“They’ll Need It for High School”

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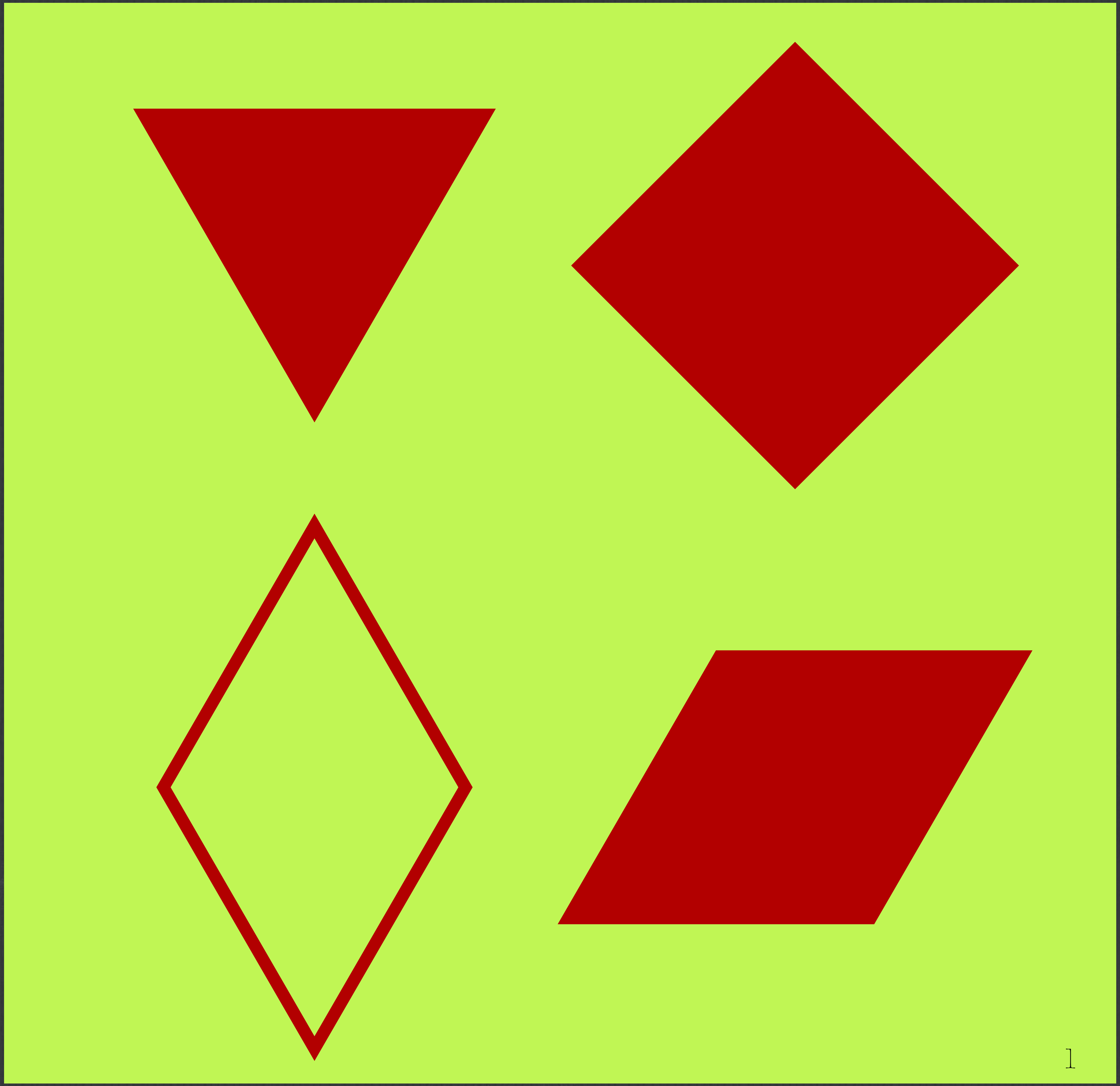
NCTM Boston · April 17, 2015

Which one doesn't belong?

A shapes book

by Christopher Danielson

A Talking Math with Your Kids production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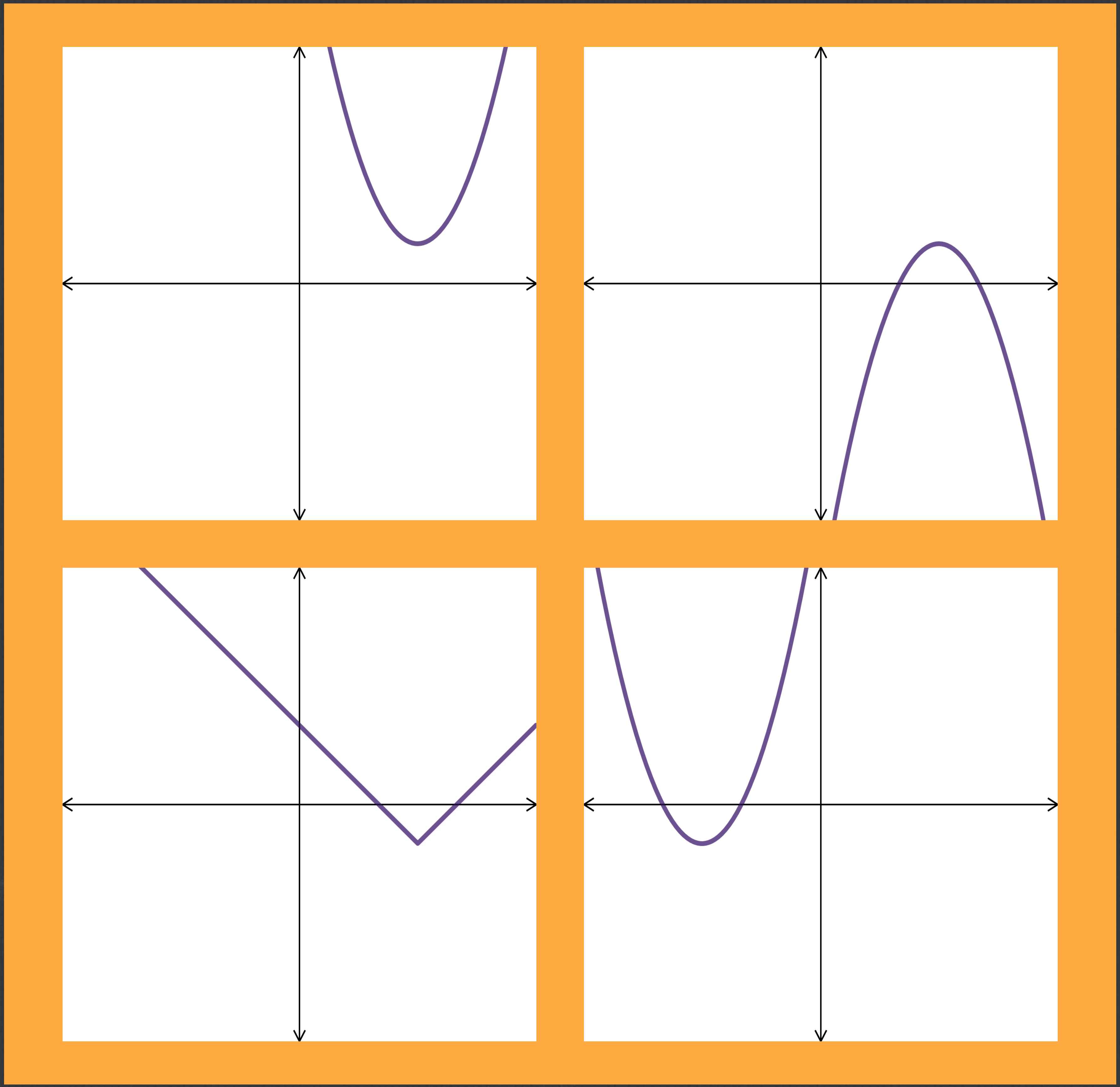

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HOMEPAGE SHAPES NUMBERS GRAPHS & EQUATIONS ABOUT

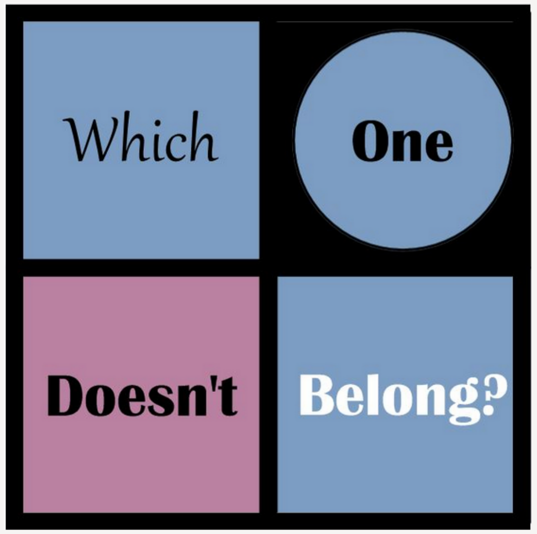
WHICH ONE DOESN'T BELONG?

THIS WEBSITE WAS INSPIRED BY THE MTBOS

with special thanks to Christopher Danielson and his Building a Better Shapes Book.

This is **Which One Doesn't Belong?**, a website dedicated to providing thought-provoking puzzles for math teachers and students alike. There are no answers provided as there are many different, correct ways of choosing which one doesn't belong. Enjoy!

These are also known as Imposter Sets. Here is a link to Steve Wyborney's blog about them.



“They’ll need it for high school.”

Cornered by the Real World

A Defense of Mathematics

Our answers to students' questions about the relevance of what we teach might paint mathematics into an undesirable corner.

Samuel Otten

"When am I ever going to use this?"
"Why do we need to learn this?"

According to beginning mathematics teachers involved in a nationwide study of secondary school teacher induction (Putnam and Britton 2009), these questions, remarkably common in mathematics classrooms, are often difficult to handle. In all likelihood, providing suitable answers is a concern for experienced teachers as well. Students feel moved to ask these questions (repeatedly, in some cases), so we should carefully consider how to go about answering them. Some teachers have developed a repertoire of responses that they draw on as needed, and many instructional materials—posters, websites, extra sections in textbooks—have been designed and marketed to aid in answering such questions. One typical response is to cite a real-world context in which the mathematical content under question can be used or at least recognized. Indeed, some teachers may fear that the failure to produce such a real-world example will damage their students' motivation or perception of the relevance of mathematics.

Here I consider many of the common answers to the student question "When am I ever going to use this?" and point out ways in which students may be dissatisfied with these answers. I then suggest a change in perspective with respect to the handling of this and similar questions. In particular, I propose that, if we are not careful, the tendency to cite real-world contexts can itself be damaging to mathematics education if it traps our discipline in a corner where all learning must be justified by something in everyday life.

CATALOGUE AND CRITIQUE OF RESPONSES TO "WHEN AM I EVER GOING TO USE THIS?"

Citing a Real-World Situation

"You would use area functions like this if you were carpeting your floors."

"The ability to solve systems of equations is important when you're comparing different phone plans."

