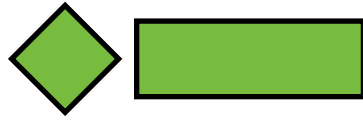
Fill in the cake and gift with Cuisenaire rods!

Now try it in a different way.

How few pieces can you use?
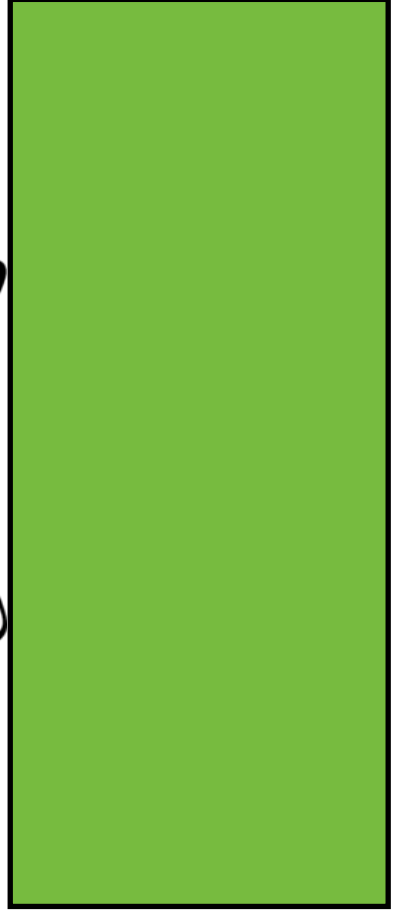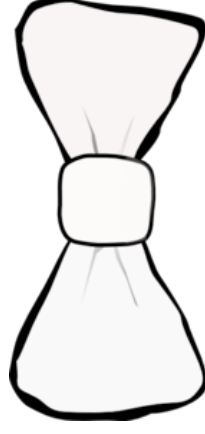
# Row Row Row Your Boat!

Fill in the boat with Cuisenaire rods!

Now try it in a different way.

# 3-2-1 Blast off!
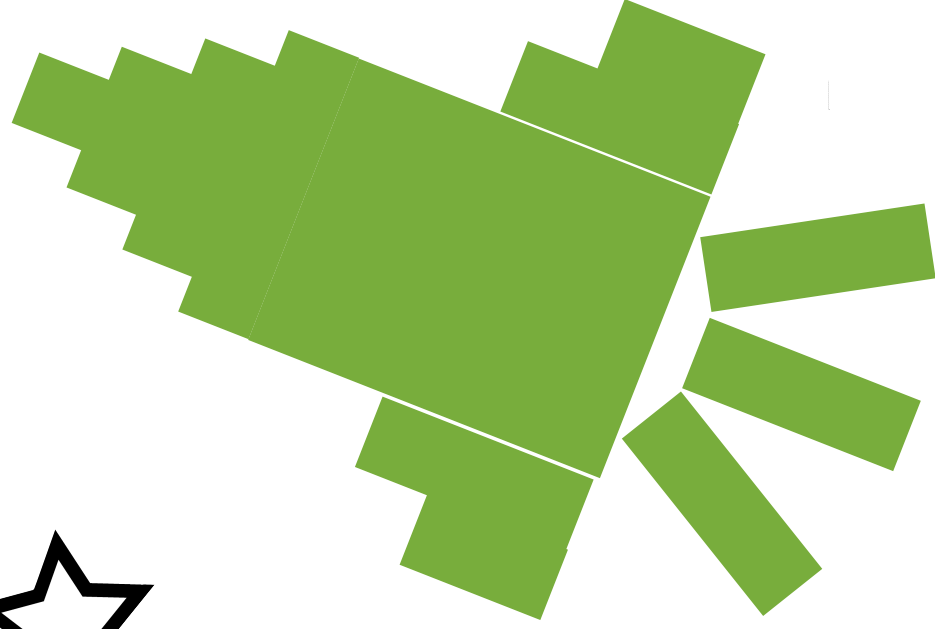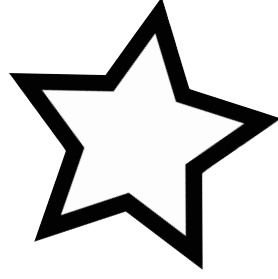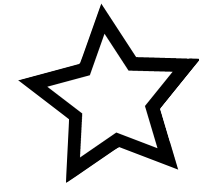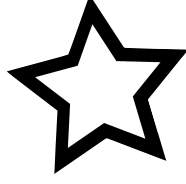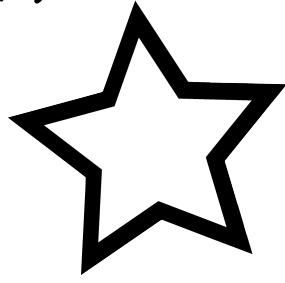
Fill in the rocket with Cuisenaire rods!

Now try it in a different way.

# King of the Jungle

Fill in the lion with Cuisenaire rods!

