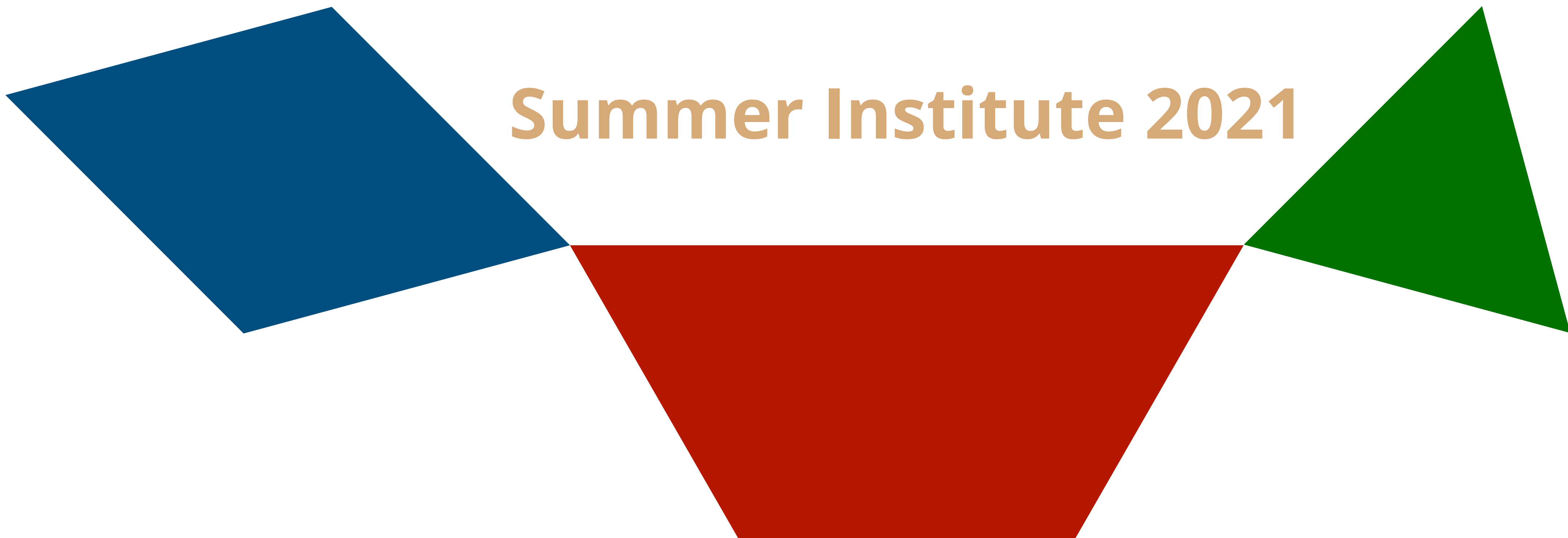


Opening Up Math Class

Summer Institute 2021



Chris Hunter

K-12 Numeracy Helping Teacher

email: hunter_c@surreyschools.ca

Twitter: [@ChrisHunter36](https://twitter.com/ChrisHunter36)

blog: chrishunter.ca

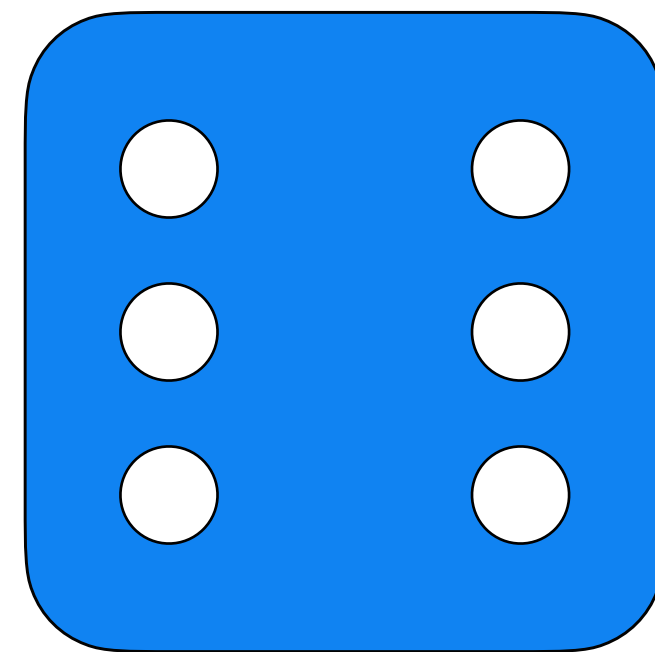
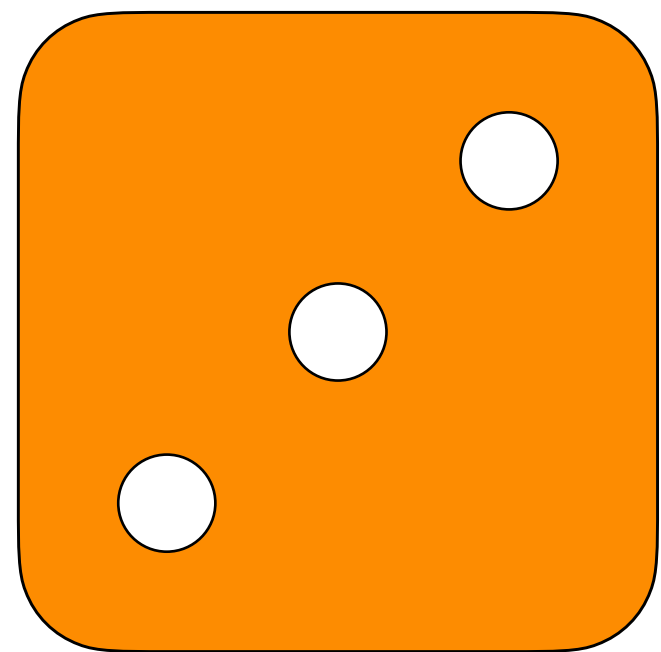




Frogs by Susan Point, descendant of the Musqueam people

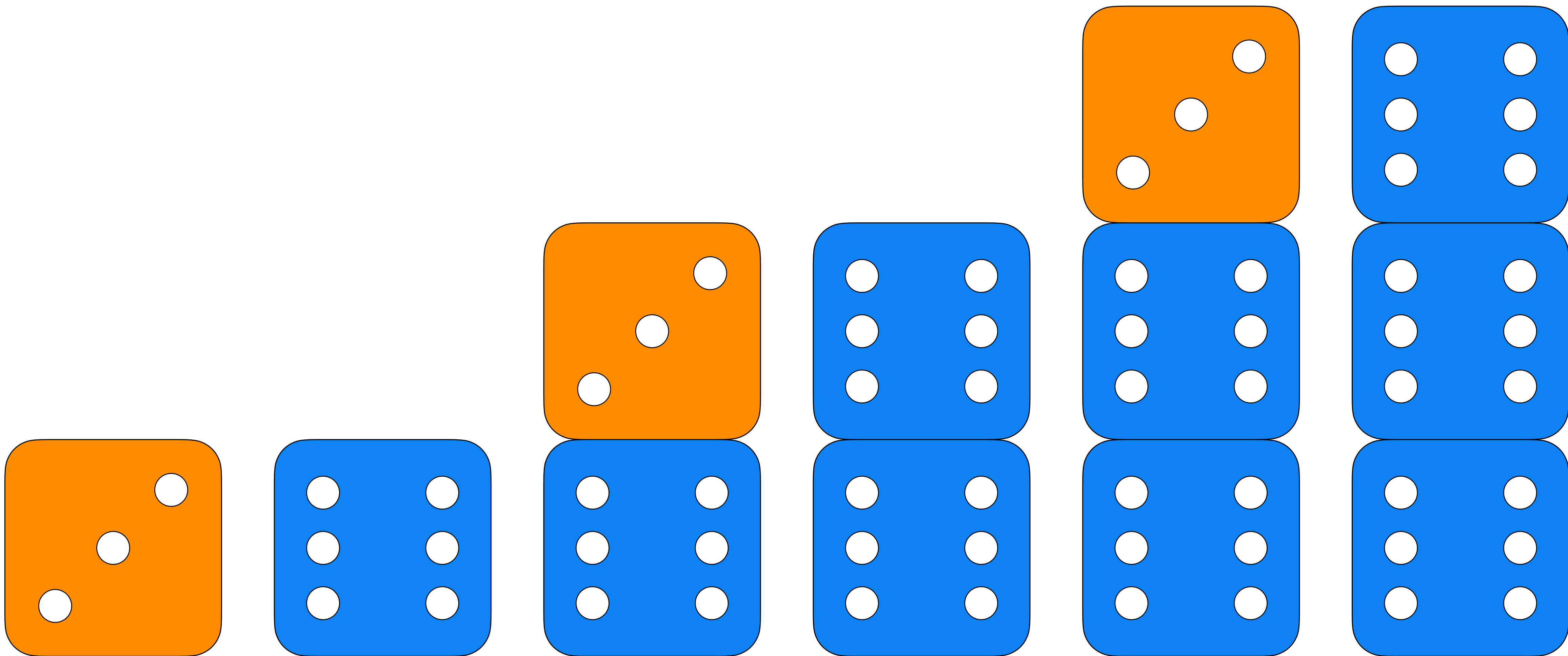
I acknowledge that I work and learn on the unceded shared territories of the Coast Salish.

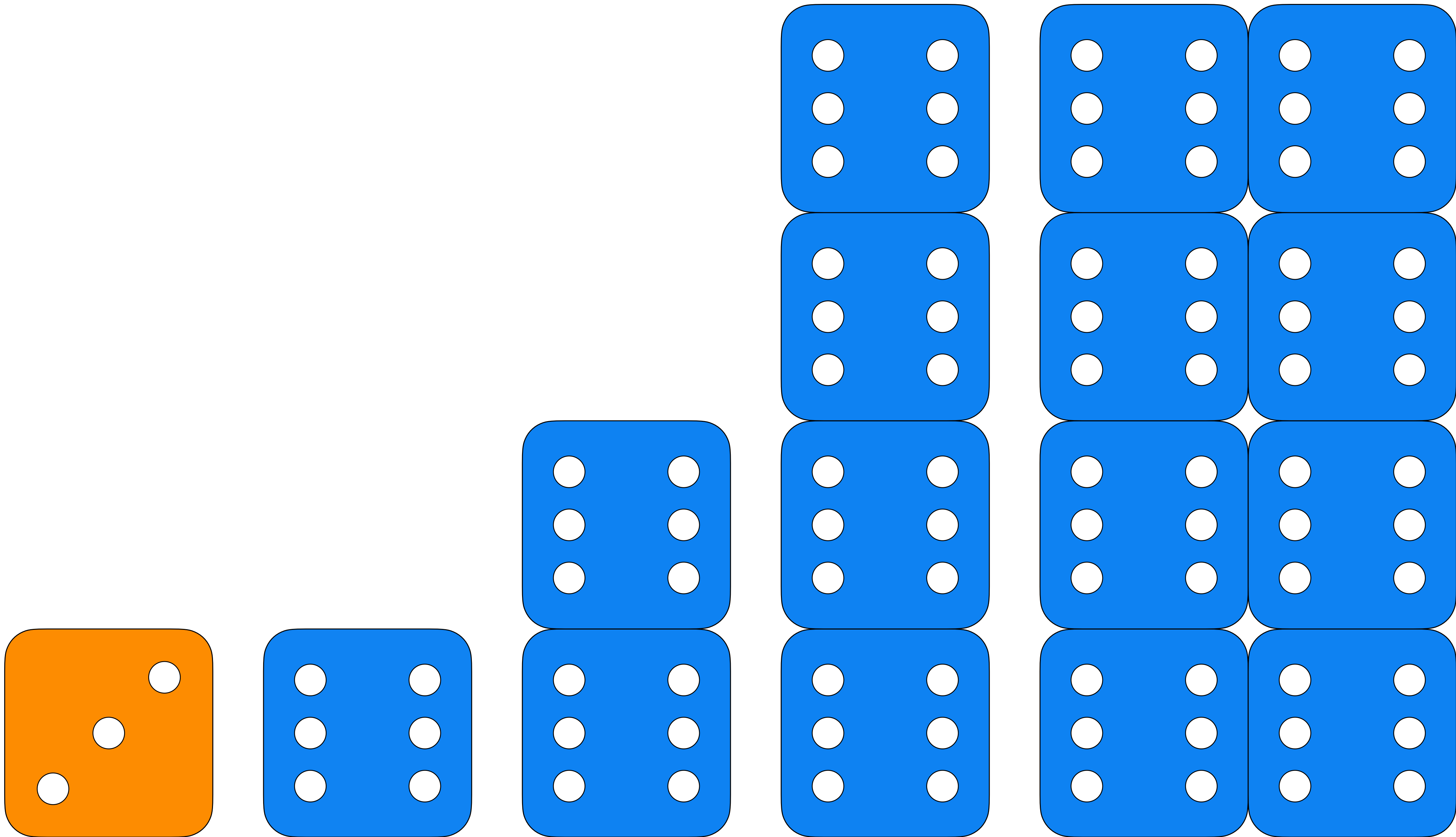
I acknowledge the Katzie, Semiahmoo, and Kwantlen First Nations
who have been stewards of this land since time immemorial.