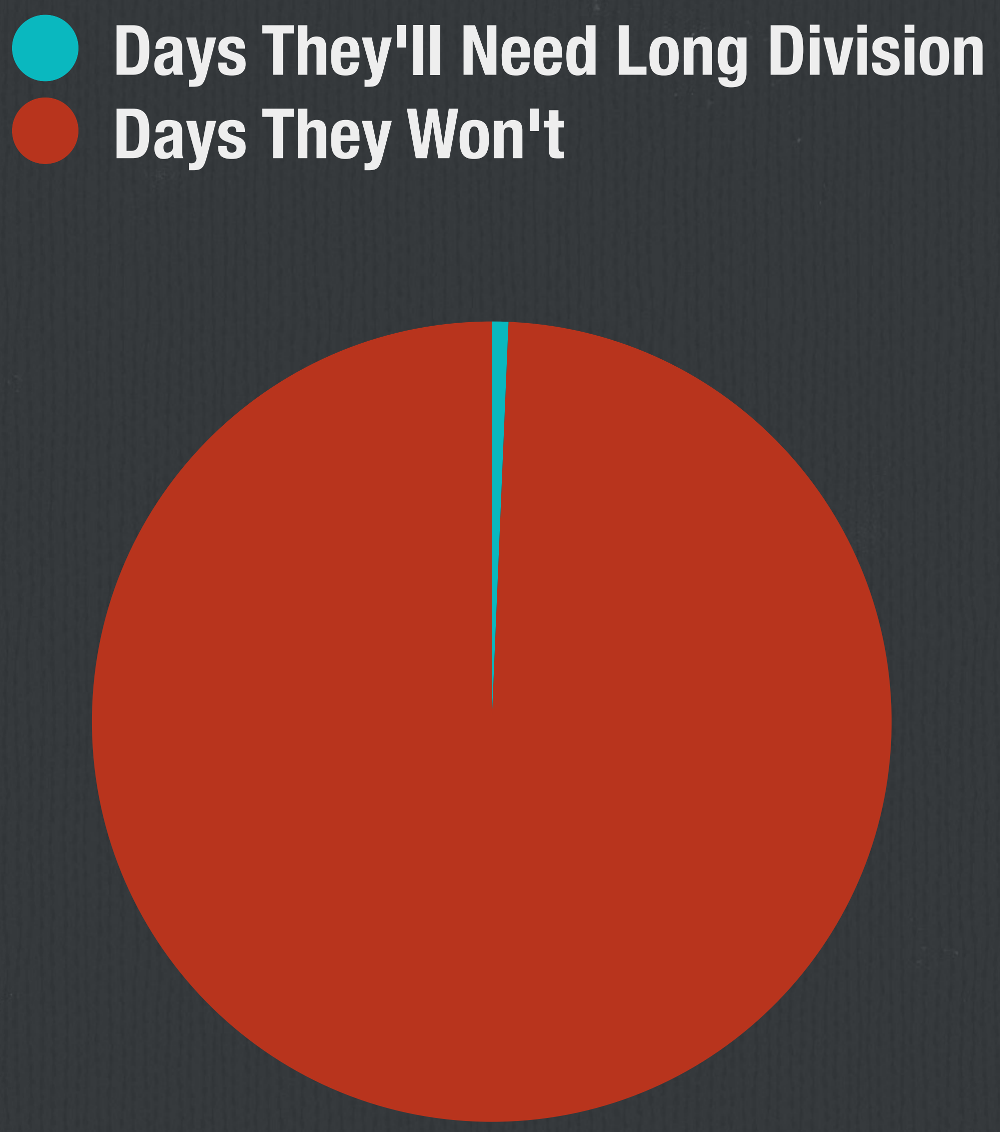
Responses of this type are perhaps the most common, but students may have several difficulties with them. First, the supplied example may be contrived and not a reflection of real life. Such a school-world disconnect is similar to the joke about scientists' attempting to help ranchers by positing a spherical cow in a frictionless field. Students are keenly aware of such detachment and may therefore consider the teacher's attempt at justification a failure.

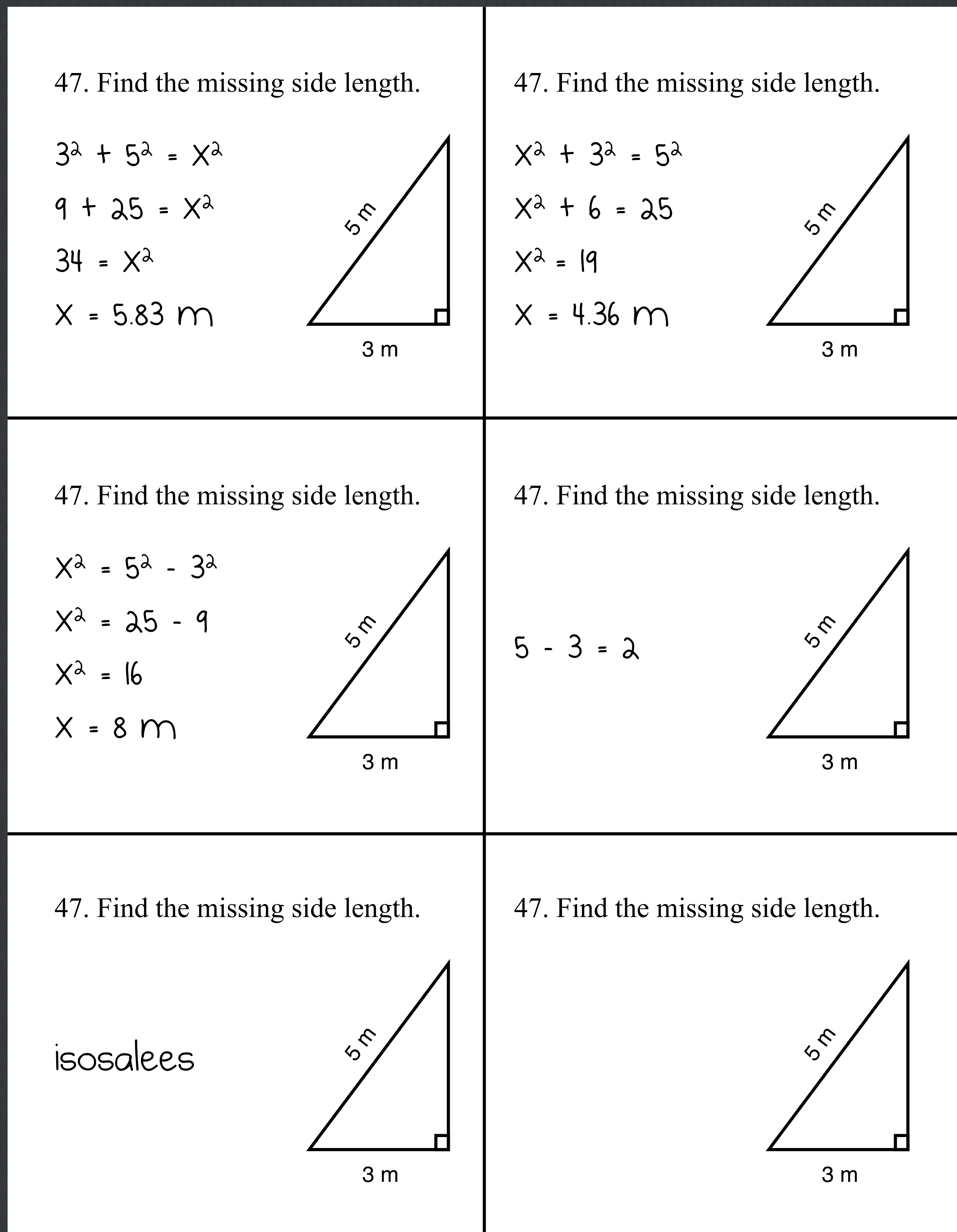
Yet even if the example very much resembles real life, people may be unlikely to solve such problems in the "school mathematics" fashion. For example, millions of people have made decisions about phone plans, but very few, I suspect, wrote functions based on the number of minutes used per month and solved the resulting system of equations. As Lave (1988) and others have shown, people are quite capable of meeting the mathematical challenges of everyday life without appealing to techniques taught in school. Thus, by pointing to real-world situations, a teacher may in fact be supplying daily evidence that people happily get by *without*

AMANDA ROHDE/ISTOCKPHOTO.COM

1. “The Basics”

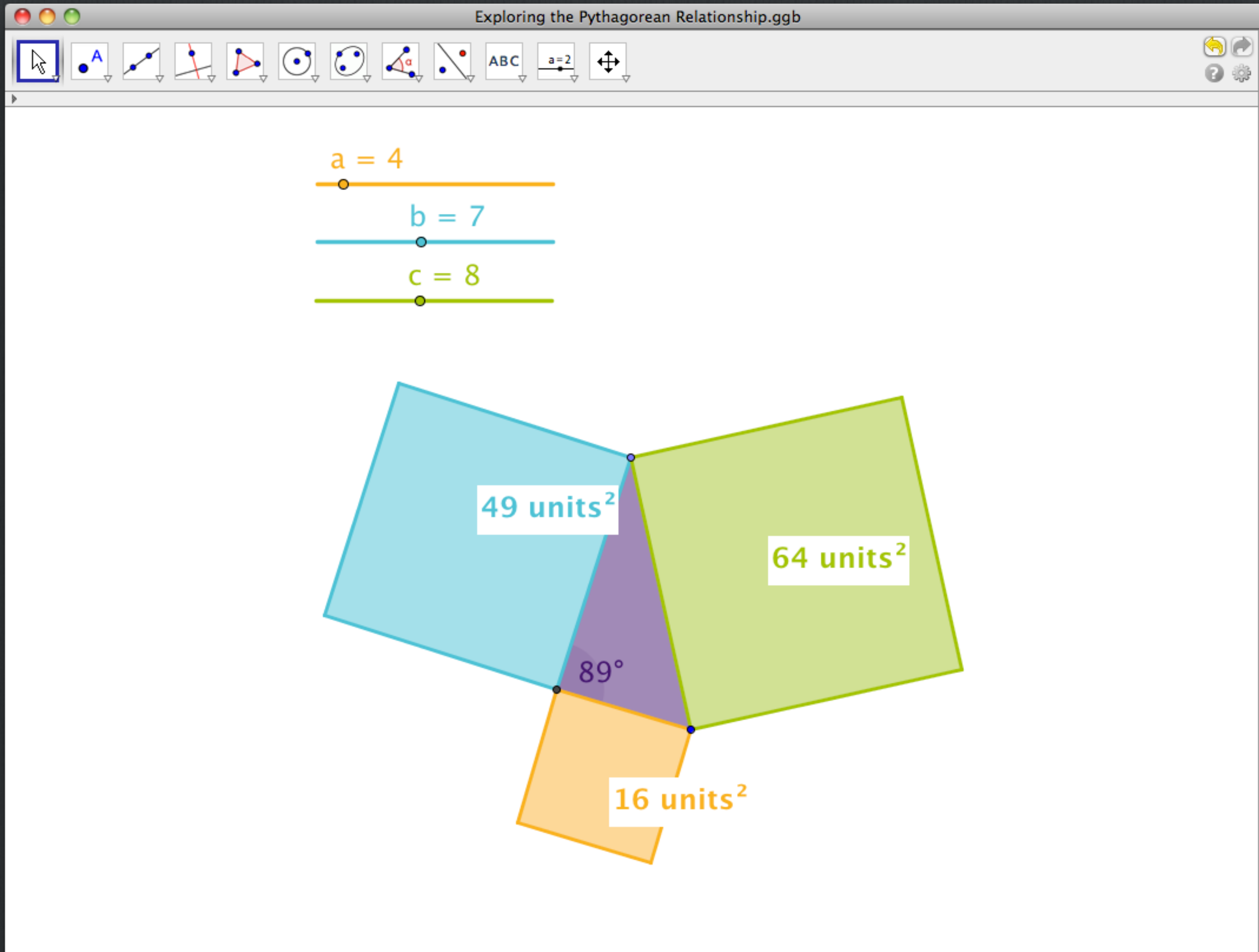
long division · times tables · fractions


Days They'll Need Long Division
Days They Won't



Pythagorean Mistakes

- ☐ What math mistake did each student make?
- ☐ What are some implications for our work?
- ☐ What role did memorization of the times table play?
- ☐ What are some implications for the conversations we could be having?