Now try it in a different way.
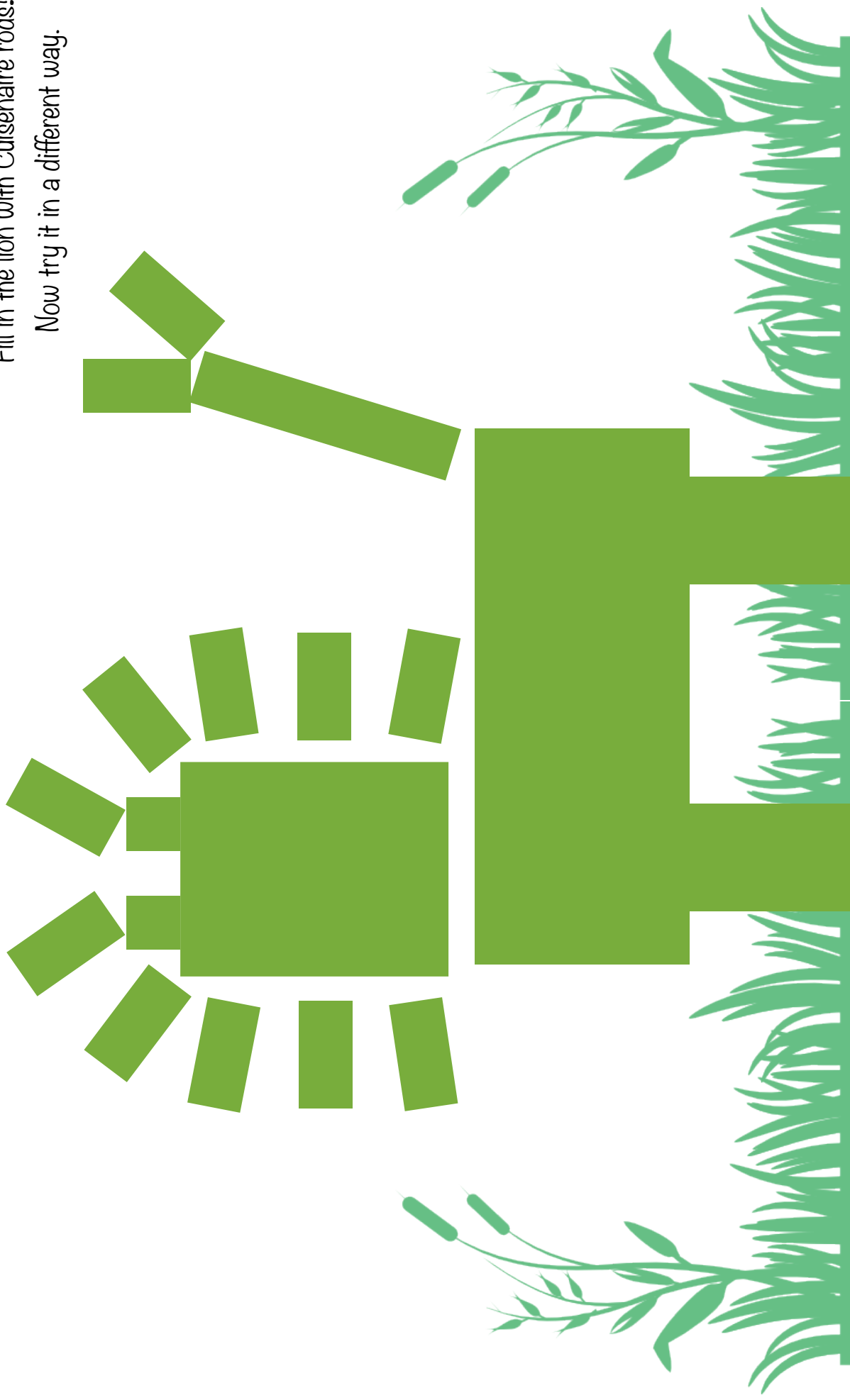
# Rabbit
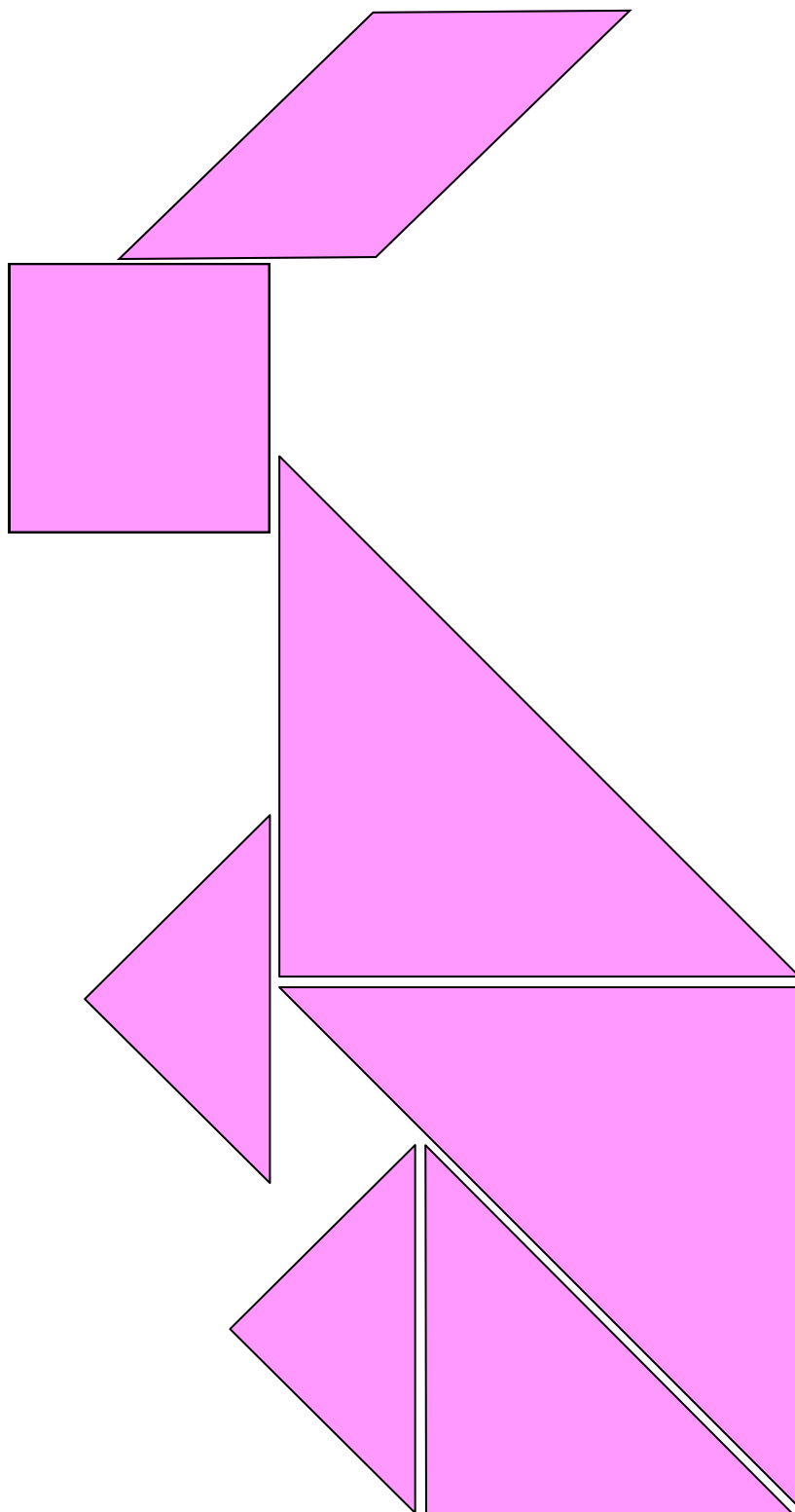