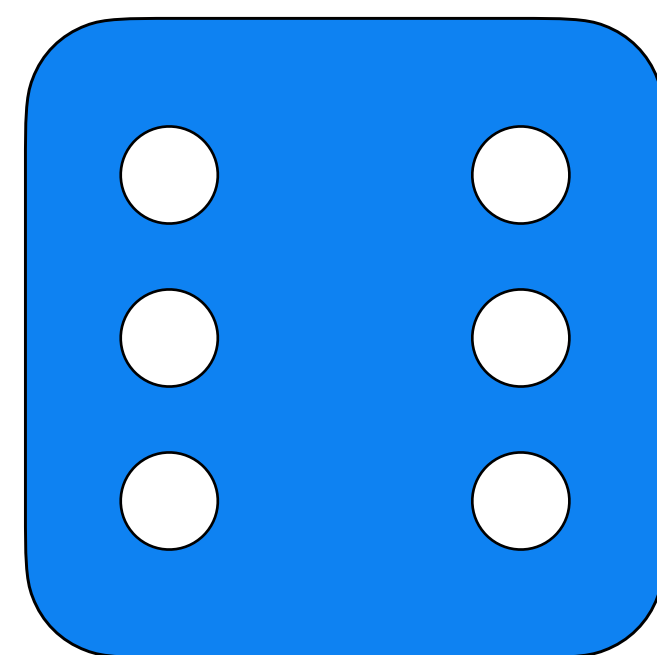
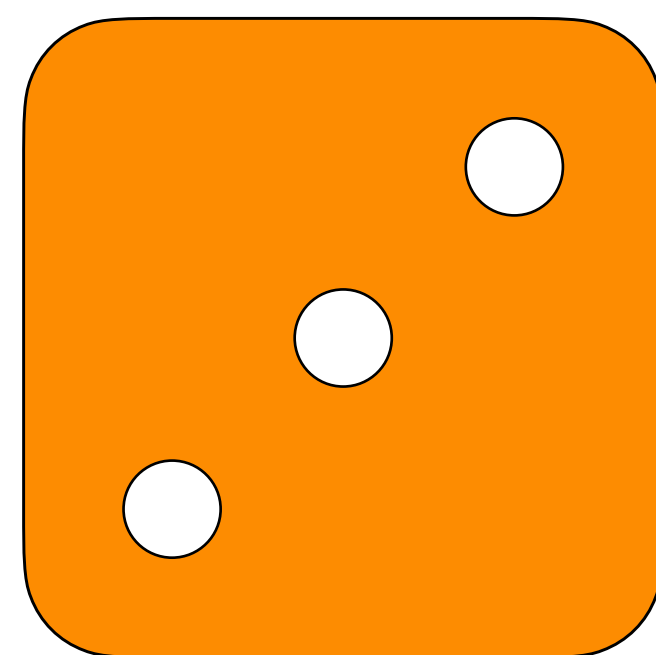
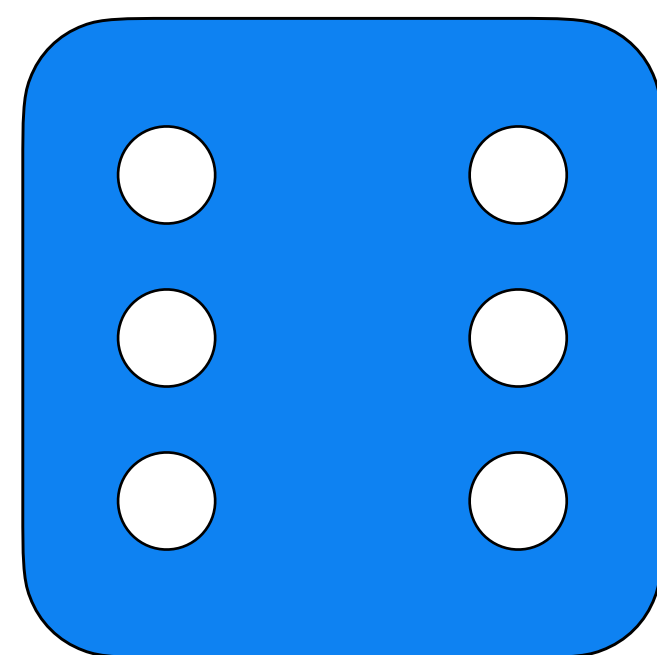
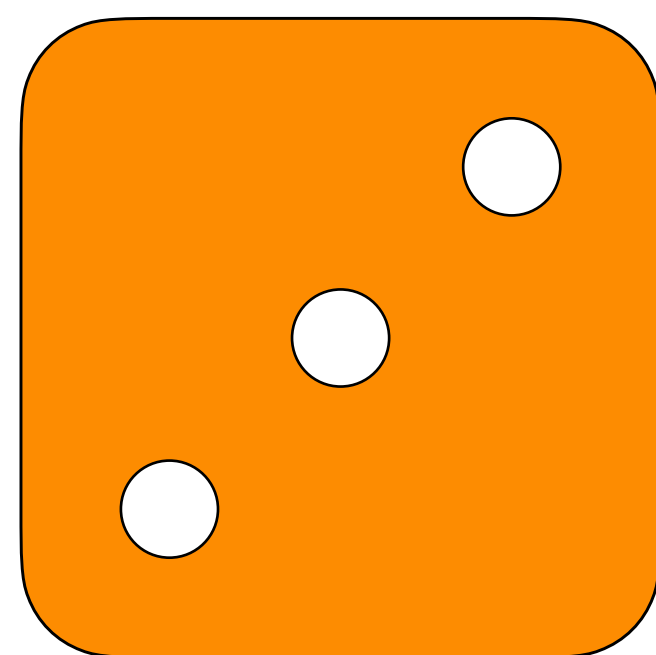
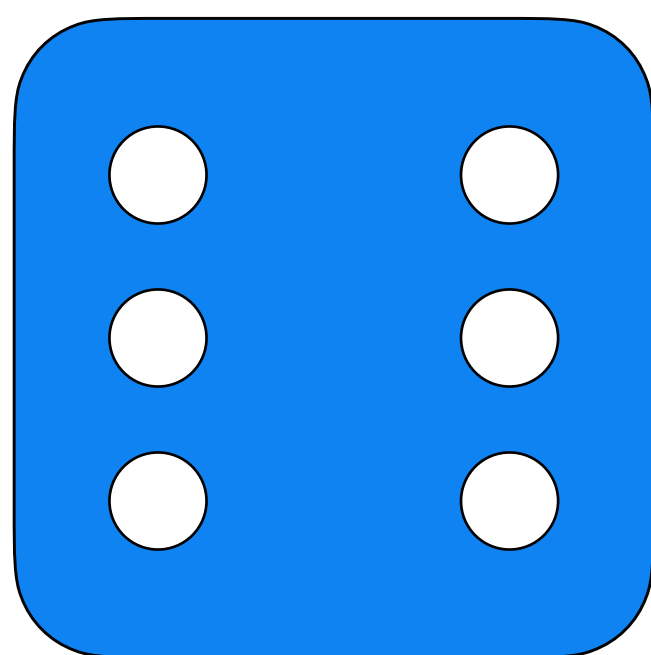
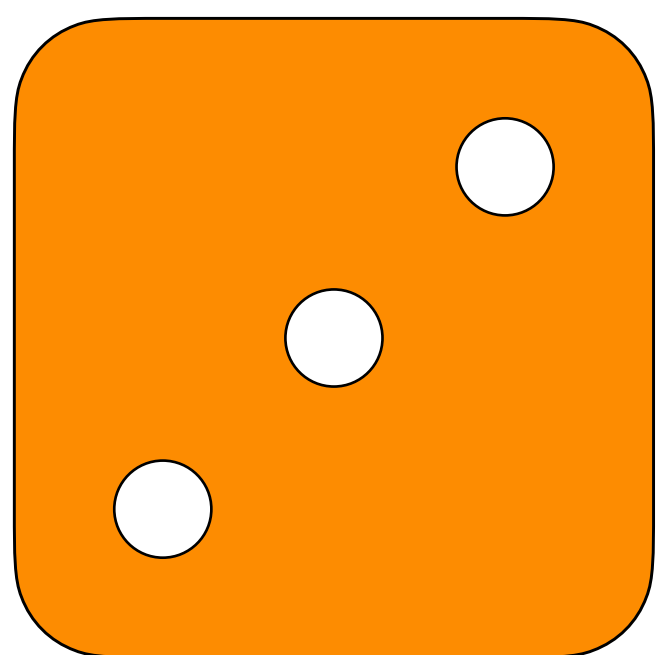