Exploring the Pythagorean Relationship.ggb
ABC
a = 4
b = 7
c = 8
49 units²
64 units²
89°
16 units²


WATSON
BAILEY
BRONCOS
PLAYOFFS

LuckyBucky

Score: 16

8:13 AM on 6/18/2013

My 13 year old nephew is an A student. I asked him what 7 times 8 equals and all I got was a blank stare. He got out his iphone to get the answer. The rest of the family laughed, but I think it's friggin sad.

8 replies

Research suggests that timed tests cause math anxiety

JO BOALER, PROFESSOR OF MATHEMATICS EDUCATION, STANFORD UNIVERSITY

Teachers in the United States are often forced to follow directives that make little sense to them and are far removed from research evidence. One of the initiatives mandated by many school districts that I place high in the category of uninformed policy is the use of timed tests to assess math facts and fluency. Teachers and administrators use these tests with the very best of intentions, but they use them without knowledge of the important evidence that is emerging from neuroscience. Evidence strongly suggests that timed tests cause the early onset of math anxiety for students across the achievement range. Given the extent of math anxiety, math failure, and innumeracy in the United States (Boaler 2009), such evidence is important for us all to consider. In this article, I summarize the evidence from neuroscience and describe an alternative pedagogical routine that teaches number sense and math fluency at the same time that it encourages mathematical understanding and excitement.

IN MY OPINION

ALEXSTAR/THINKSTOCK

Math anxiety

Occurring in students from an early age, math anxiety and its effects are exacerbated over time, leading to low achievement, math avoidance, and negative experiences of math throughout life (Ramirez et al. 2013; Young, Wu, and Menon 2012). Educators have witnessed the impact of math anxiety for decades, but only in recent years have timed math tests been shown to be one cause of the early onset of math anxiety. Indeed, researchers now know that students experience stress on timed tests that they do not experience even when working on the same math questions in untimed conditions (Engle 2002).

In a recent study of 150 first and second graders, researchers measured students' levels of math anxiety, finding that children as young as first grade experienced it and that levels of math anxiety did not correlate with grade level, reading level, or parental income (Ramirez et al. 2013). Other researchers analyzed brain-imaging data from forty-six seven- to nine-year-old children while they worked on addition and subtraction problems and found that those students who "felt panicky" about math had increased activity in brain regions associated with fear. When those areas were active, decreased activity took place in the brain regions that are involved in problem solving (Young, Wu, and Menon 2012).

Beilock and her colleagues conducted brain scans to study the ways in which anxiety affects individuals, showing that children compute with math facts—such as those required in timed tests—by recalling information that is held in the working memory (Beilock 2011). The more working memory an individual

2. Pedagogy Preparation

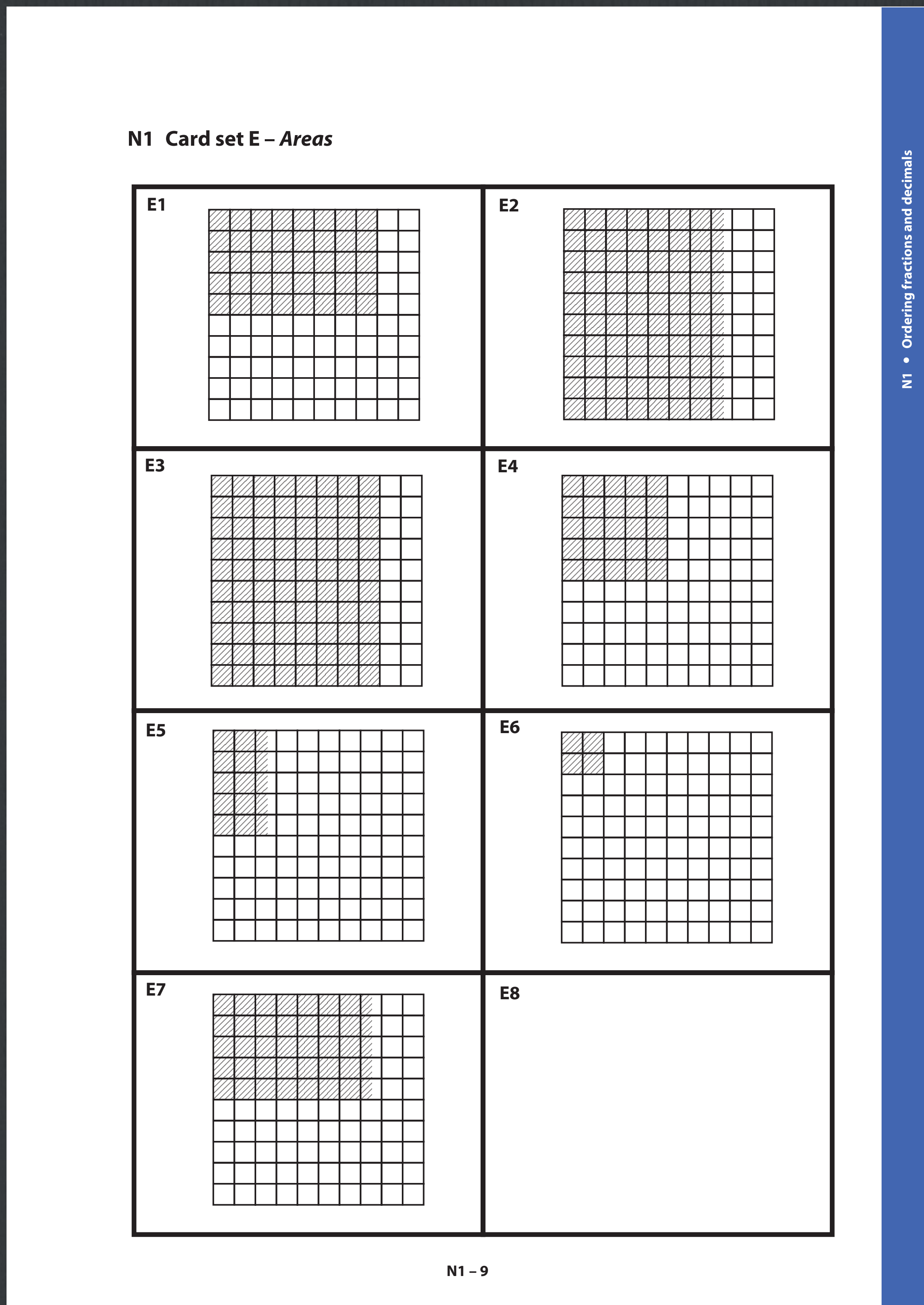
"I want them to get used to it."

N1 Card set A – *Decimals*

0.8	**0.04**
0.25	**0.375**
0.4	**0.125**
0.75	

A $y = 2\sin x$	B $y = 3\cos\frac{1}{2}(x + 90°) - 1$
C $y = \cos x + 2$	D $y = -2\cos 3(x - 60°)$
E $y = \frac{1}{2}\sin(x + 60°)$	F $y = \cos 2x - 3$
G $y = -3\sin 2x$	H $y = \sin 3(x - 90°) + 1$