# Rabbit
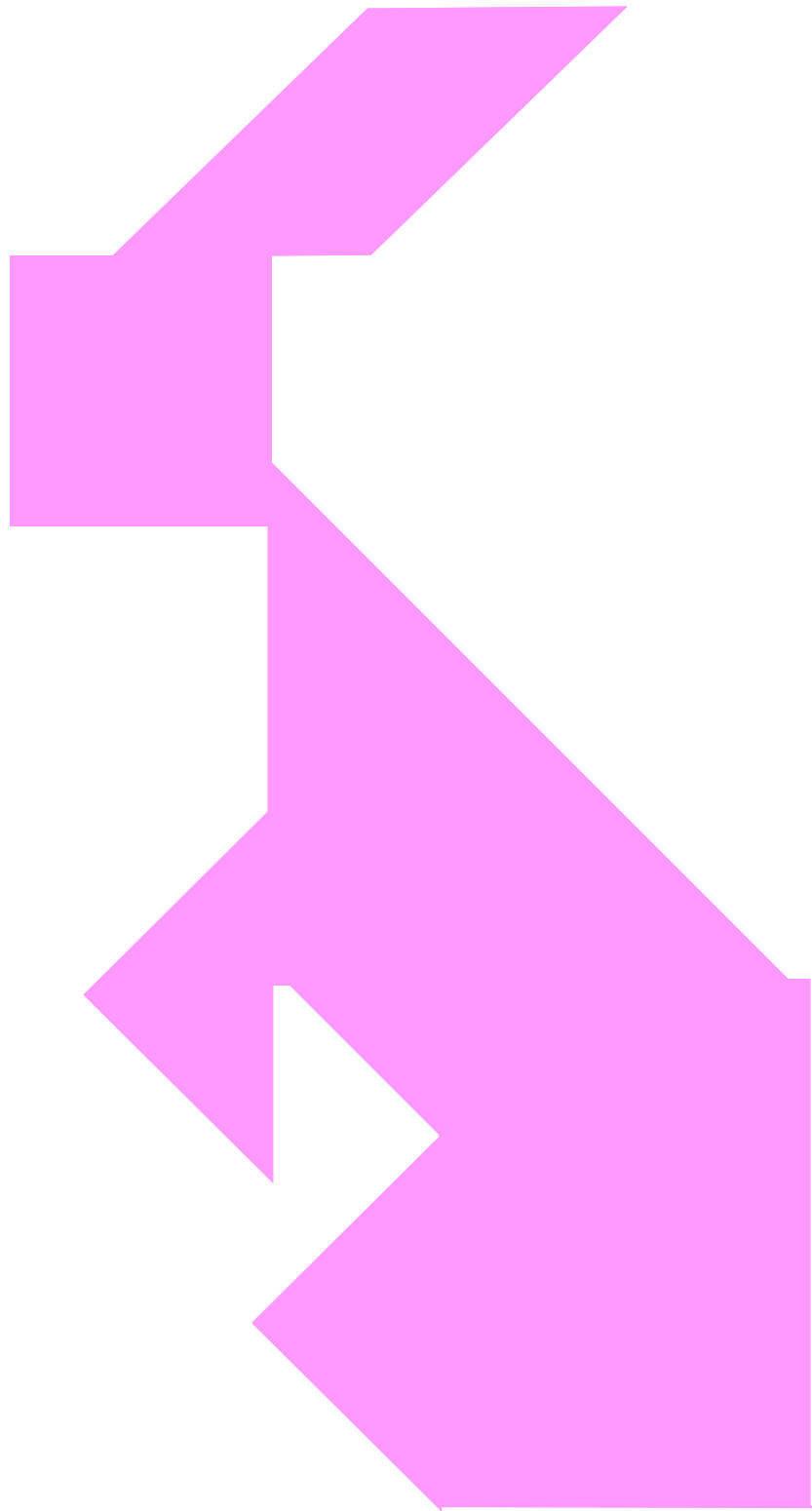
# Dog
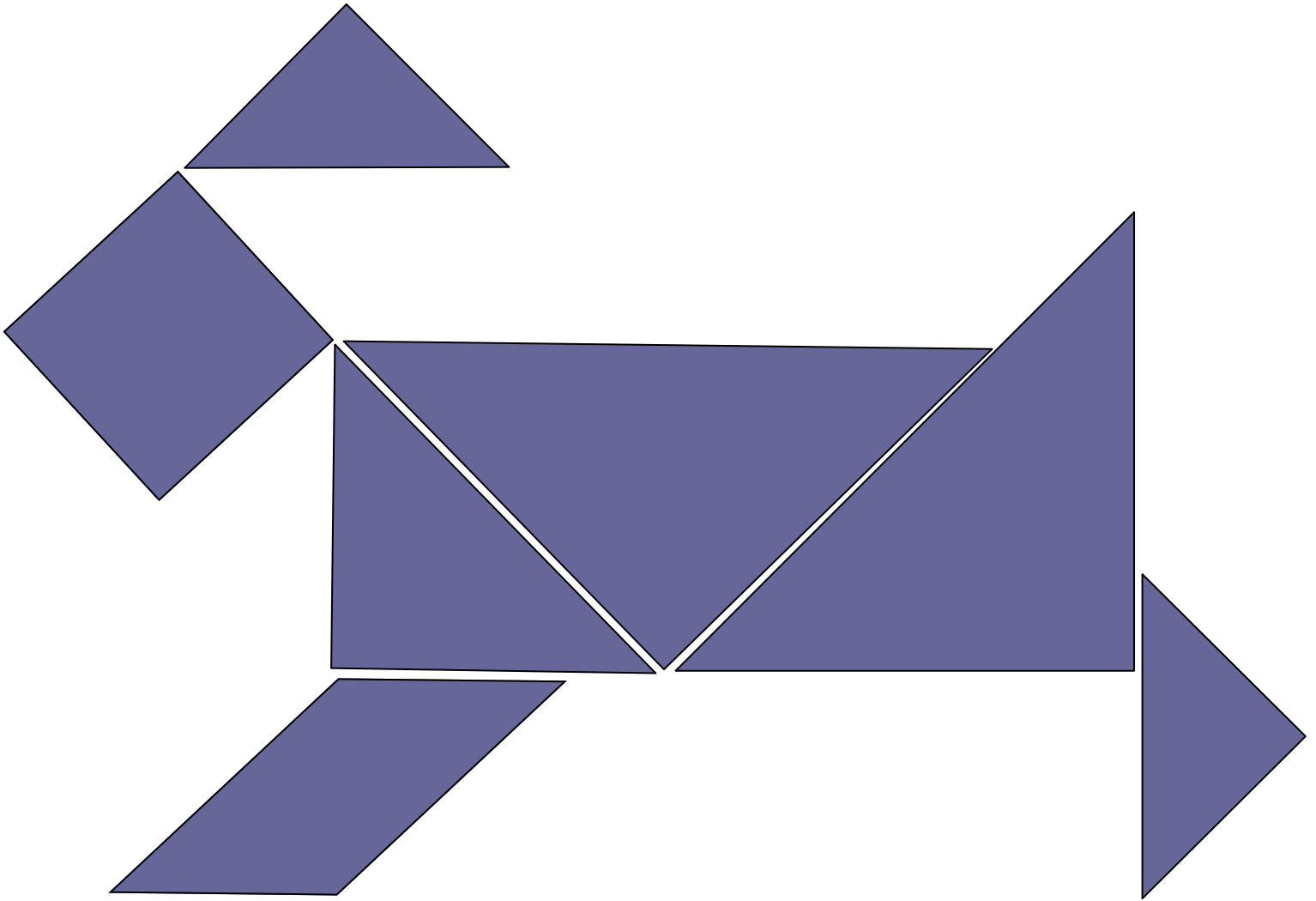