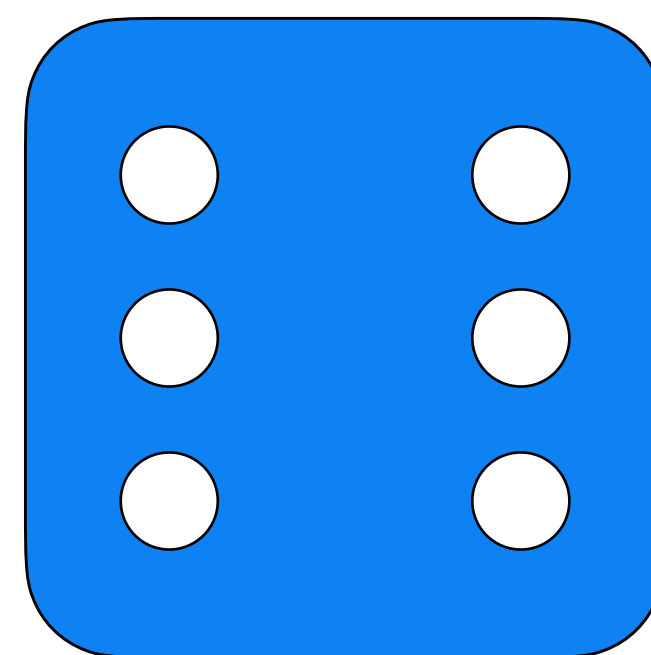
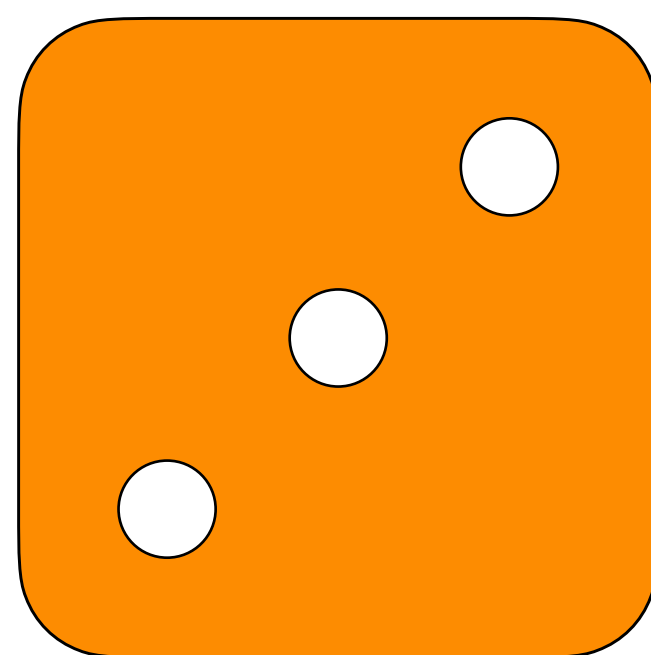
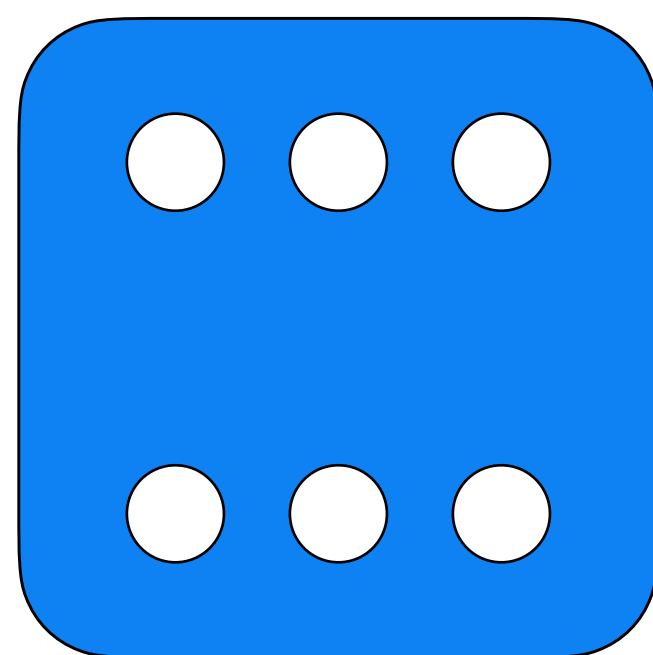
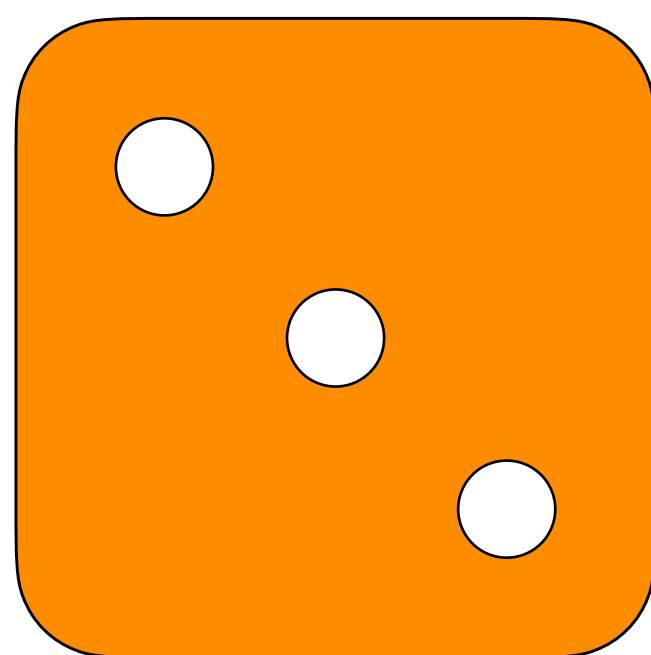
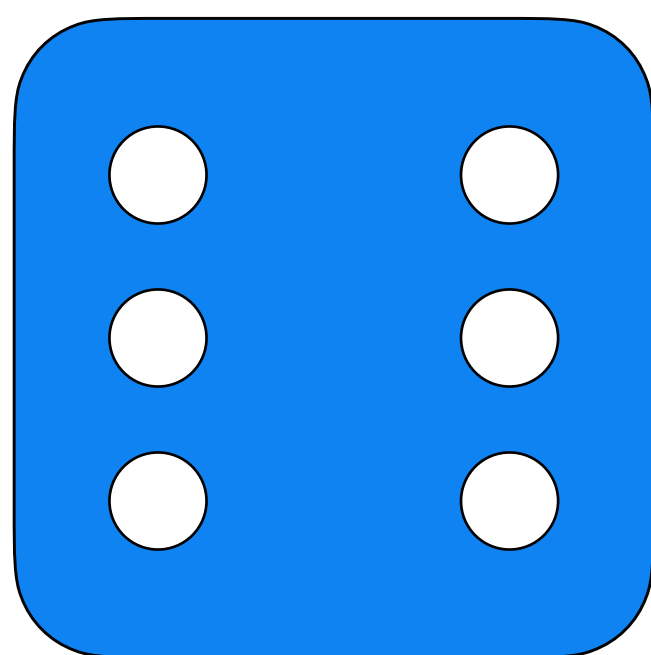
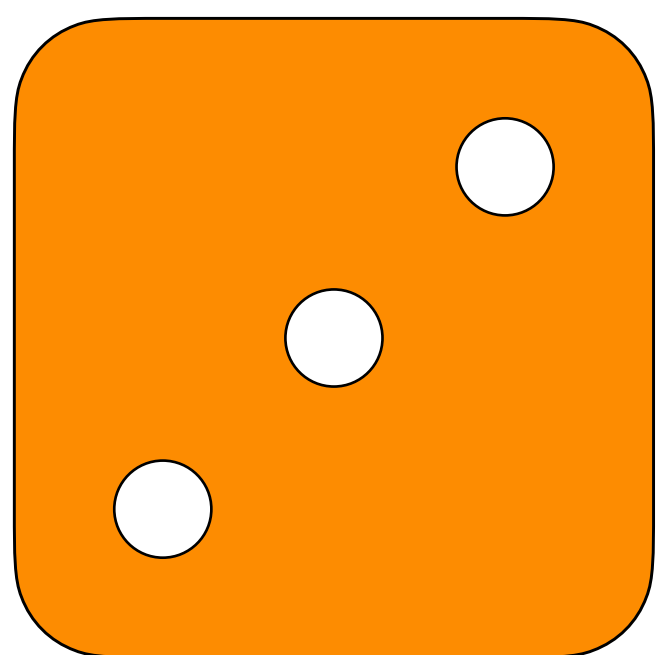
**what
comes
next?**

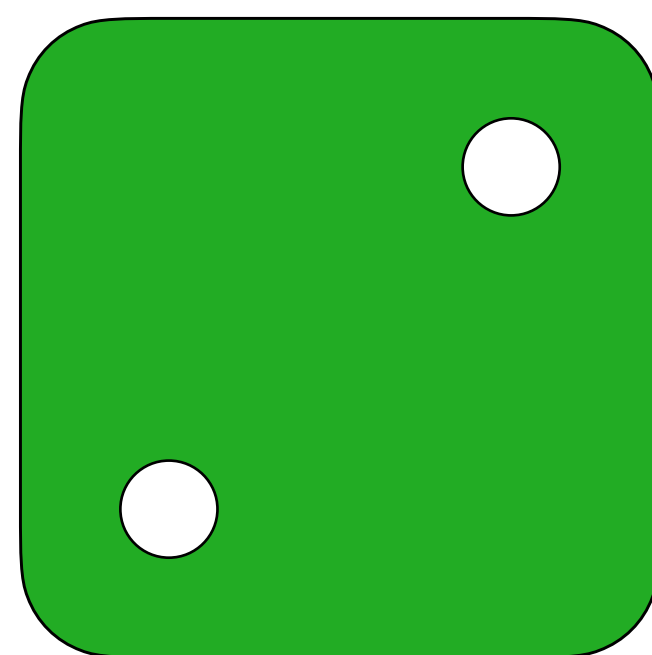
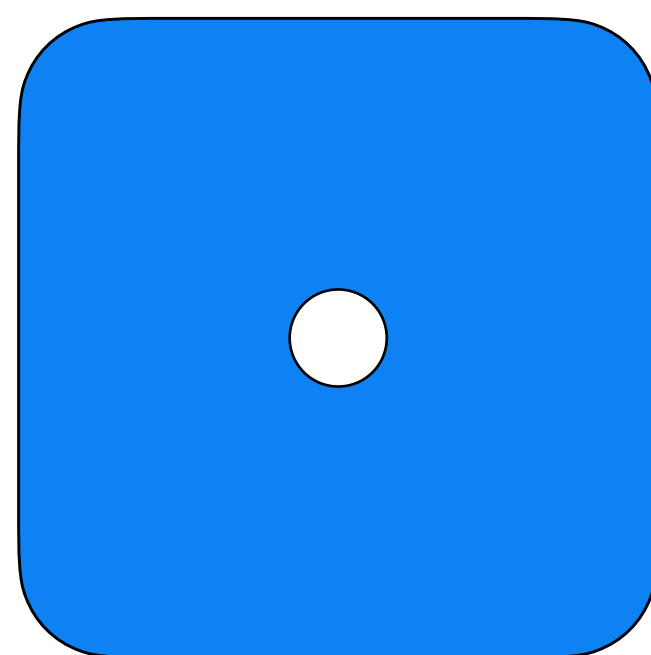
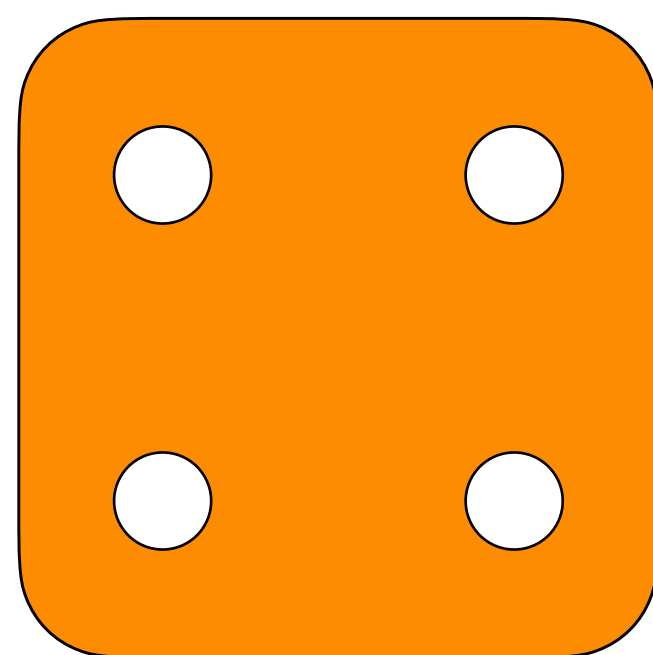
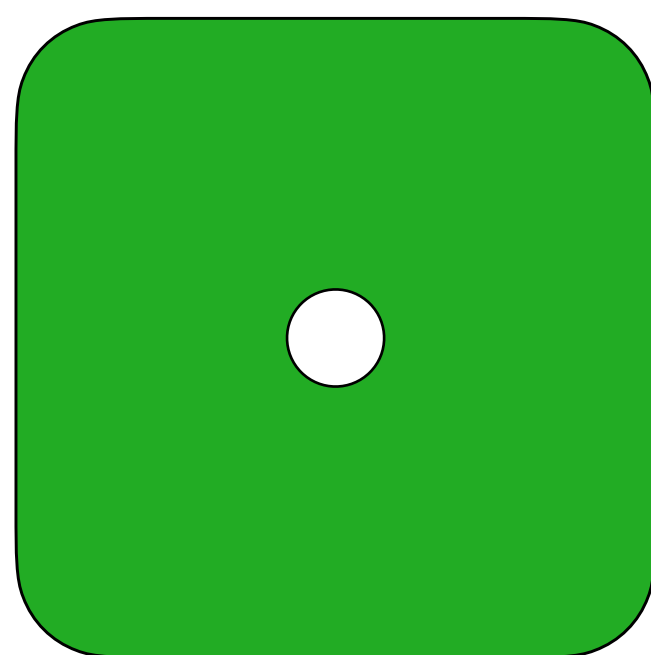
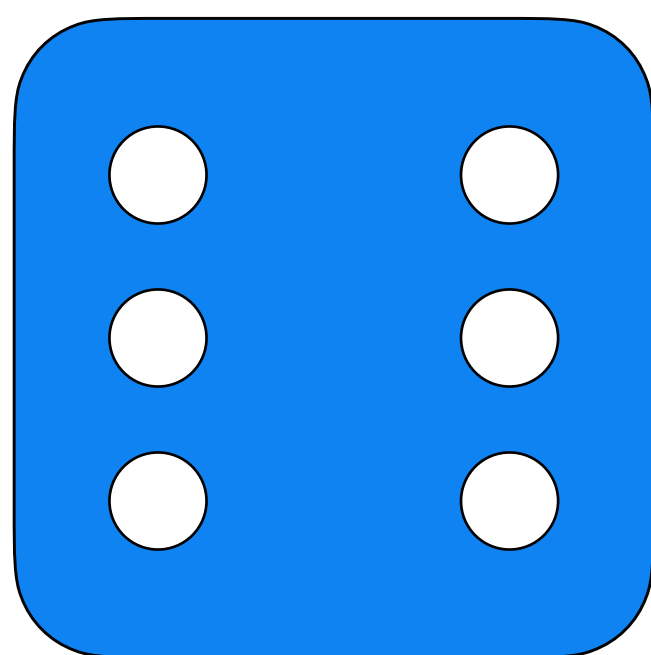
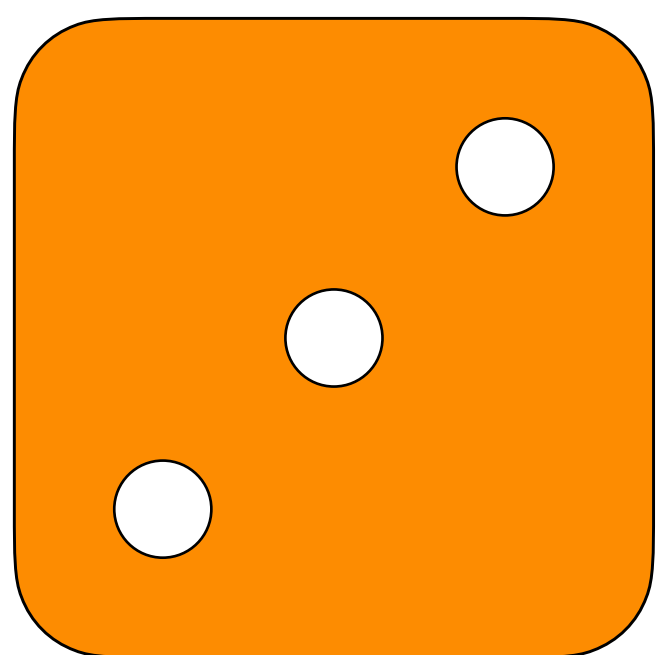
Extend the pattern in as many ways as you can.