Malcolm Swan · Standards Unit


N1 Card set E – *Areas*

E1

E2

E3

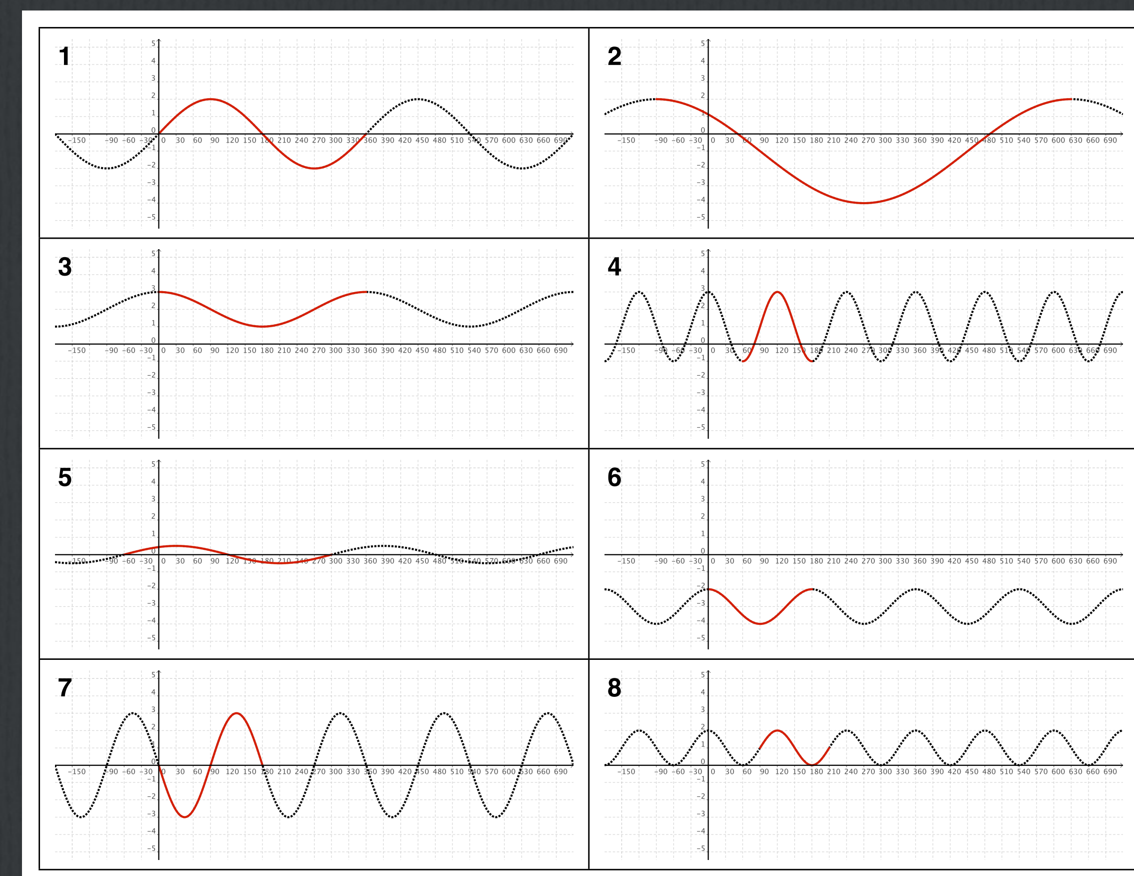
E4

E5

E6

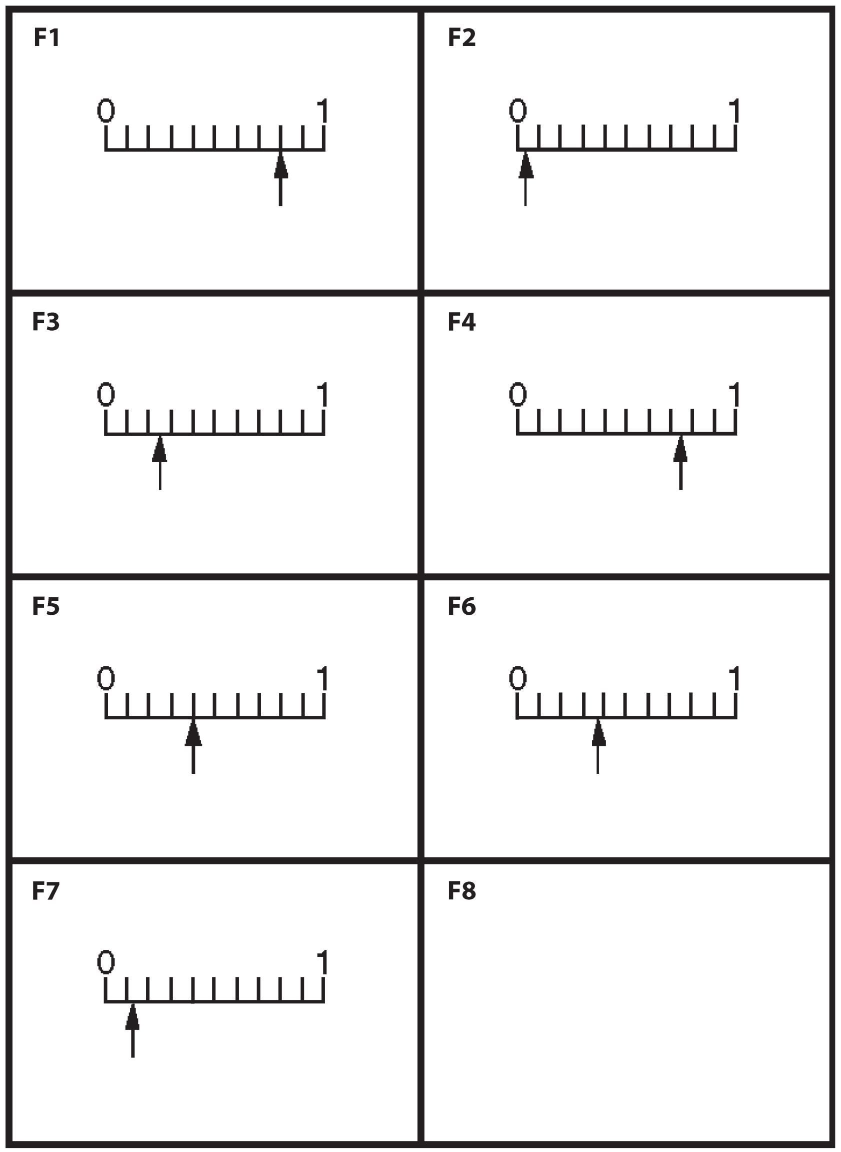
E7

E8



Malcolm Swan · Standards Unit

N1 Card set F – *Scales*



i amplitude: 1 period: 360° maximum: 3 minimum: 1 range: $1 \le y \le 3$ vertical translation: up 2	**ii** amplitude: 1 period: 180° maximum: -2 minimum: -4 range: $-4 \le y \le -2$ vertical translation: down 3
iii amplitude: $\frac{1}{2}$ period: 360° maximum: $\frac{1}{2}$ minimum: $-\frac{1}{2}$ range: $-\frac{1}{2} \le y \le \frac{1}{2}$ horizontal translation: left 60°	**iv** amplitude: 3 period: 180° maximum: 3 minimum: -3 range: $-3 \le y \le -3$ reflection: x-axis
v amplitude: 3 period: 720° maximum: 2 minimum: -4 range: $-4 \le y \le 2$ horizontal translation: left 90° vertical translation: down 1	**vi** amplitude: 2 period: 120° maximum: 2 minimum: -2 range: $-2 \le y \le 2$ horizontal translation: right 60° reflection: x-axis
vii amplitude: 1 period: 120° maximum: 2 minimum: 0 range: $0 \le y \le 2$ horizontal translation: right 90° vertical translation: up 1	**viii** amplitude: 2 period: 360° maximum: 2 minimum: -2 range: $-2 \le y \le 2$

Malcolm Swan · Standards Unit

3. Affective Domain

“Give me a student with a positive attitude towards mathematics, who’s persistent, who’s curious, … and she will be successful in high school.”

4. Mathematical Thinking

Habits of Mind · Processes · Practices

Chris | Jeff | Marc

$75 | $60 | $45

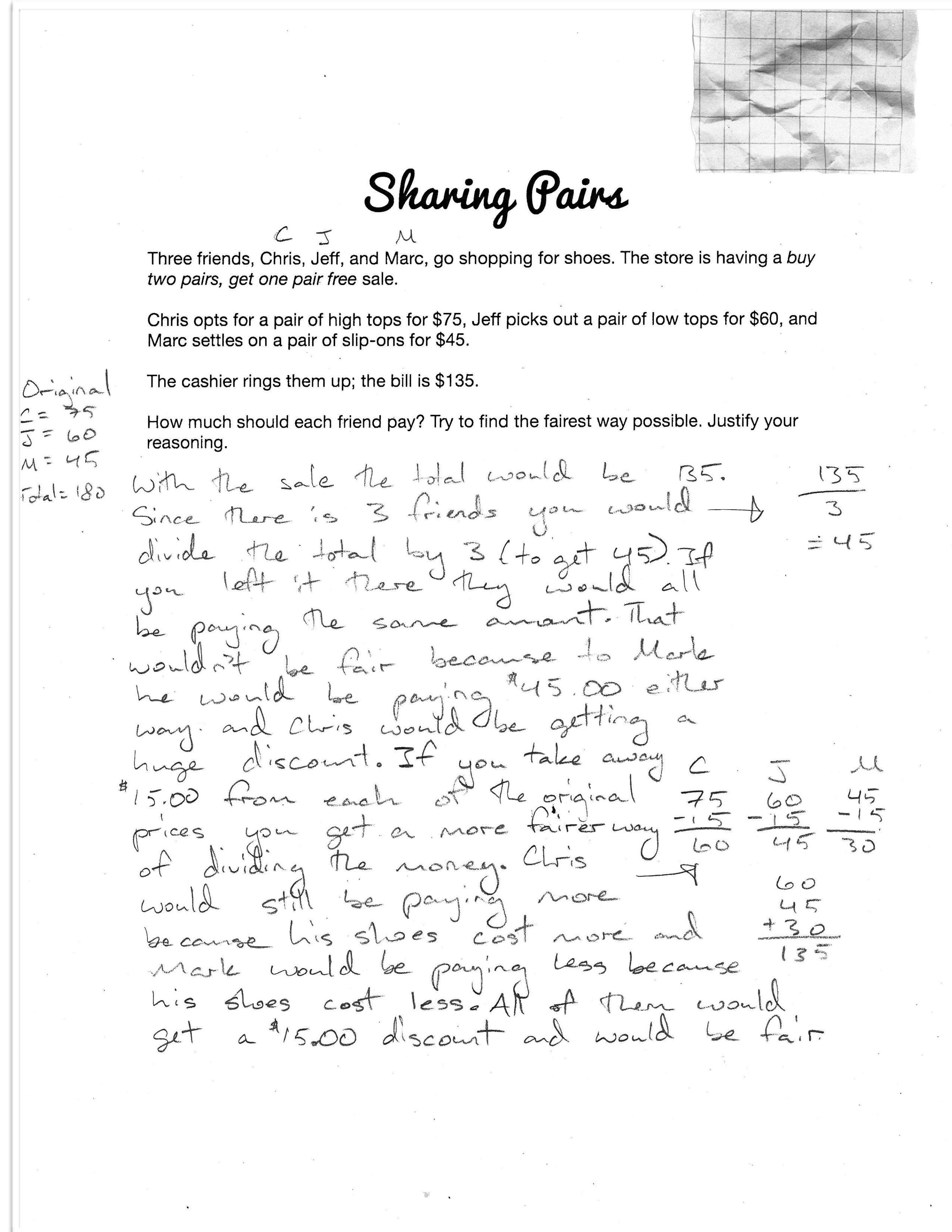
BUY TWO PAIRS, GET ONE PAIR FREE!

3RD PAIR MUST BE OF EQUAL OR LESSER VALUE

Chris Jeff Marc

? ? ?

Chris	Jeff	Marc
		
$45	$45	$45



Sharing Pairs

Three friends, Chris, Jeff, and Marc, go shopping for shoes. The store is having a *buy two pairs, get one pair free* sale.

Chris opts for a pair of high tops for $75, Jeff picks out a pair of low tops for $60, and Marc settles on a pair of slip-ons for $45.

The cashier rings them up; the bill is $135.

How much should each friend pay? Try to find the fairest way possible. Justify your reasoning.

Original
C = 75
J = 60
M = 45
Total = 180

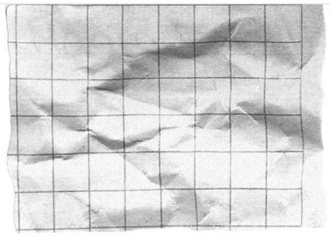
With the sale the total would be 135. Since there is 3 friends you would divide the total by 3 (to get 45). If you left it there they would all be paying the same amount. That wouldn't be fair because to Mark he would be paying $45.00 either way and Chris would be getting a huge discount. If you take away $15.00 from each of the original prices you get a more fairer way of dividing the money. Chris would still be paying more because his shoes cost more and Mark would be paying less because his shoes cost less. All of them would get a $15.00 discount and would be fair.

$\frac{135}{3} = 45$

C	J	M
75	60	45
−15	−15	−15
60	45	30

60 + 45 + 30 = 135

Chris	Jeff	Marc
		
$60	$45	$30



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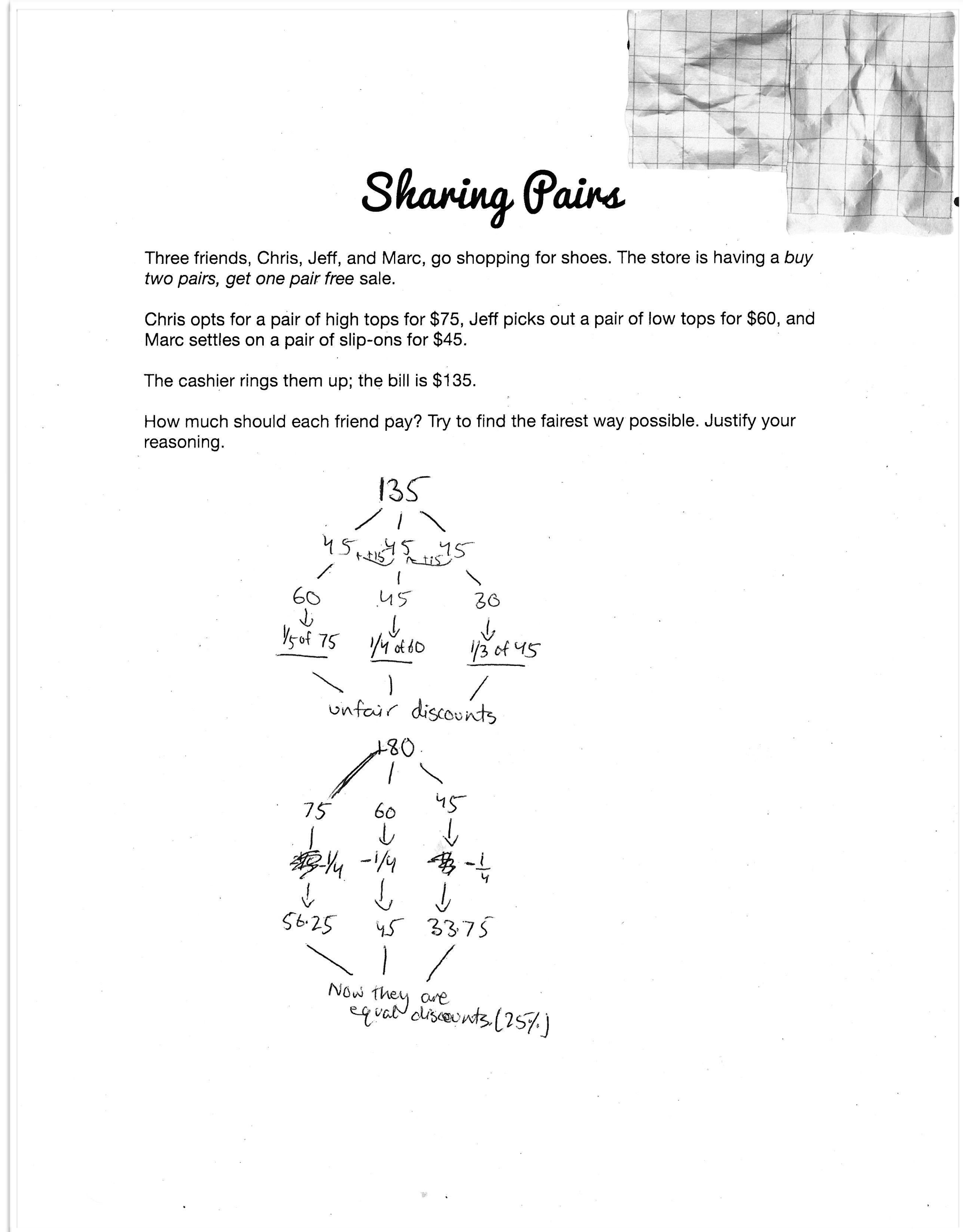
Chris	Jeff	Marc
$75	$60	$45
− $15	− $15	− $15
$60	$45	$30

This is the fairest way because in the beginning all their shoes prices were a $15 difference. so basically they subtract $15 from the orginal price. chris would then pay $60, Jeff would pay $45 and Marc would pay $30. It is fair because Chris' shoes are the most expensive so he should pay more than Jeff and Marc. since Marc's shoes were the least, thats why he pay's the least amount. Also, in the beginning all the prices had a difference of $15 and with the prices $60, $45, $30, there is a difference of $15. I think this is the fairest way to split up the money.