# Dog
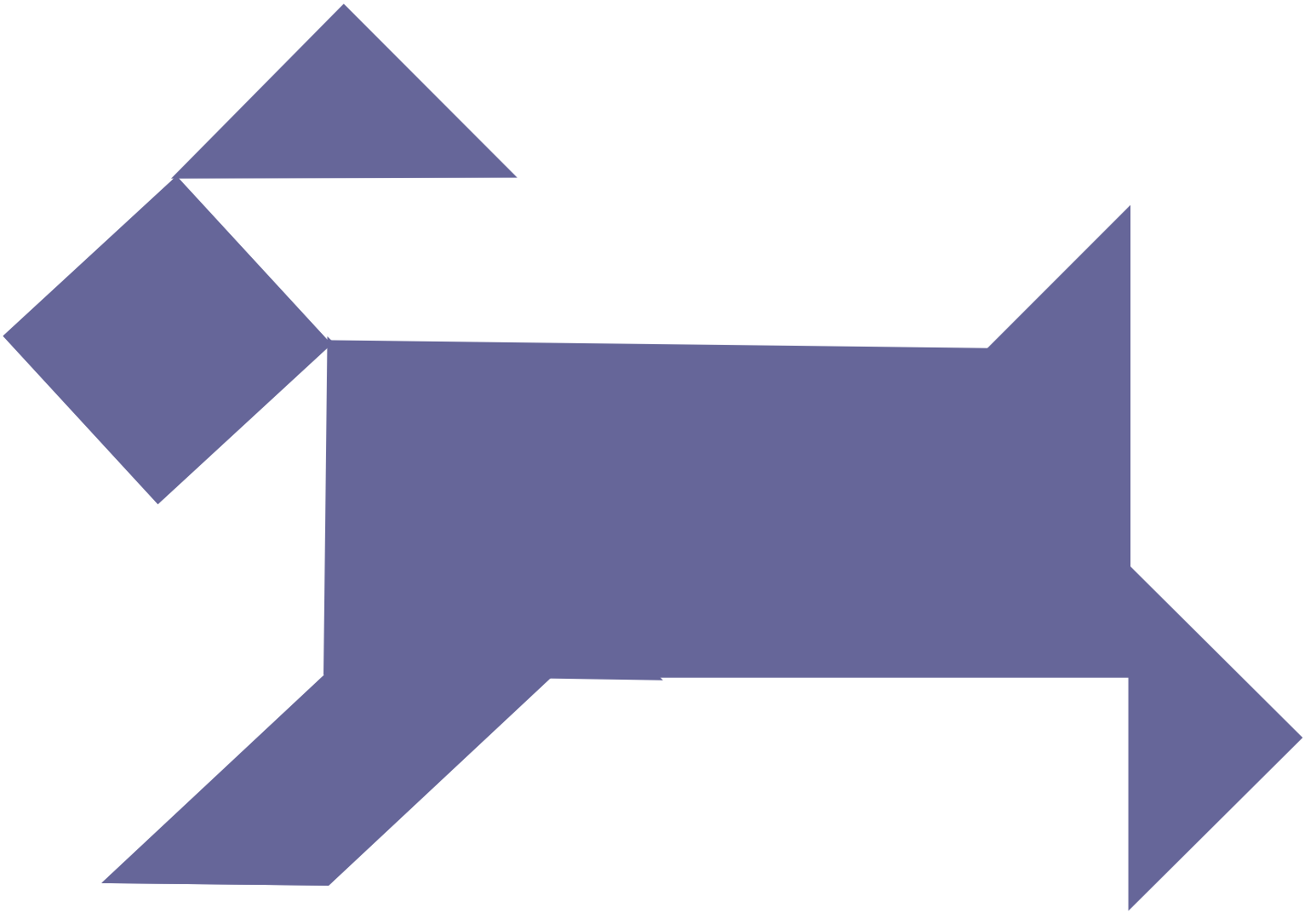
# Pattern Block Dragonfly
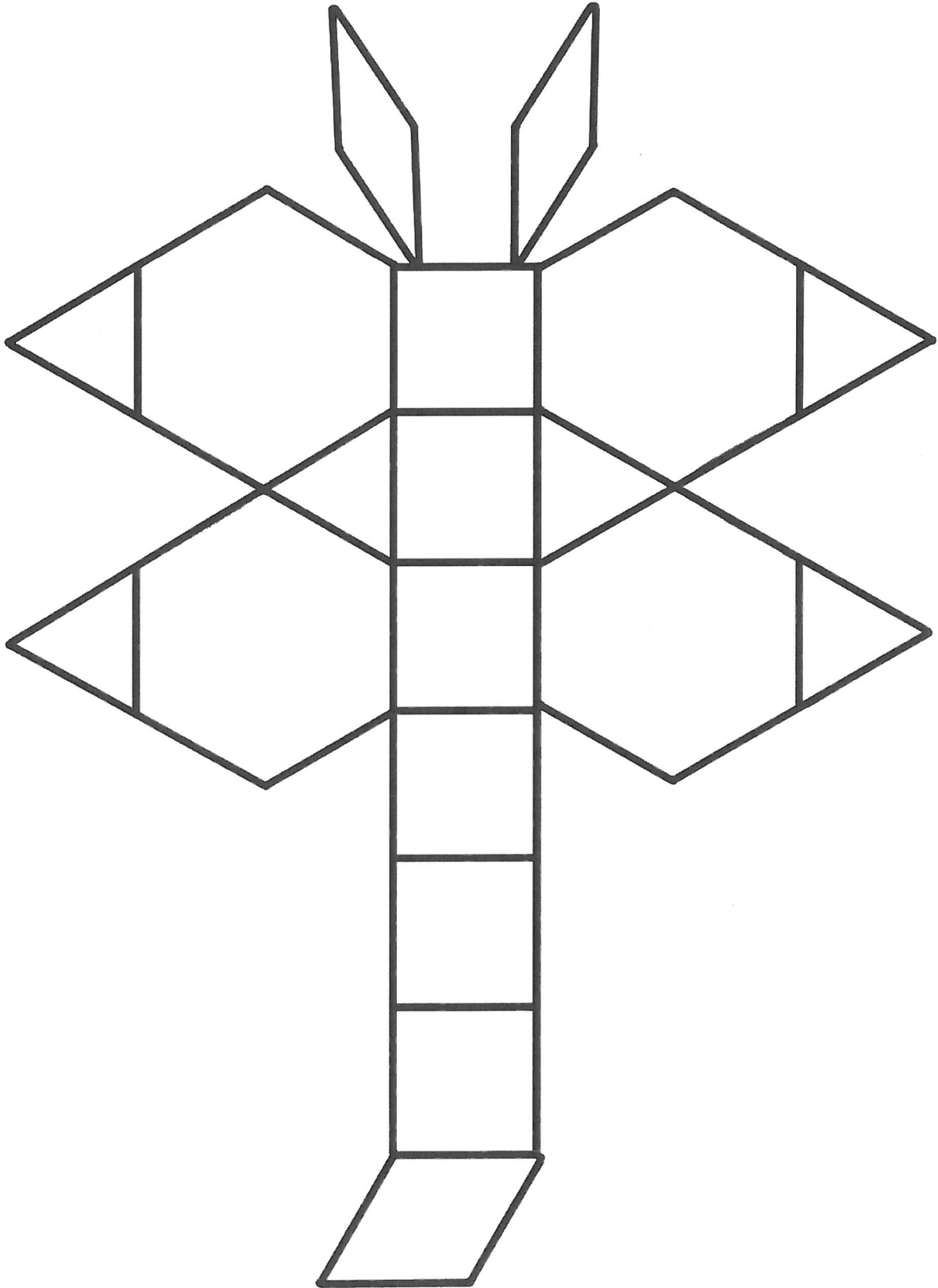