Equity: Principles

Guiding Principles for School Mathematics	
Teaching and Learning.	An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically.
Access and Equity.	An excellent mathematics program requires that all students have access to a high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential.
Curriculum.	An excellent mathematics program includes a curriculum that develops important mathematics along coherent learning progressions and develops connections among areas of mathematical study and between mathematics and the real world.
Tools and Technology.	An excellent mathematics program integrates the use of mathematical tools and technology as essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking.
Assessment.	An excellent mathematics program ensures that assessment is an integral part of instruction, provides evidence of proficiency with important mathematics content and practices, includes a variety of strategies and data sources, and informs feedback to students, instructional decisions, and program improvement.
Professionalism.	In an excellent mathematics program, educators hold themselves and their colleagues accountable for the mathematical success of every student and for their personal and collective professional growth toward effective teaching and learning of mathematics.

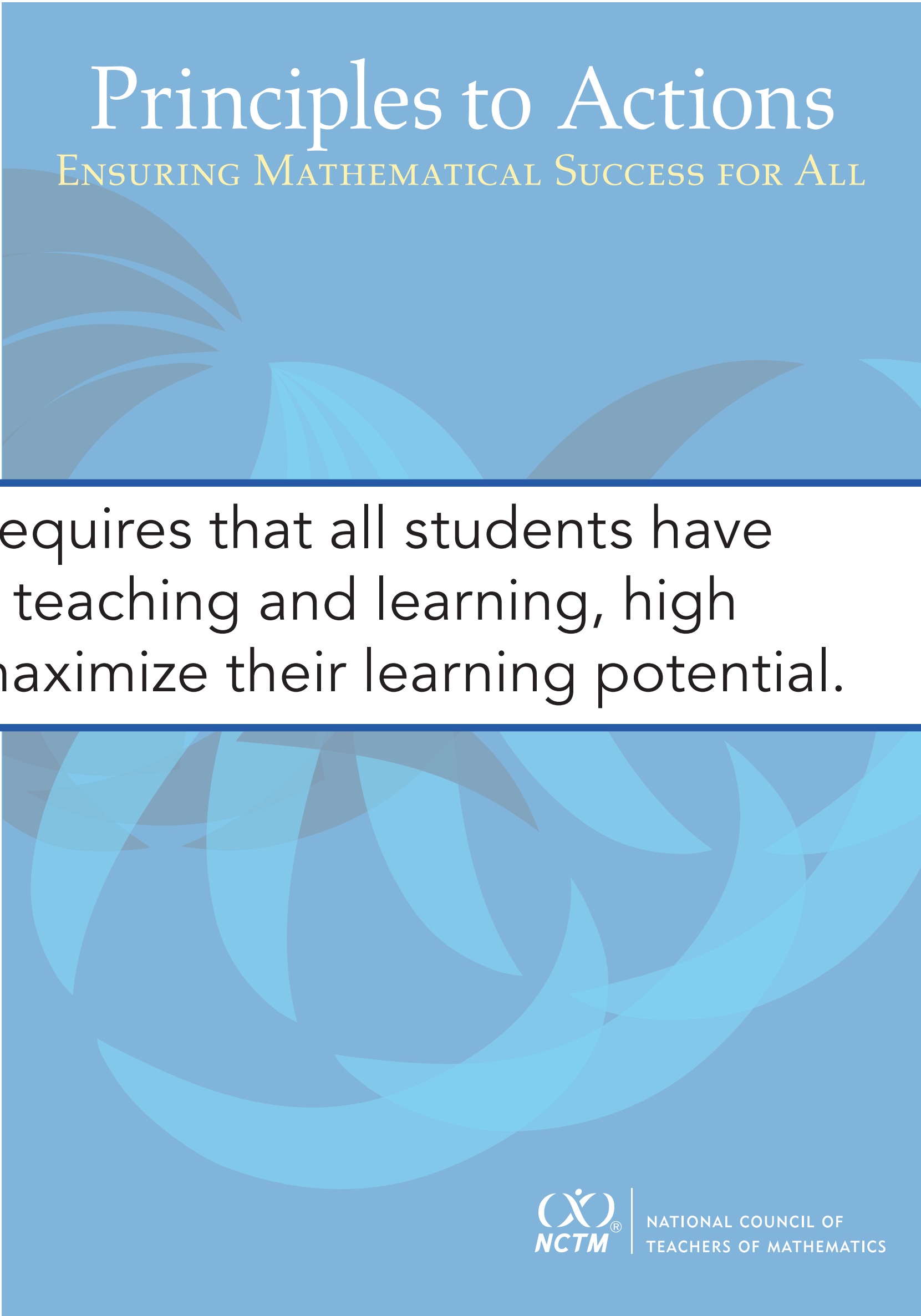


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Principles to Actions

ENSURING MATHEMATICAL SUCCESS FOR ALL

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Equity: Practices

Effective Mathematics Teaching Practices

- ▶ Establish mathematics goals to focus learning
- ▶ Implement tasks that promote reasoning and problem solving
- ▶ Use and connect mathematical representations
- ▶ Facilitate meaningful discourse
- ▶ Pose purposeful questions
- ▶ Build procedural fluency from conceptual understanding
- ▶ Support productive struggle in learning mathematics
- ▶ Elicit and use evidence of student thinking

(NCTM 2014)



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Equity-Based Mathematics Teaching Practices

- ▶ Go deep with mathematics
- ▶ Leverage multiple mathematical competencies
- ▶ Affirm mathematics learners' identities
- ▶ Challenge spaces of marginality
- ▶ Draw on multiple resources of knowledge

The Impact of Identity in K-8 Mathematics
(Aguirre, Mayfield-Ingraham, and Martin 2013)

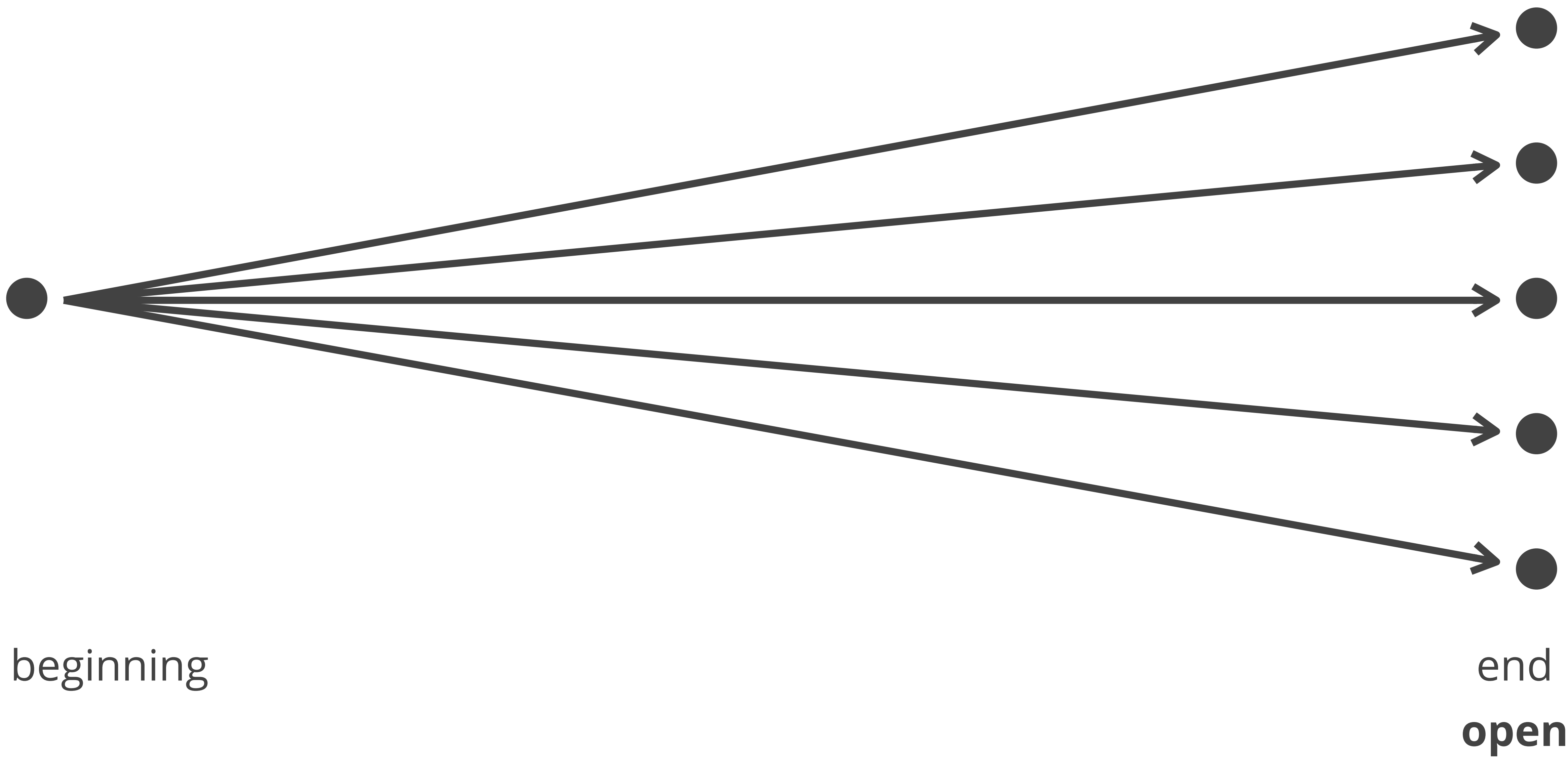
Video Games and Making Math More Like Things Students Like (Meyer 2014)