Chris	Jeff	Marc
		
$56.25	$45.00	$33.75



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5. Key Concepts & Big Ideas

on my mind

readers speak out

They'll Need It for Calculus

Christopher Danielson

VITALLIY/THINKSTOCK

Imagine a self-propelled lawn mower that is tied to a stake by a rope. As the lawn mower runs, the rope wraps around the stake, pulling the mower closer with each revolution. My Calculus 2 students viewed a video (Atterberry 2010) of this scenario one recent semester. The lawn mower is already under way as the video begins, and the viewer sees only a few revolutions before the video ends.

We worked to answer this question: How long will it take to complete the mowing?

As my students undertook the challenge of answering this question, I came to understand that they were struggling to write an equation to describe the lawn mower's path. This surprised me, and it forced me to reflect on what it means to be ready for calculus.

Although it is reasonable to question whether secondary mathematics ought to be a pipeline to calculus, this is certainly part of its present function in U.S. schools. This pipeline has many components. They include middle school mathematics, high school algebra, geometry courses, college placement exams, college courses prior to calculus, and so on, right up to—and in the case of my Calculus 2 students working on the Lawn Mower problem—the previous day's lesson.

This article focuses on the big question of what it means to be ready for calculus; it also explores the role of the middle school curriculum in preparing students to study calculus later. This should not be construed as an endorsement of the pipeline to calculus or as an assignment of responsibility for these ideas to the middle school curriculum and to teachers exclusively. Instead, this article is written for an audience of middle school teachers from the perspective of a former middle school teacher and current college teacher. In fact, it is my hope that middle school teachers (many of whom may not have

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An Alternative Perspective on What They'll Need

on my mind

readers speak out

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VITALLY/THINKSTOCK

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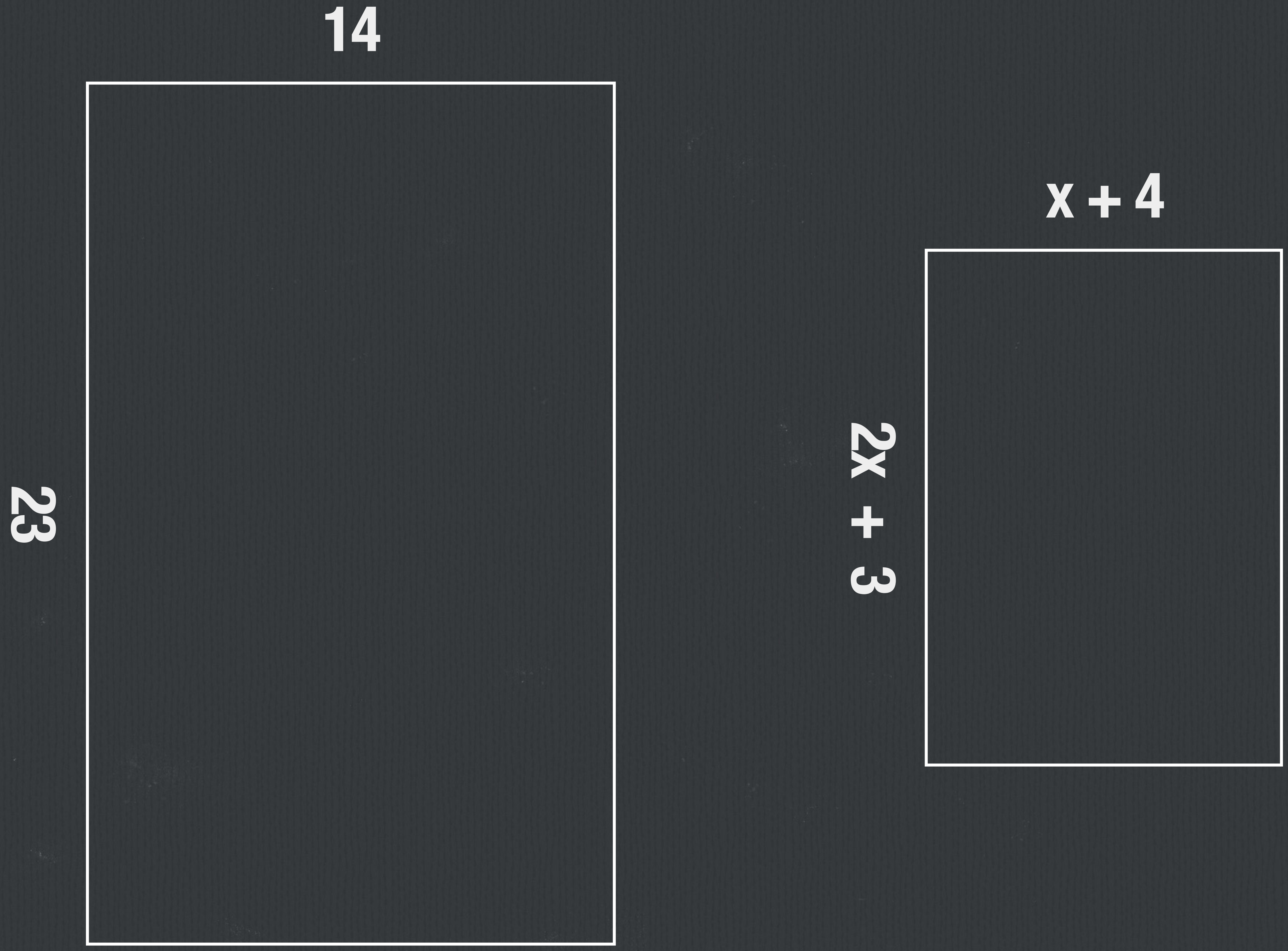
1. A function is a relationship between two variables; and
2. Slope is a rate of change.

1. The operations of addition, subtraction, multiplication, and division hold the same fundamental meaning no matter the domain in which they are applied.

Addition	Subtraction
$231+145$ $2.31+1.45$ $(2x^2+3x+1)+(x^2+4x+5)$	$1\frac{1}{4}-\frac{1}{2}$ $5x-2x$ $5\sqrt{2}-\sqrt{8}$
Multiplication	**Division**
23×14 $2\frac{3}{10}\times 1\frac{4}{10}$ $(2x+3)(x+4)$	$6\div 3$ $(-6)\div(+3)$ $\frac{6}{5}\div\frac{3}{5}$

How are they the same?