# Pattern Block Butterfly
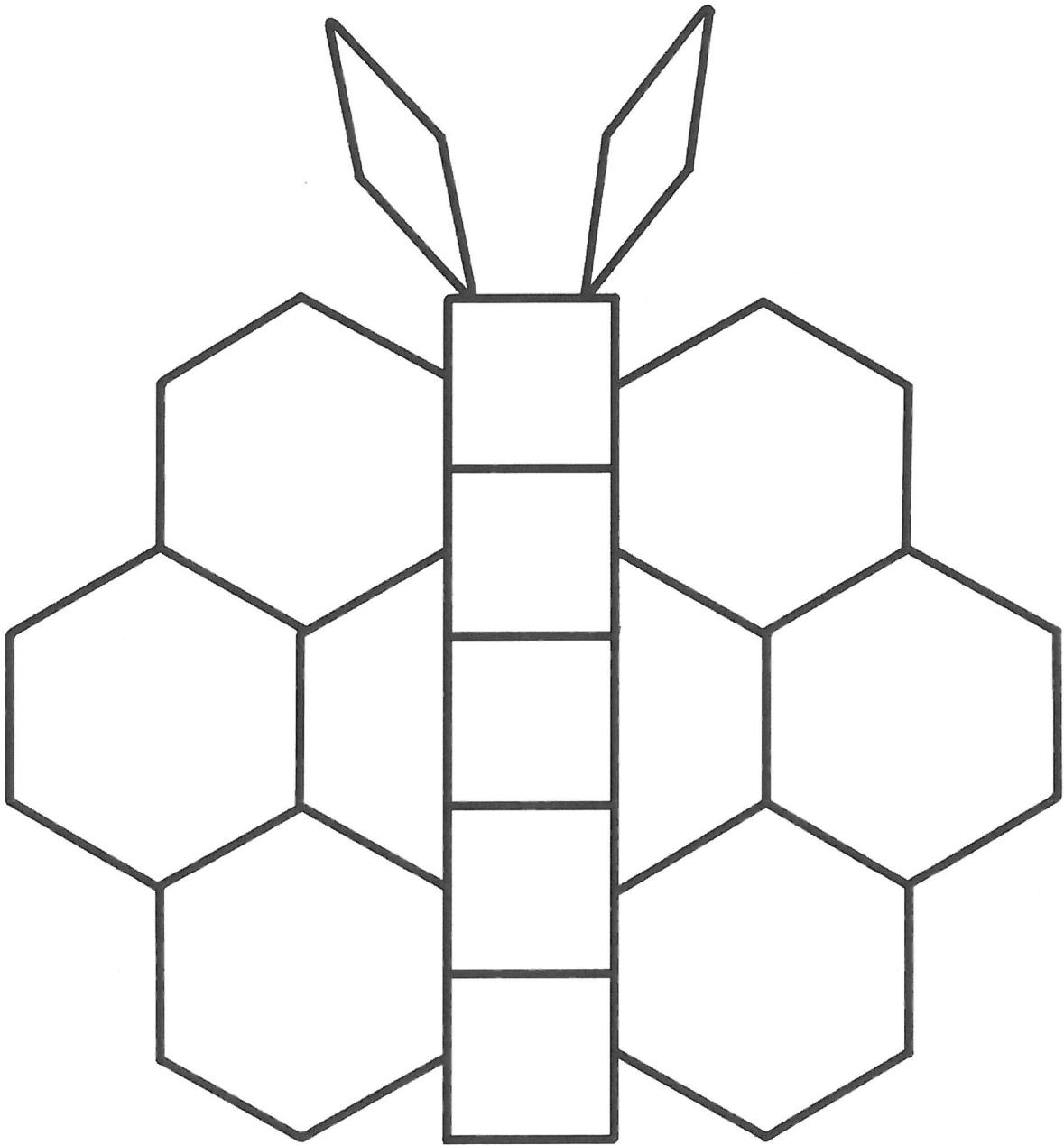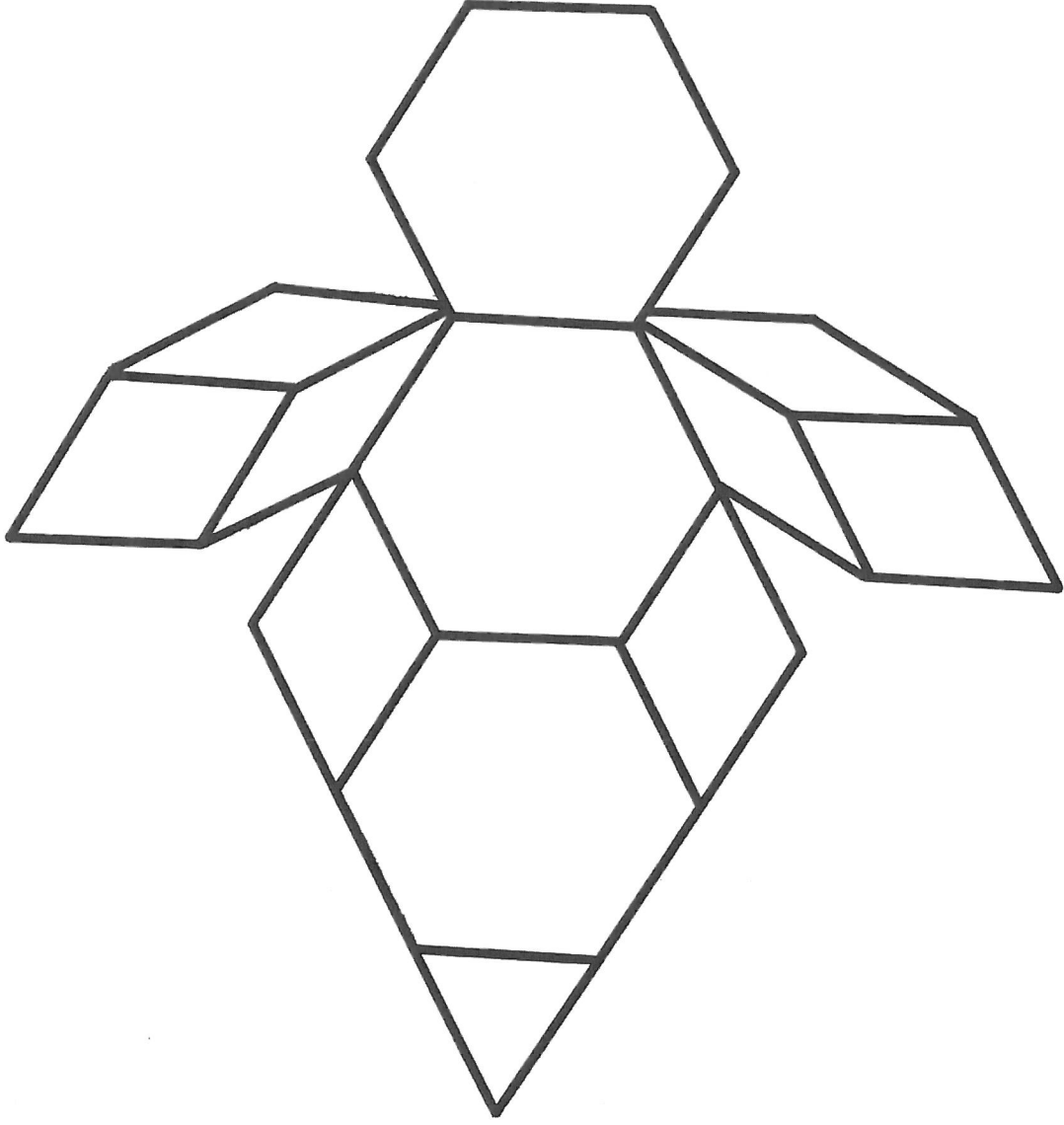
Pattern Block Caterpillar