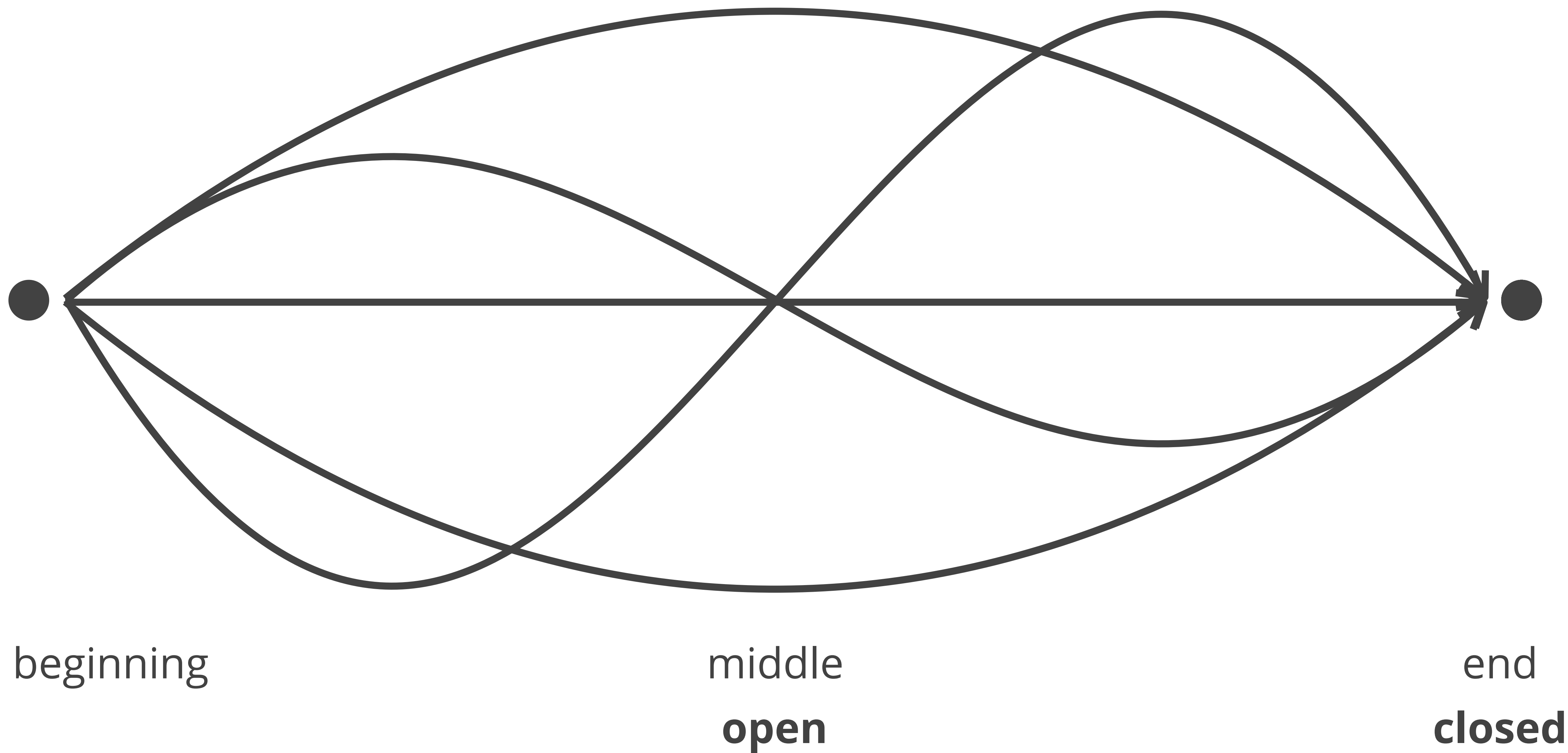


beginning

end
closed

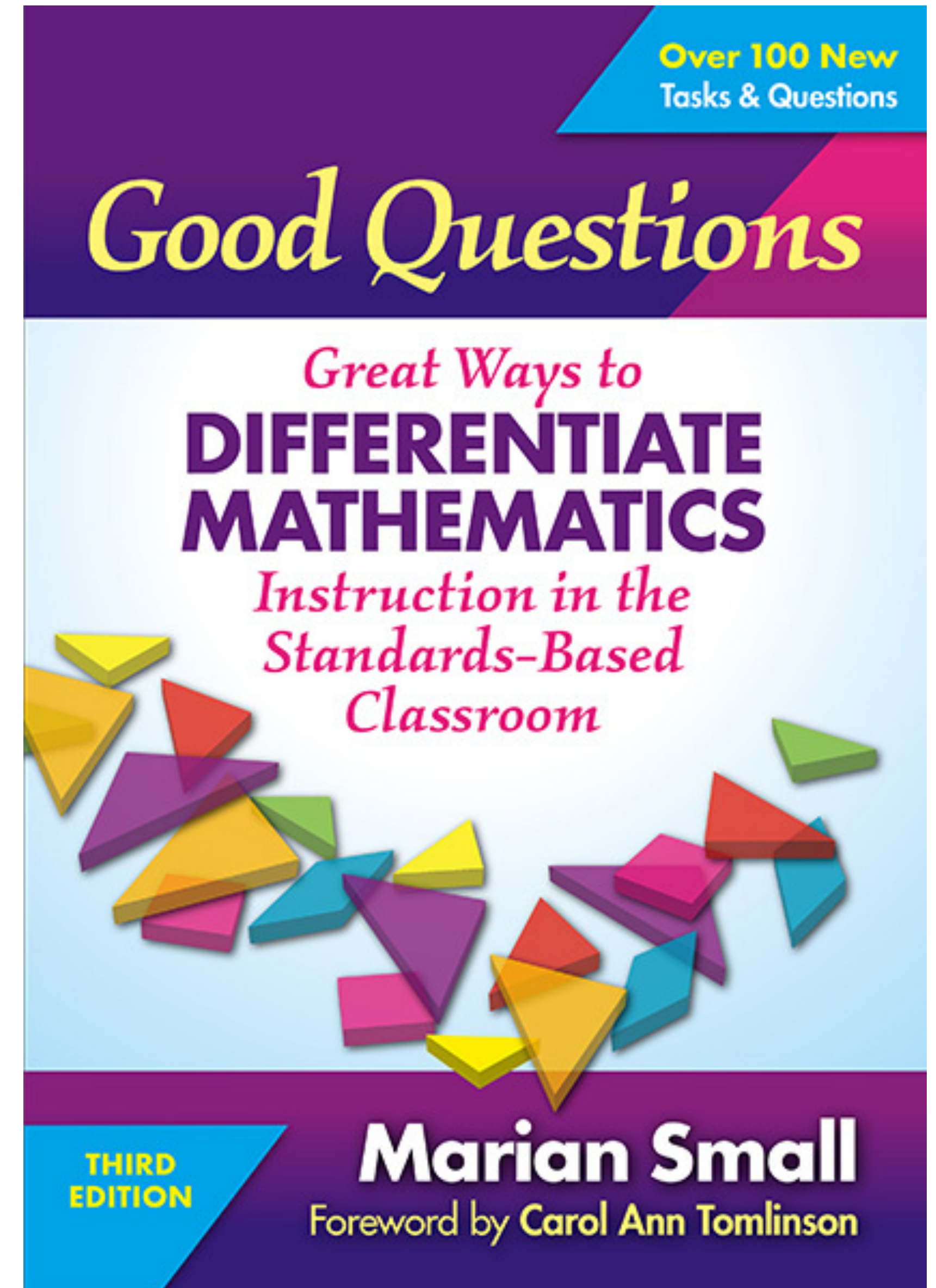
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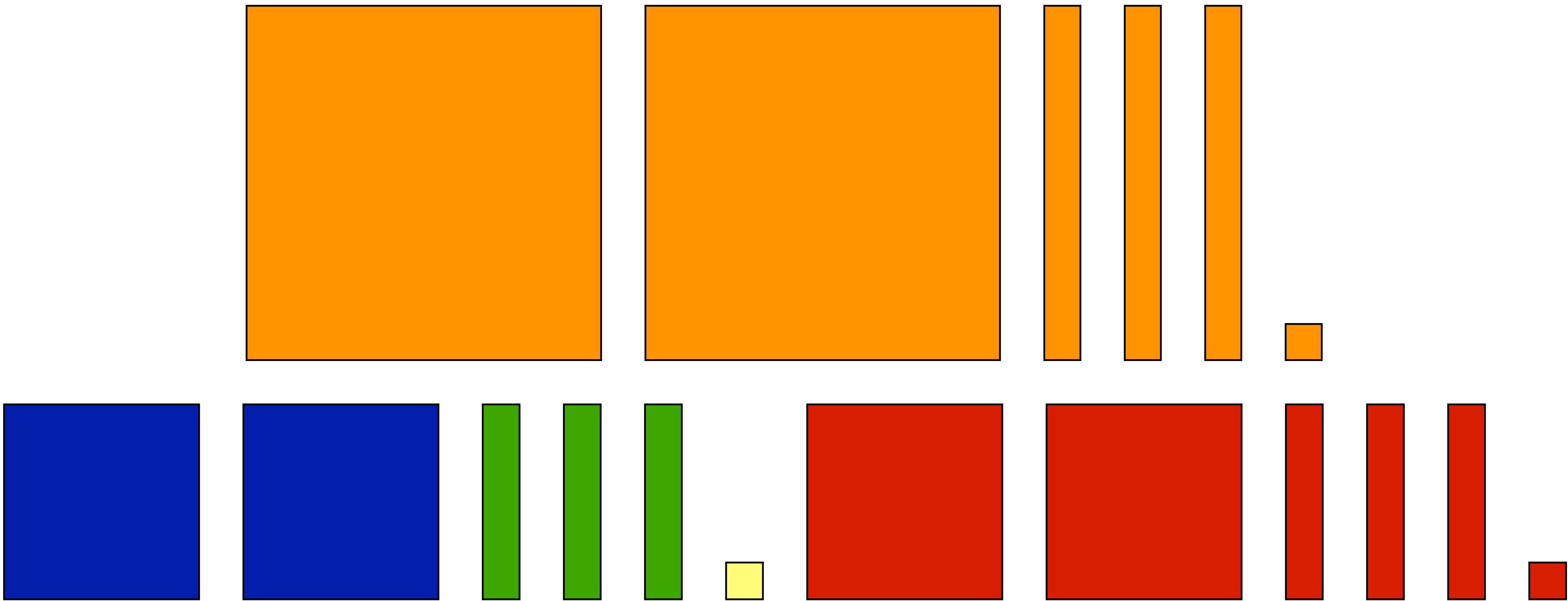


What are Open Questions?

A question is **open** when it is framed in such a way that a **variety** of **responses** or **approaches** are possible.



What **numbers** can you represent using thirteen **base ten blocks**?