- ☐ Evaluate, or simplify, each set of expressions
- ☐ Make as many connections as you can:
  - ☐ conceptually & procedurally
  - ☐ pictorially & symbolically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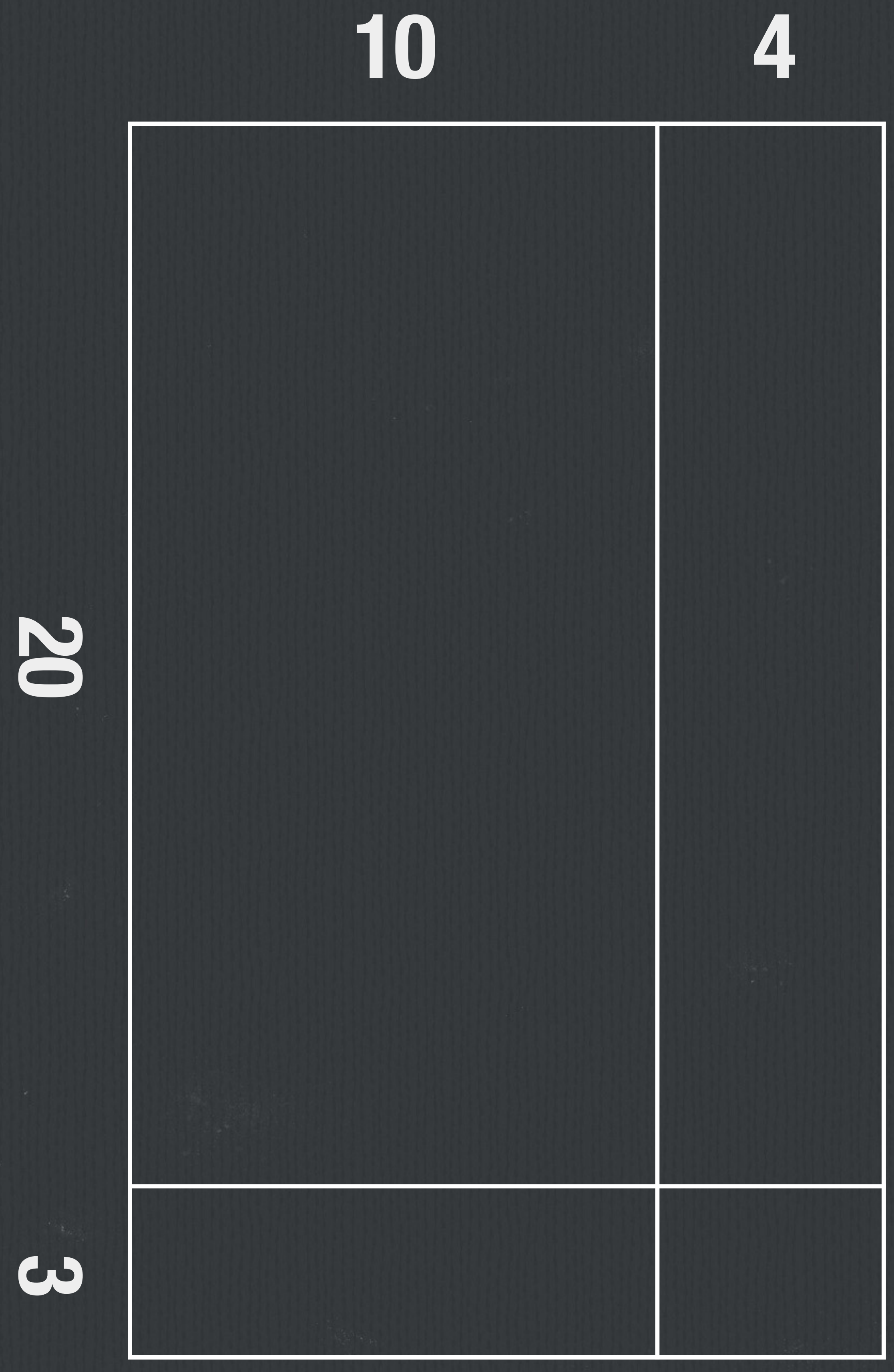
2x + 3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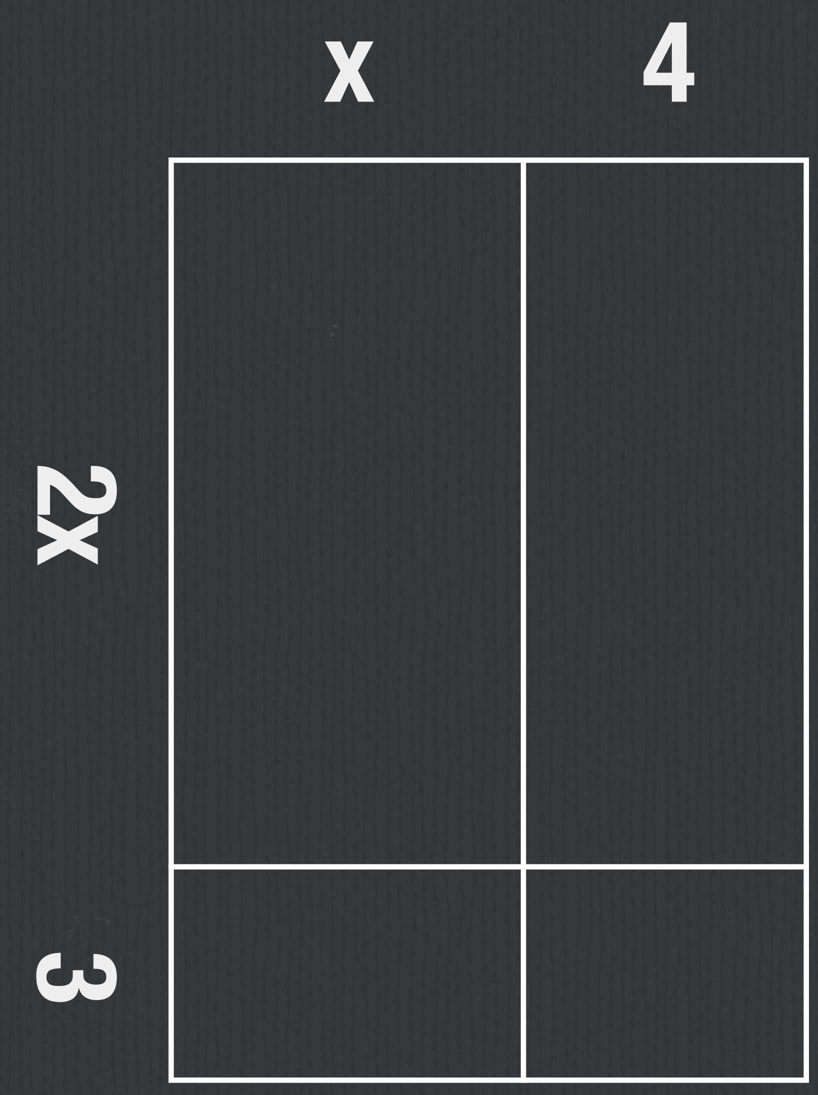
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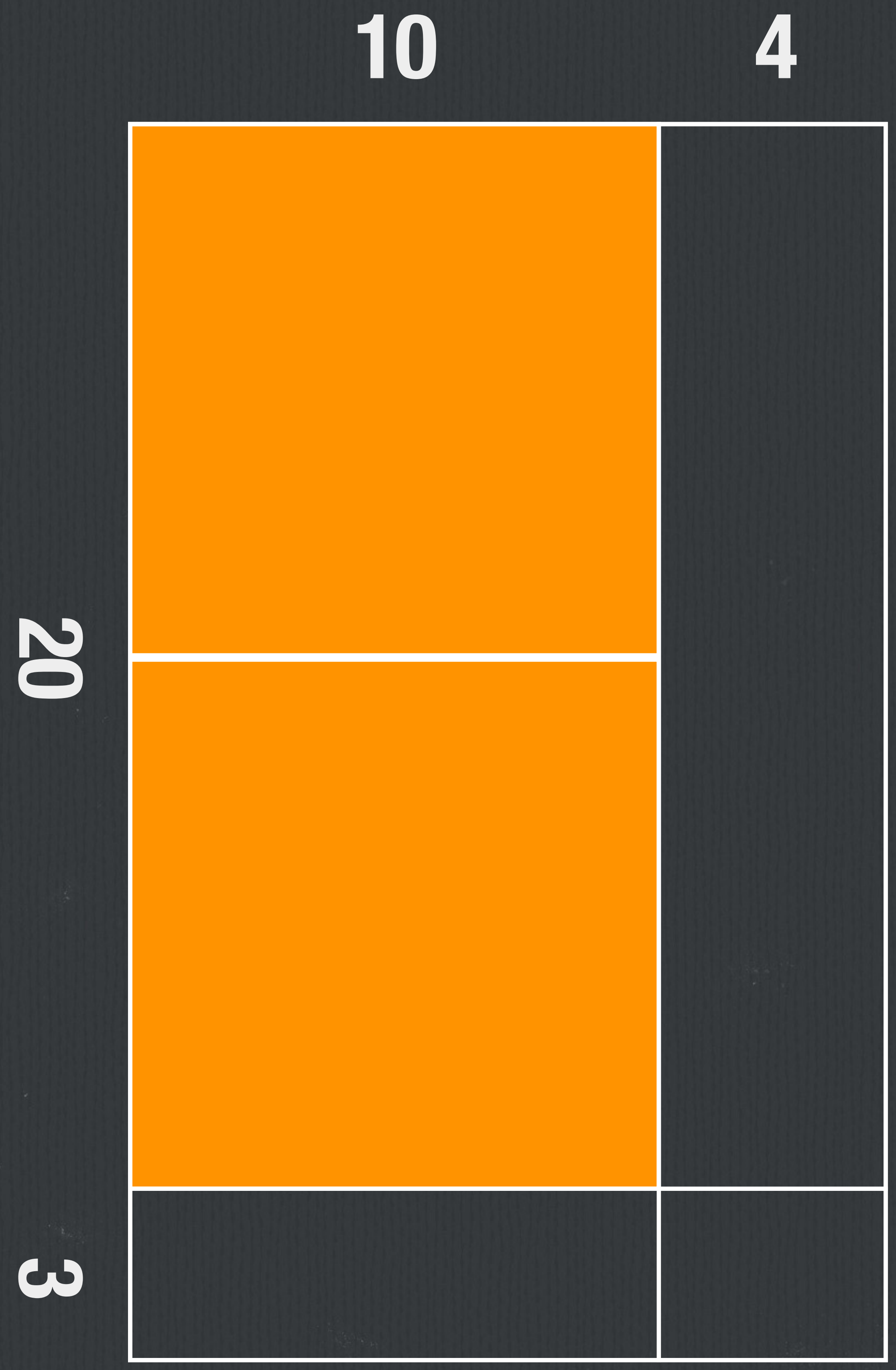
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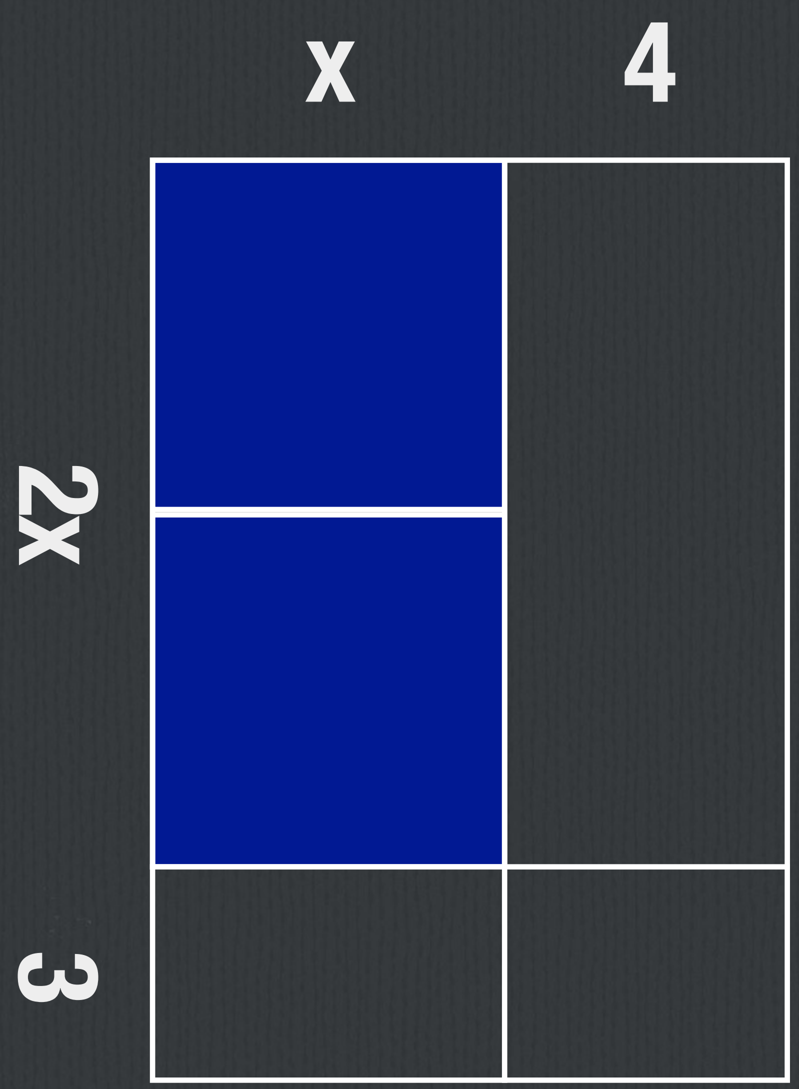
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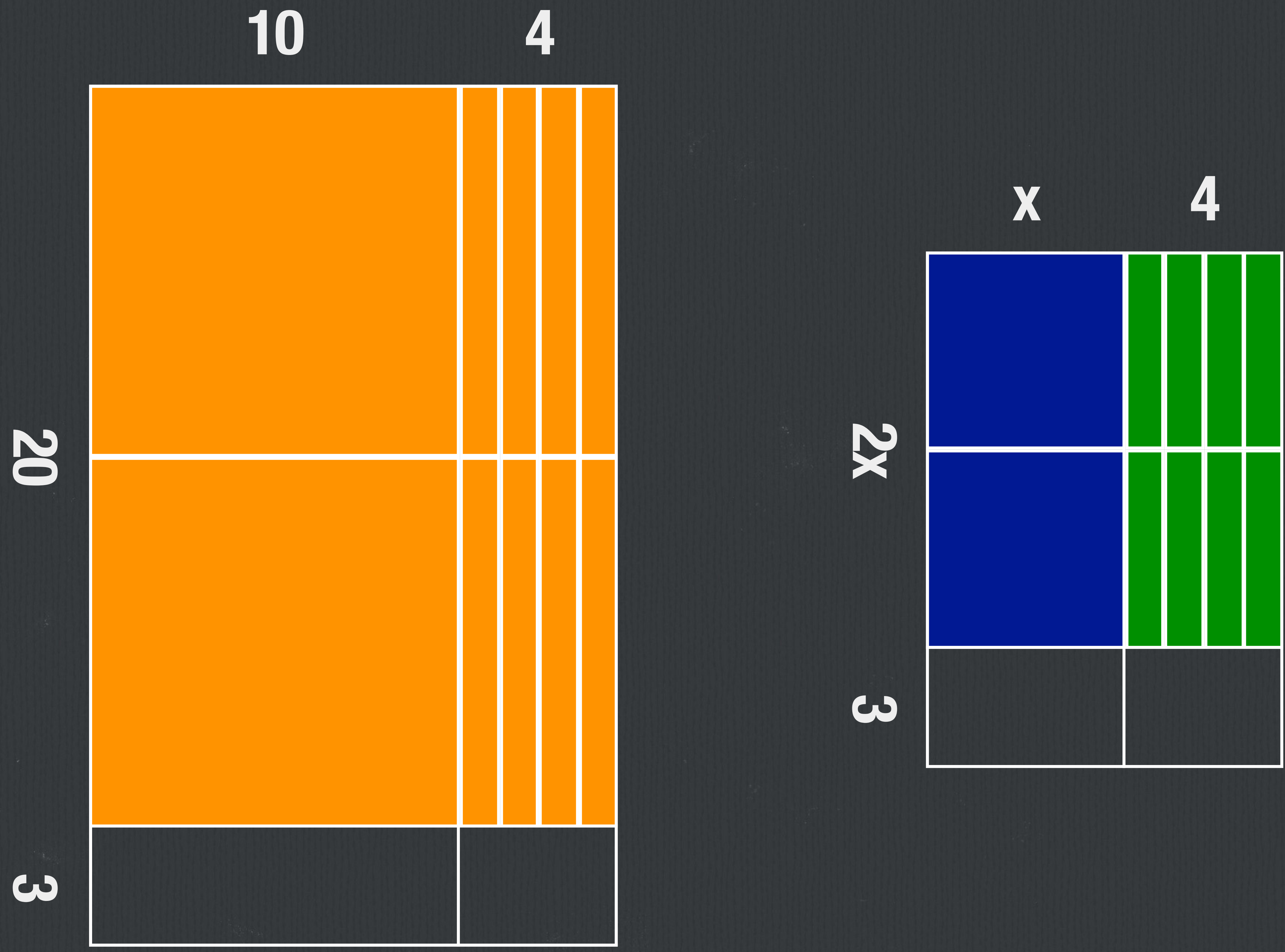