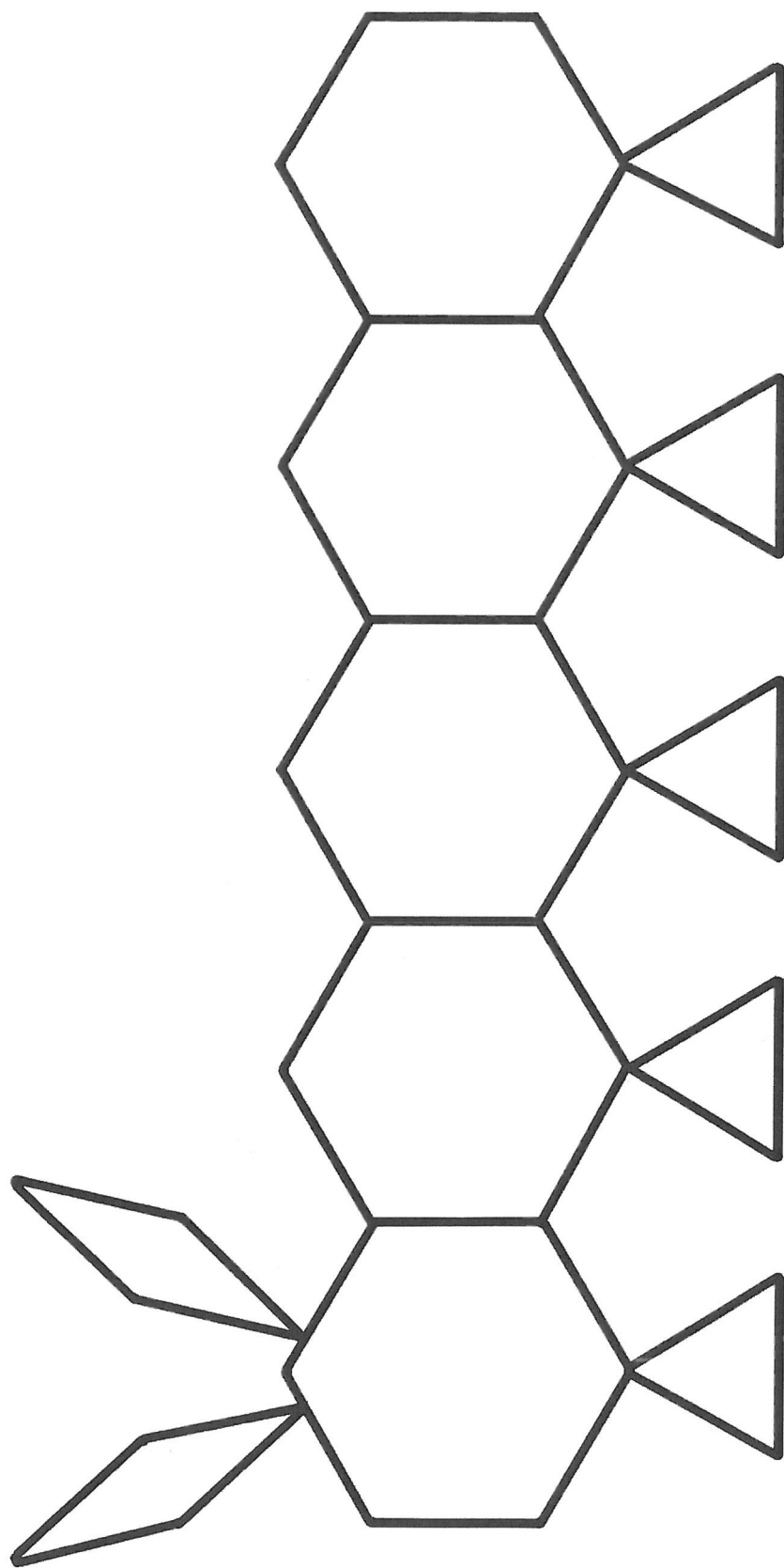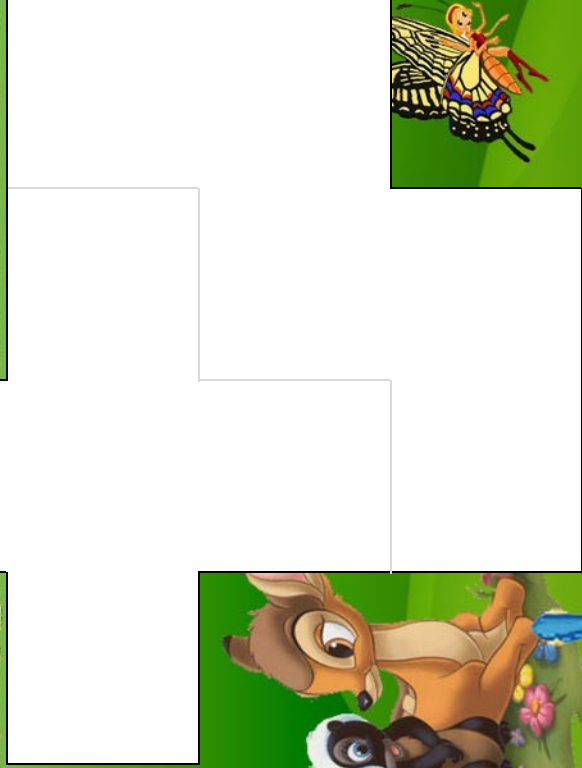
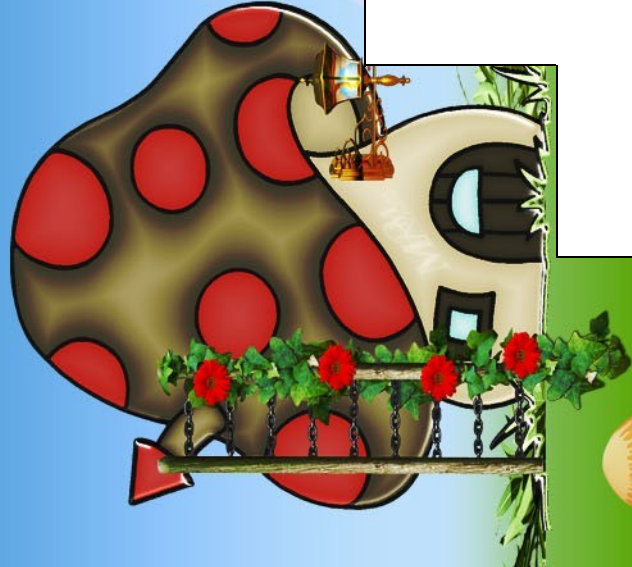How many pentominoes do you need to cover each shape?
Now cover each shape.
Now cover each shape.