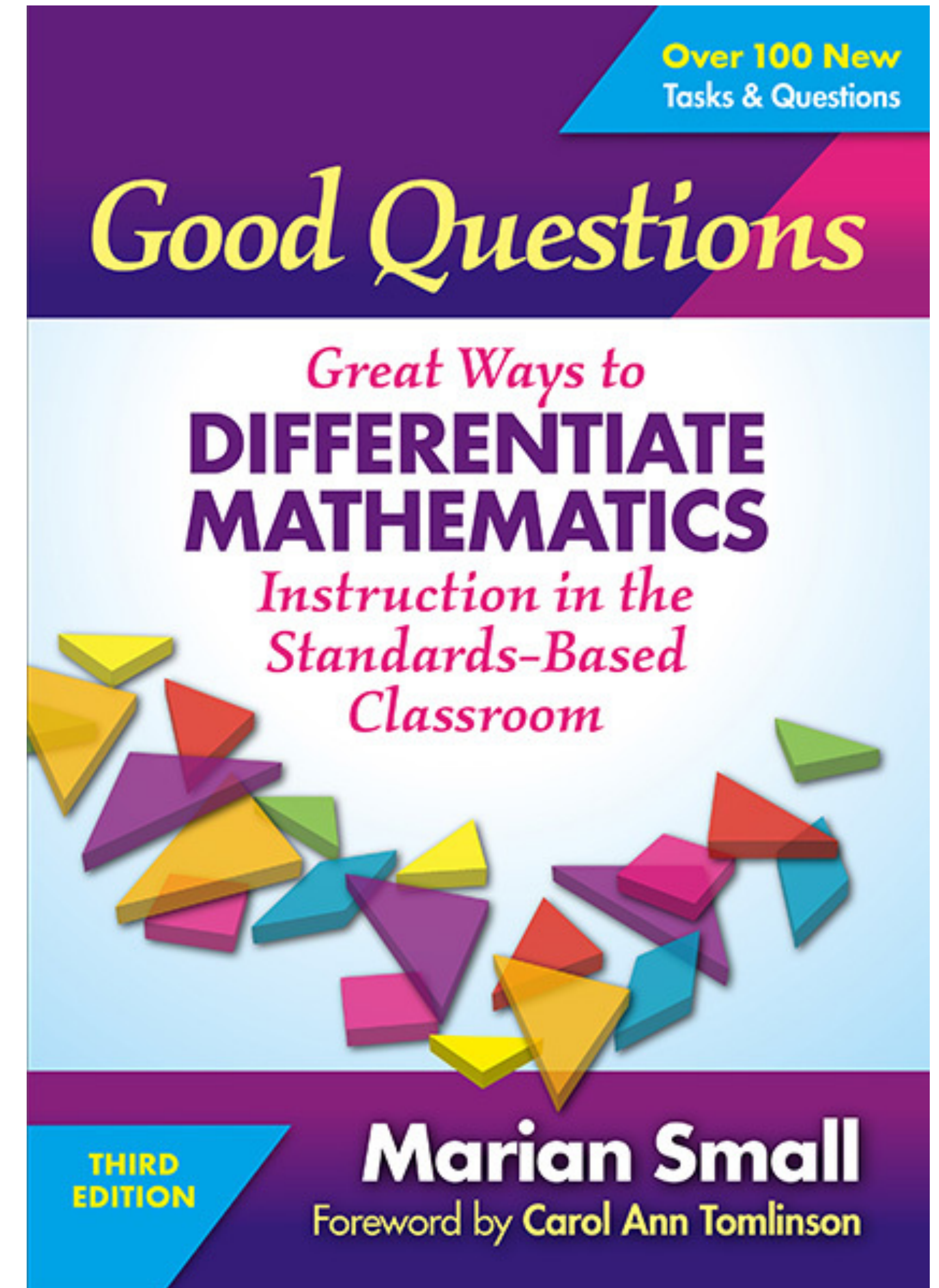
What **expressions** can you represent using thirteen **algebra tiles**?

Why Should I Ask Open Questions?

inclusive

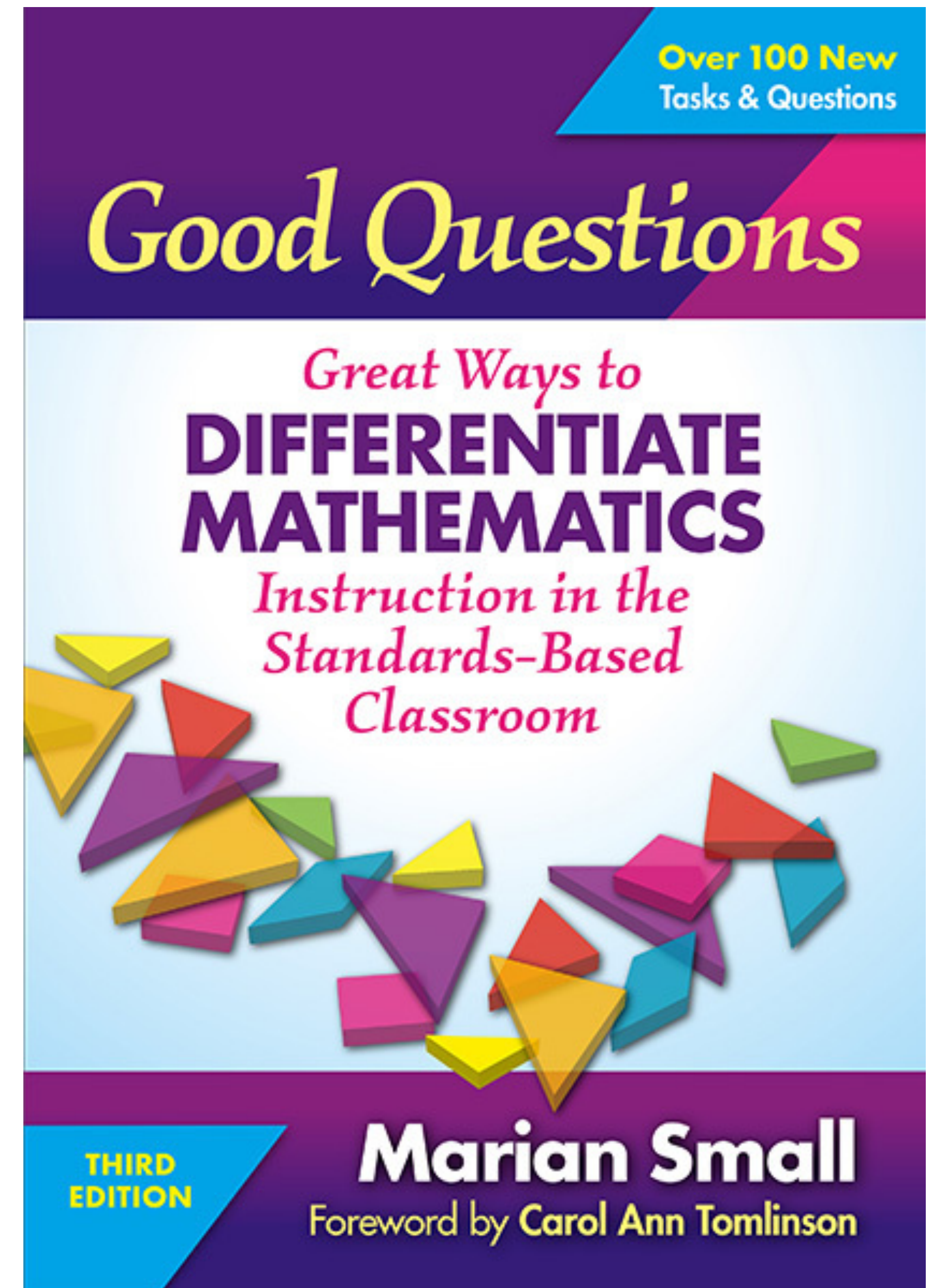
- ▶ allows for **different students** to approach a **single question** or **task** by using **different processes** or **strategies**
- ▶ allows for **all students** to **benefit** and **grow** from attention to the **task**
- ▶ allows for **all students** to **participate** fully and **gain** from the **classroom discussion**

another benefit...



Why Should I Ask Open Questions?

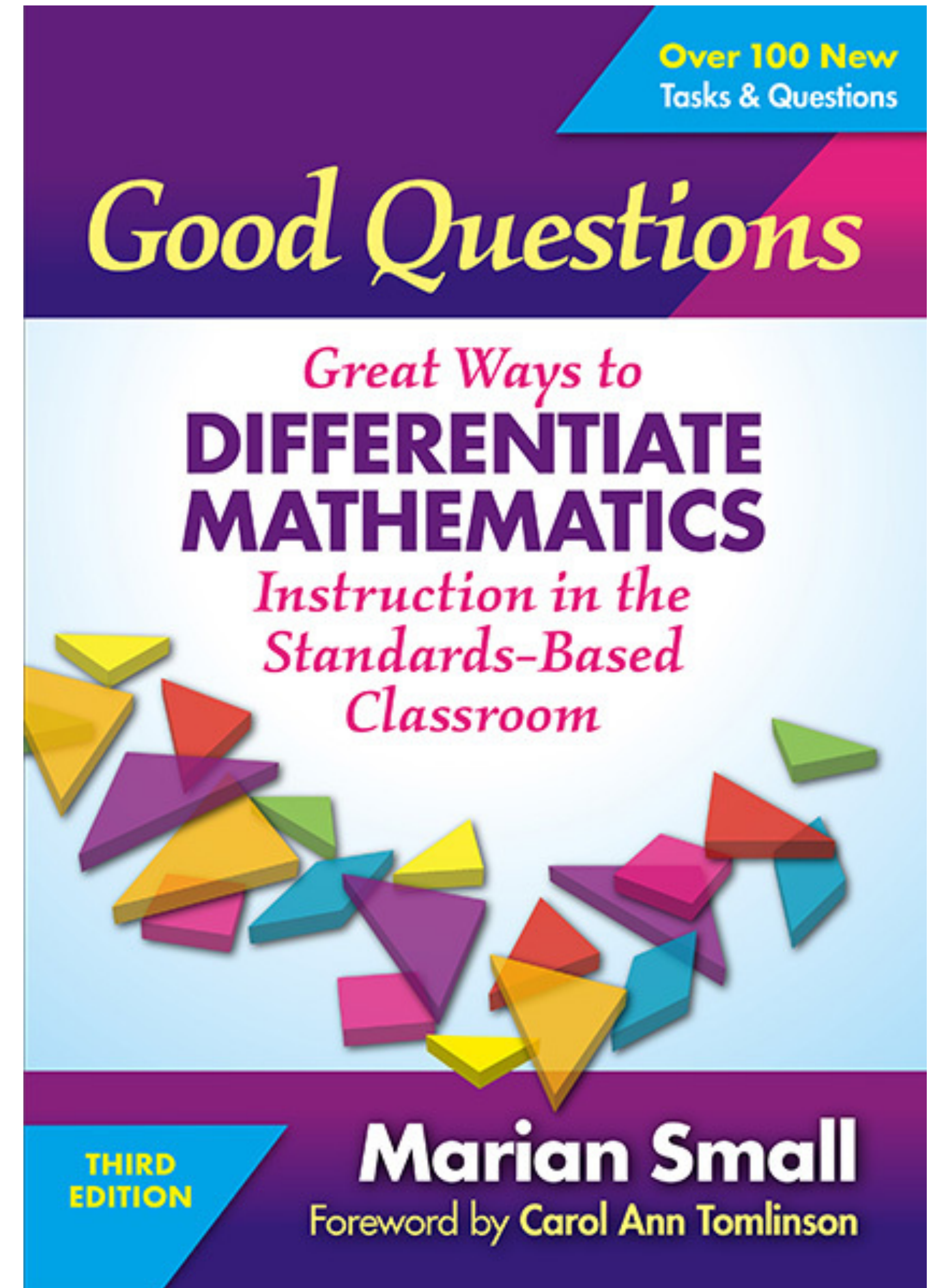
*It is the job of teachers to help see that **mathematics is multifaceted**. Any mathematical concept can be considered from a variety of perspectives, and those multiple perspectives actually enrich its study. Open questions provide the opportunity to demonstrate this.*



How Do I Create Open Questions?

Four Strategies

1. Turning Around a Question
2. Asking for Similarities and Differences
3. Replacing a Number with a Blank
4. Asking for a Number Sentence



Ways to Make $\sqrt[3]{36} + 6x$

*“Write as many ways to make 36 **as you can think of.**”*

Ways to Make 36

*“Write as many ways to make 36 **as you can think of.**”*

*“Write ways to make 36 with **three** addends.”*

*“Think of ways to make 36 using **subtraction.**”*

*“Think of ways to make 36 using **multiplication.**”*

Ways to Make 36

From freedom...