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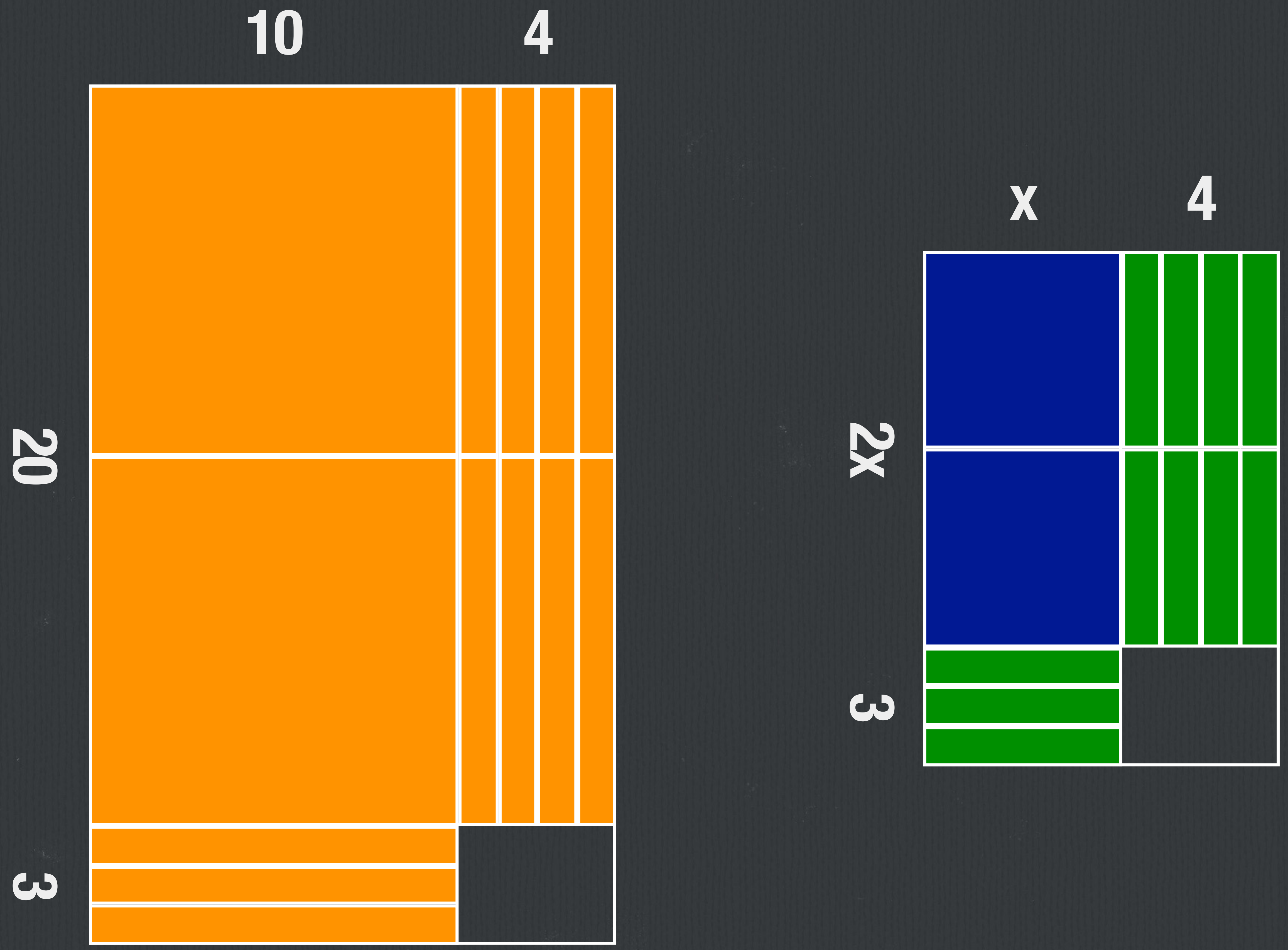
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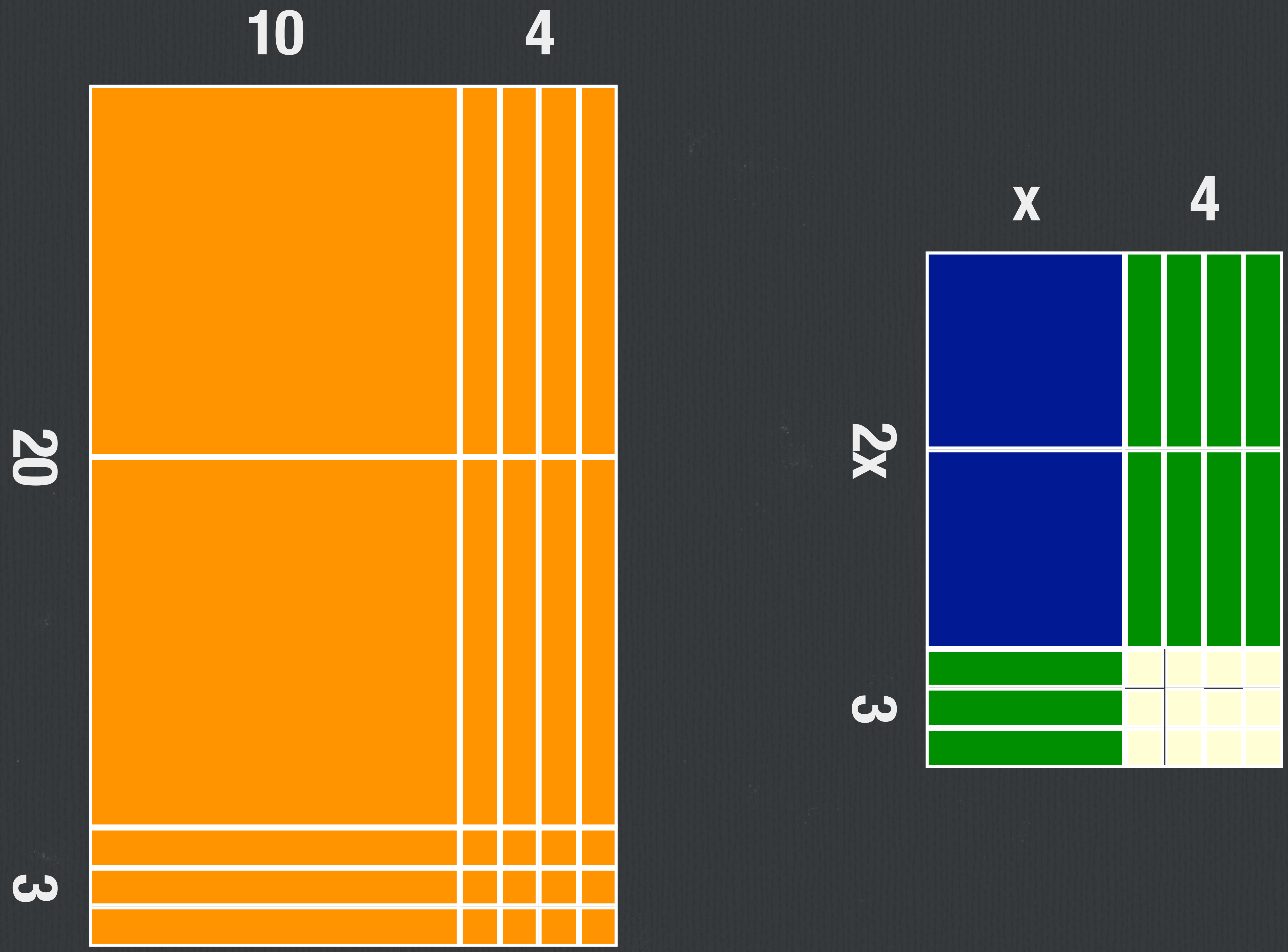
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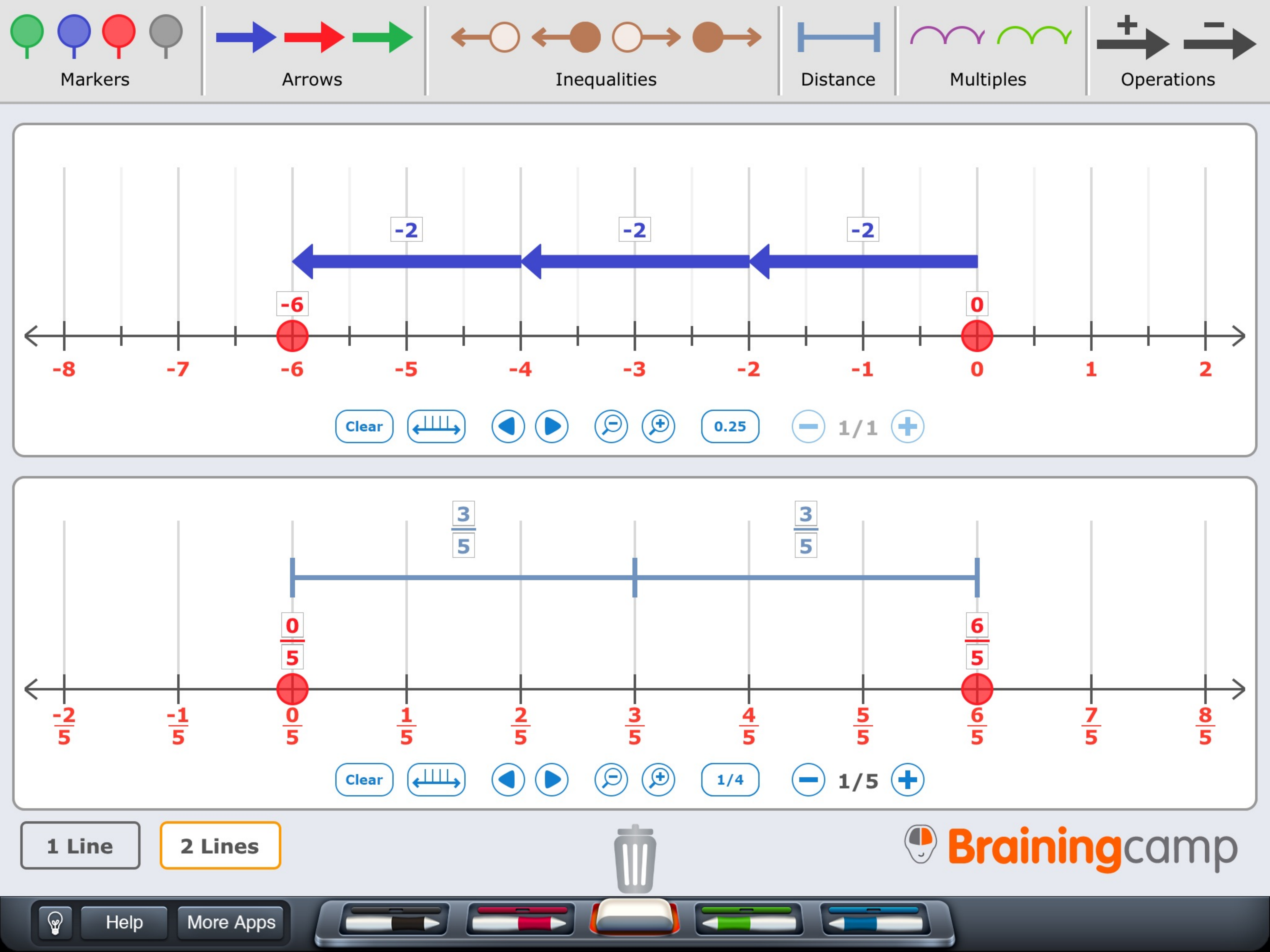
Markers
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Inequalities
Distance
Multiples
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6/5
-2/5
-1/5
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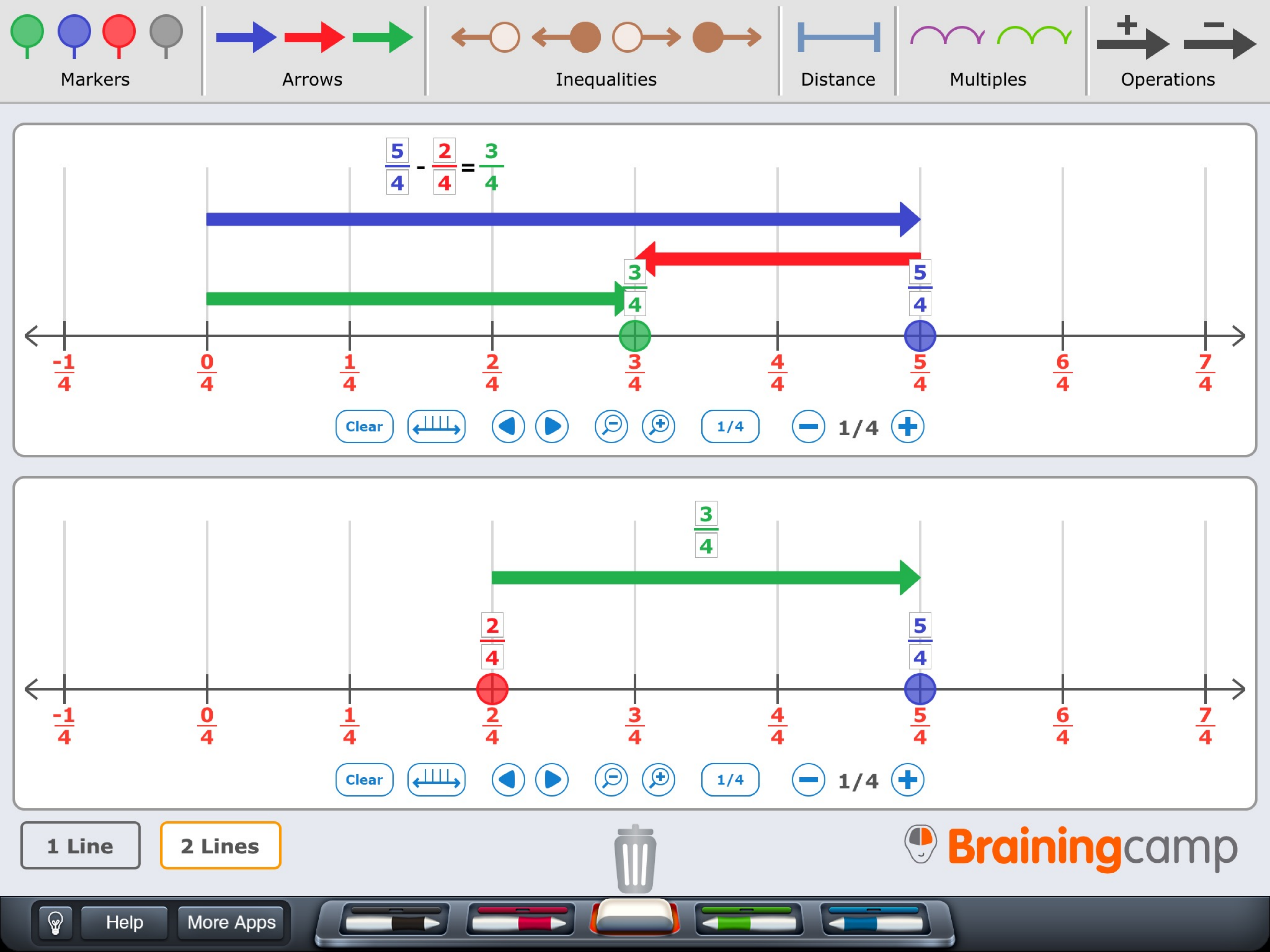
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Clear
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1 Line
2 Lines
Brainingcamp
Help
More Apps