# Target Domino

|  |  |
|---|---|
|  |  |

| Less Than | Same As | More Than |
|---|---|---|
|  |  |  |

**Use all of your 0-9 tiles!**

$\square$ + 2 = 9

2 + $\square$ = 8

$\square$ + 5 = 9

$\square$ + $\square$ = 7

6 + $\square$ = 6

$\square$ + $\square$ = 4

$$\begin{array}{r} 5 \\ + \ 3 \\ \hline \square \end{array}$$

$$\begin{array}{r} 8 \\ + \ 1 \\ \hline \square \end{array}$$

Puzzle

**A**

Carole Fullerton 2013

**Use all of your 0-9 tiles!**

$\square$ + 1 = $\square$

$\square$ + $\square$ = 9

$\square$ + 5 = 7

$\square$ + 2 = 6

1 + $\square$ = 2

9 + $\square$ = $\square$

$$\begin{array}{r} 2 \\ +\ 3 \\ \hline \square \end{array}$$

Puzzle

**B**

Carole Fullerton 2013

**Use all of your 0-9 tiles!**

☐ + 3 = 10

9 + ☐ = 15

☐ + 8 = 13

☐ + ☐ = 6

16 + ☐ = 16

☐ + ☐ = 11

```
   15
+  ☐
―――――
   16
```

```
    4
+  ☐
―――――
   13
```

Puzzle

C

**Use all of your 0-9 tiles!**

☐ > ☐          3 > ☐

9 > ☐          8 > ☐

1 > ☐          5 > ☐

☐ > ☐          ☐ > 7

Puzzle

**D**

Carole Fullerton 2013

**Use all of your 0-9 tiles!**

☐ < ☐

☐ < 1

☐ < ☐

4 < ☐

☐ < ☐

☐ < ☐

Puzzle

# E

Carole Fullerton 2013

**Use all of your 0-9 tiles!**

$\square + \square = 10 + 7$

$\square + 7 = 6 + 2$

$7 + \square = 10 + 4$

$7 + \square = 5 + \square$

$\square + 1 = 4 + \square$

$\square + 3 = 6 + \square$

Puzzle

**F**

# Thinking About Number

_____ is more than _____

_____ is less than _____

_____ is the same as _____

## My picture

# Thinking About Number

_____ is too many

_____ is too few

_____ is just enough

## My picture

# Thinking About Number

_____ is about _____

_____ is close to _____

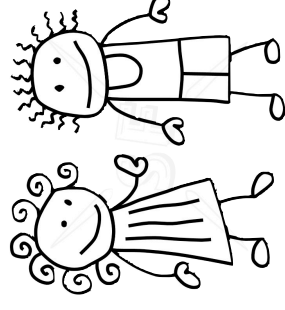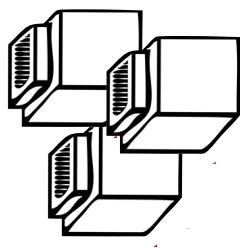_____ but far from _____

## My picture

# Thinking About Number

_____ people could _____.
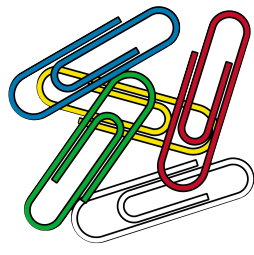
# My picture

# Thinking About Number

## unifix would _____

## My picture

# Thinking About Number

## _____ paperclips could

•

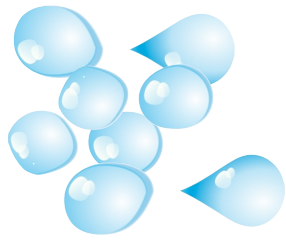# My picture

# Thinking About Number

_____ drops of water would

.

## My picture

# Thinking About Number
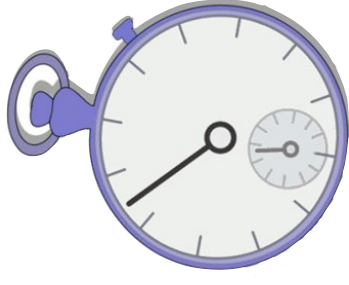
_____ jellybeans would _____

.

**My picture**

# Thinking About Number

____ seconds is about the same as

____ seconds is too short for

____ seconds is too long for

# Thinking About Number

$ _____ is enough for

_____

$ _____ is not enough for

_____

# Thinking About Number

$ _____ is more than _____.

$ _____ is less than _____.

$ _____ is the same as _____.

# Hundred Chart

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

playground

bus

beach

bakery

smaller than

about the same as

bigger than

lighter than

heavier than

about the same as

glue stick

eraser

unifix cube

pencil

foot

arm

hand

thumb

glue stick

chair

book

paper towel

20 counters

100 counters

10 counters

5 counters