*“Write as many ways to make 36 **as you can think of.**”*

... to constraints

*“Write ways to make 36 with **three** addends.”*

*“Think of ways to make 36 using **subtraction.**”*

*“Think of ways to make 36 using **multiplication.**”*

From freedom...



... to constraints



Ways to Make a Number

What to Look For

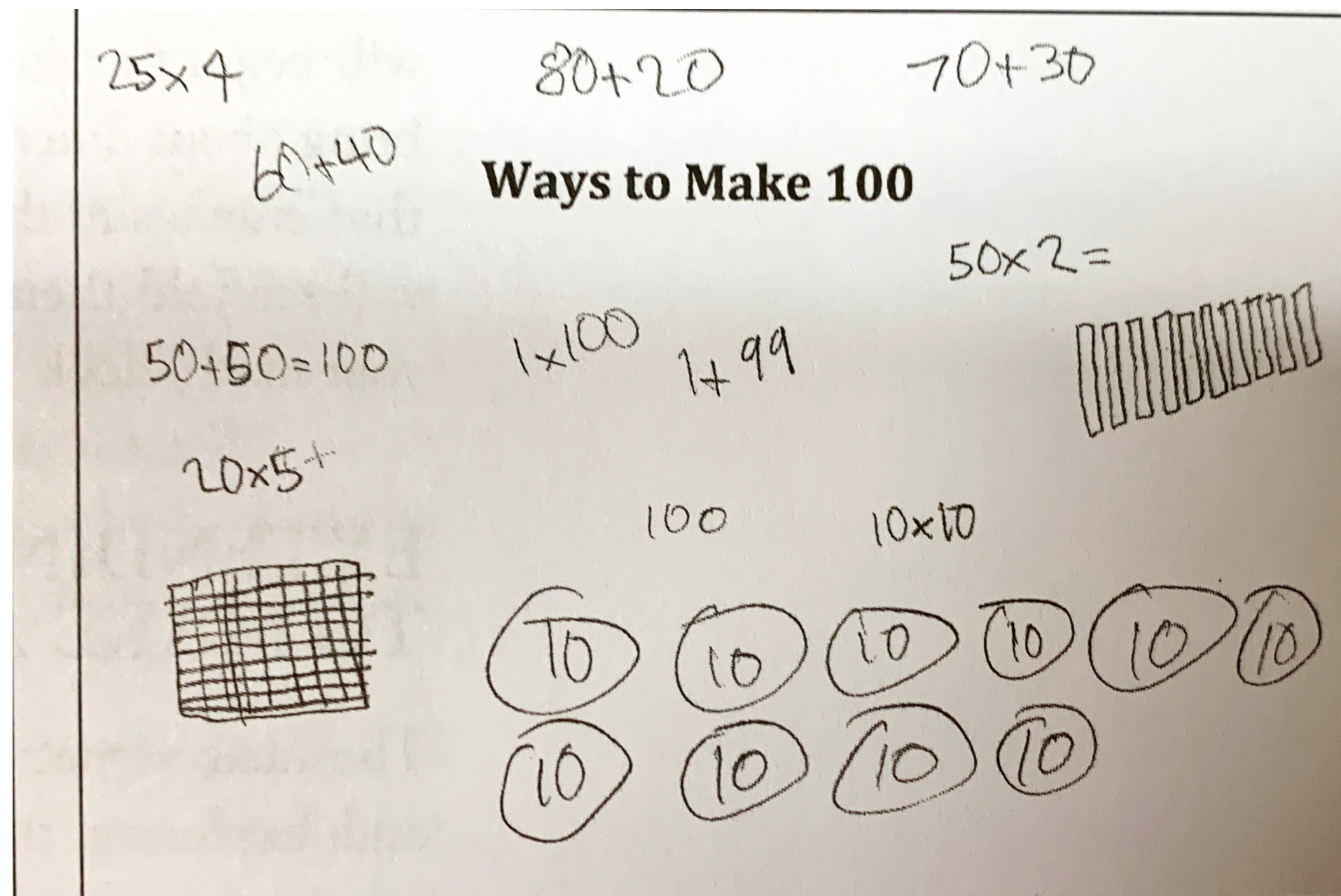
- **decomposing**

- expanded notation, place value, unit fractions, benchmarks or friendly numbers, etc.

- using **patterns**

- **visual** ways of thinking about numbers

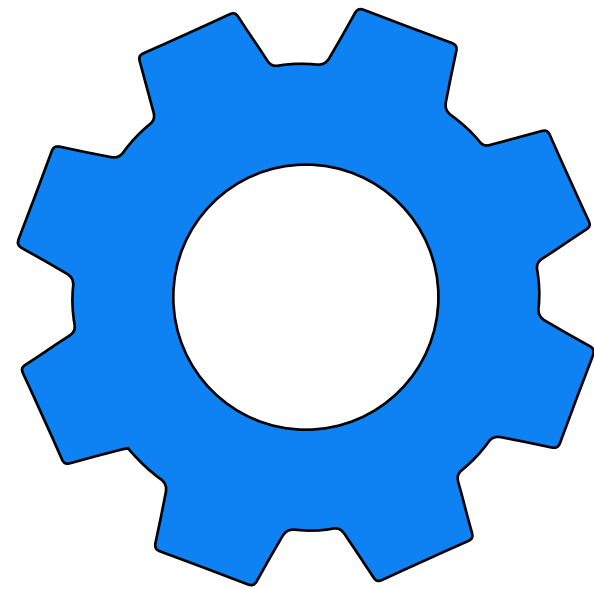
- pictures, number lines, tallies, base ten blocks, etc.



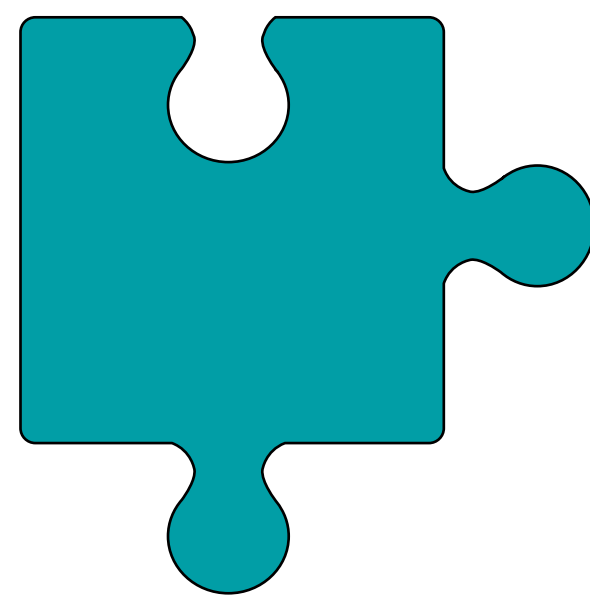
When Should I Ask Open Questions?



Getting Started



Working On It



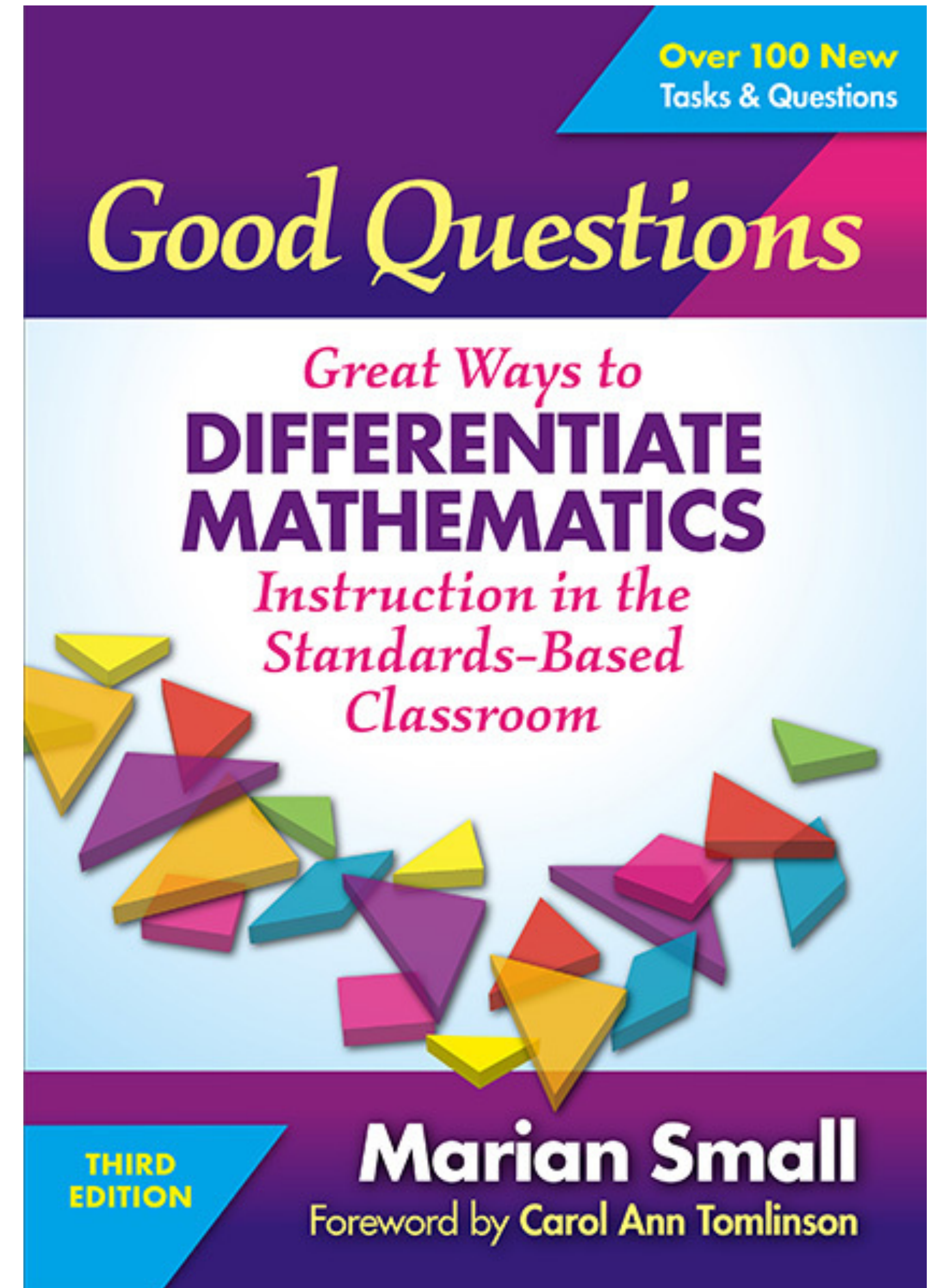
Consolidating



How Do I Create Open Questions?

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How Do I Create Open Questions?

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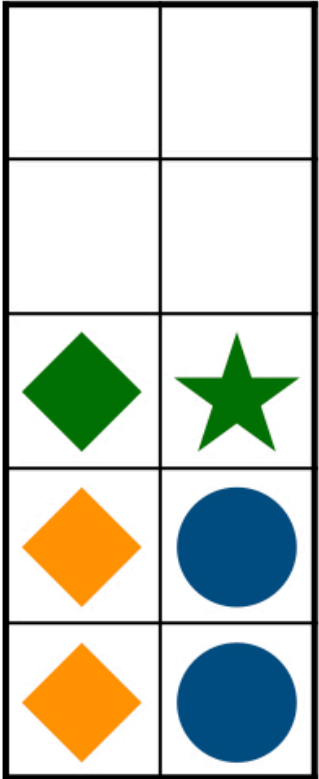
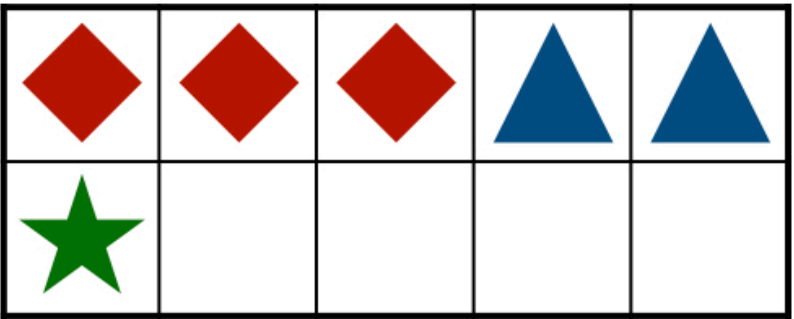
2. Asking for Similarities and Differences

3. Replacing a Number with a Blank

4. Asking for a Number Sentence



What is the same? What's different?