Which meaning is more meaningful?

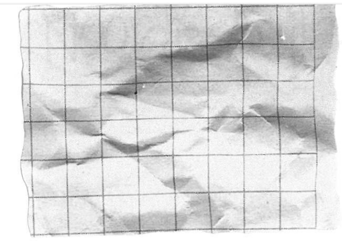
Simplify $(1.89t+15)-(1.49t+12)$, where t represents the number of pizza toppings

Determine $(F_2-F_1)(C)$, where $F_1(C)=\frac{9}{5}C+32$ and $F_2(C)=2C+30$

Solve: $|x-5|=2$

2. Proportional reasoning involves the use of multiplicative relationships to solve problems.

March 27th, 2015



Sharing Pairs

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The cashier rings them up; the bill is $135.

How much should each friend pay? Try to find the fairest way possible. Justify your reasoning.

Ex.1 $\frac{\$135}{3}$ = $45 per person

Ex.2 CHRIS 60 + JEFF 45 + MARC 30 = $135 $15 difference

Ex.3 55 (73.3%) + 45 (75%) + 35 (77.7%) = $135 $10 difference

FINAL ANSWER : Ex.4 56.45 (75.27%) + 45 (75%) + 33.55 (74.56%) = $135 ← 75% paid per person from the original price

Chris

Jeff

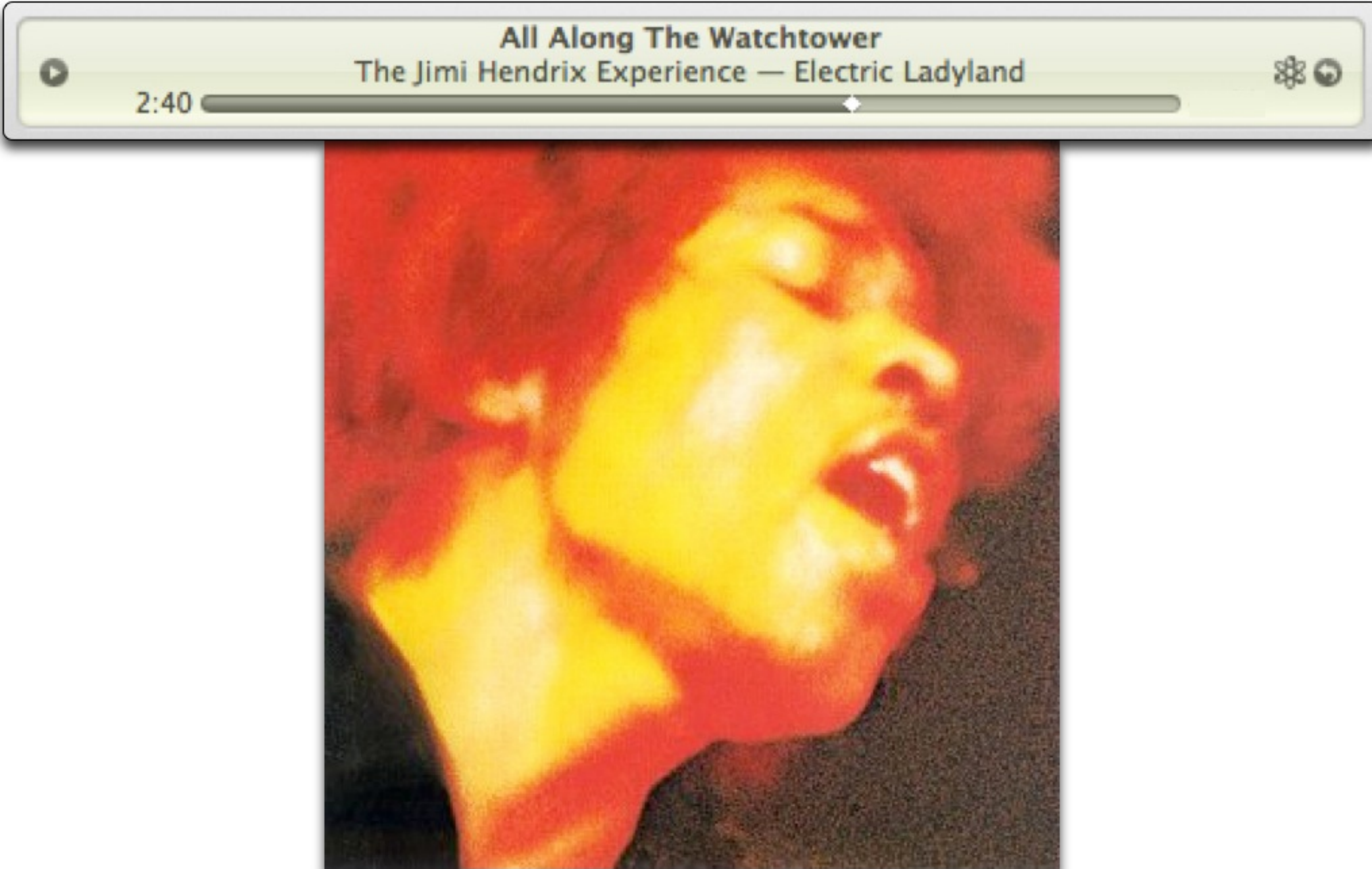
Marc

$150

$90

$60


ESTIMATION180.COM
19' 3"
5.87 M
6' 4"
1.93 M



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#25 GRAPE JUICE JUNGLE

Figure This! Math Challenges for Families

Which tastes JUICIER?

Figure This! If all grape juice concentrates are the same strength, which recipe would you expect to have the strongest grape taste?

Hint: For each recipe think about how much water should be used with 1 cup (c.) of concentrate, or how much concentrate should be used with 1 cup of water.

Ratios are fractions that compare two or more quantities. Shoppers use ratios to compare prices; cooks use them to adjust recipes. Architects and designers use ratios to create scale drawings.