How Do I Create Open Questions?

Four Strategies

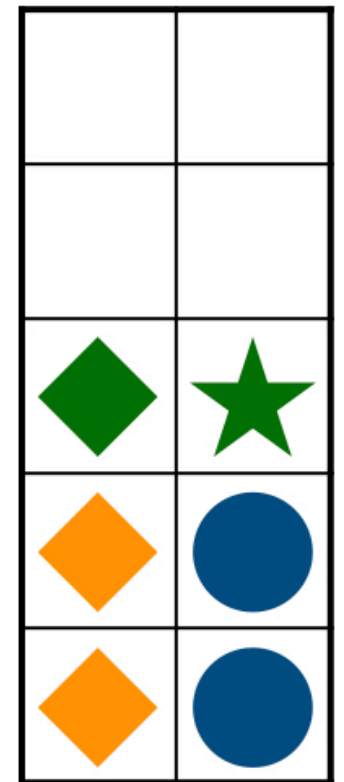
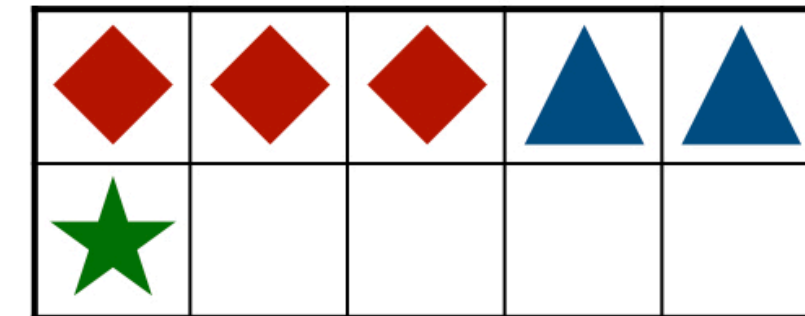
1. Turning Around a Question

2. Asking for Similarities and Differences

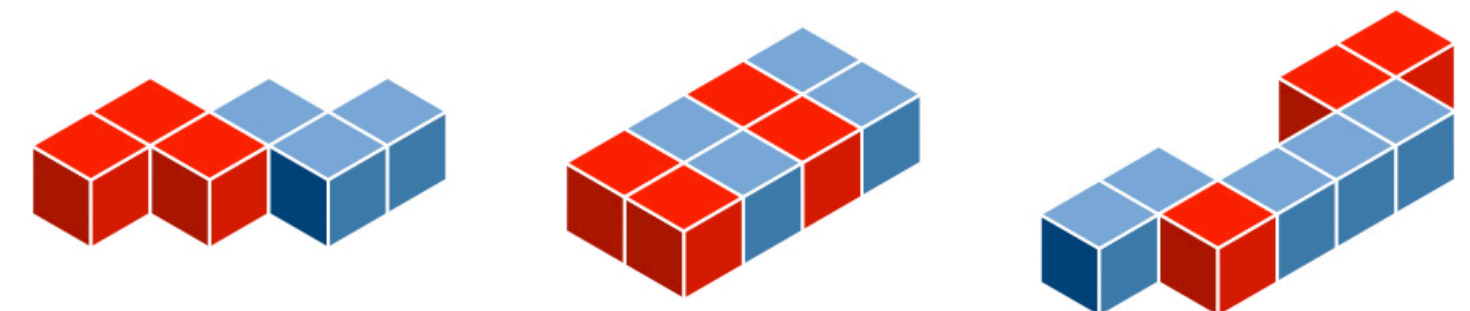
3. Replacing a Number with a Blank

4. Asking for a Number Sentence

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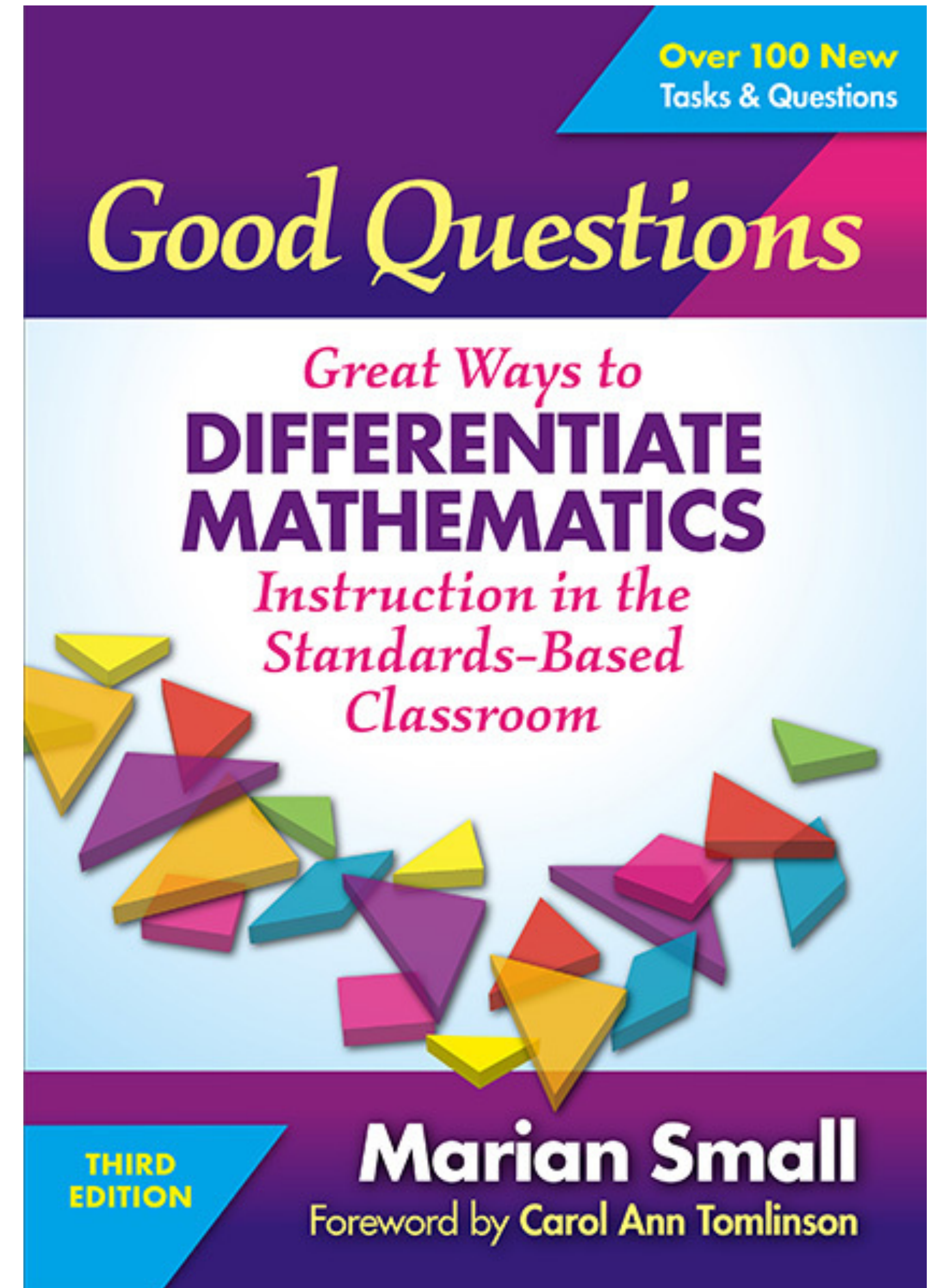
Which two are most alike?



How Do I Create Open Questions?

Four Strategies

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NUMBER TILES

[Printable PDF with the digits 0 to 9](#)[Printable PDF with the integers -9 to 9](#)

BROWSE BY COMMON CORE STATE STANDARDS

Place the digits from 0 to 9 in the
boxes below to make the statement true.

$$\frac{\boxed{}}{\boxed{}} < \frac{\boxed{}}{\boxed{}\boxed{}} < \frac{1}{2} < \frac{\boxed{}\boxed{}}{\boxed{}\boxed{}\boxed{}}$$

0

1

2

3

4

5

6

7

8

9

ORDER NUMBERS

Directions: Using the digits 0 through 9 at most one time each, place a digit in each box to make a two-digit number that has a value between given numbers.

, 35, , 41, , , 88, , 107

COMPARING AND ORDERING RADICALS

Directions: Using the digits 1-9 at most one time each, create a sequence that is in numerical order and cannot be simplified anymore.

$\sqrt{\text{$ }, $\sqrt{\text{$ }, $\sqrt{\text{$

Place the digits from 0 to 9 in the
boxes below to make the statement true.

$$\frac{\boxed{}}{\boxed{}} < \frac{\boxed{}}{\boxed{}\boxed{}} < \frac{1}{2} < \frac{\boxed{}\boxed{}}{\boxed{}\boxed{}\boxed{}}$$

0

1

2

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$$\frac{\boxed{}}{\boxed{}} < \frac{\boxed{}}{\boxed{}\boxed{}} < \frac{1}{2} < \frac{\boxed{}\boxed{}}{\boxed{1}\boxed{}\boxed{}}$$

0

2

3

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3

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7

8

9

Place the digits from 0 to 9 in the boxes below to make the statement true.

$$\frac{\boxed{}}{\boxed{}} < \frac{\boxed{}}{\boxed{2}\boxed{0}} < \frac{1}{2} < \frac{\boxed{}\boxed{}}{\boxed{1}\boxed{}\boxed{}}$$

3 4 5 6 7 8 9

Place the digits from 0 to 9 in the boxes below to make the statement true.

$$\frac{\boxed{}}{\boxed{}} < \frac{\boxed{9}}{\boxed{2}\boxed{0}} < \frac{1}{2} < \frac{\boxed{}\boxed{}}{\boxed{1}\boxed{}\boxed{}}$$

3 4 5 6 7 8

Place the digits from 0 to 9 in the boxes below to make the statement true.

$$\begin{array}{|c|} \hline \\ \hline 8 \\ \hline \end{array} < \begin{array}{|c|c|} \hline 9 \\ \hline 2 & 0 \\ \hline \end{array} < \frac{1}{2} < \begin{array}{|c|c|c|} \hline & \\ \hline 1 & & \\ \hline \end{array}$$

3 4 5 6 7

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5 6 7

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5

Place the digits from 0 to 9 in the boxes below to make the statement true.

$$\begin{array}{|c|} \hline 3 \\ \hline 8 \\ \hline \end{array} < \begin{array}{|c|c|} \hline 9 \\ \hline 2 & 0 \\ \hline \end{array} < \frac{1}{2} < \begin{array}{|c|c|c|} \hline 7 & 5 \\ \hline 1 & 4 & 6 \\ \hline \end{array}$$

Place the digits from 0 to 9 in the boxes below to make the statement true.

$$\begin{array}{|c|} \hline 1 \\ \hline 4 \\ \hline \end{array} < \begin{array}{|c|c|} \hline \\ \hline \\ \hline \end{array} < \frac{1}{2} < \begin{array}{|c|c|c|} \hline \\ \hline \\ \hline \end{array}$$

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$$\begin{array}{|c|} \hline 1 \\ \hline 4 \\ \hline \end{array} < \begin{array}{|c|c|} \hline \\ \hline 3 & 2 \\ \hline \end{array} < \frac{1}{2} < \begin{array}{|c|c|c|} \hline & \\ \hline & & \\ \hline \end{array}$$

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5

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Place the digits from 0 to 9 in the boxes below to make the statement true.

$$\begin{array}{|c|} \hline 1 \\ \hline 4 \\ \hline \end{array} < \begin{array}{|c|c|} \hline 9 \\ \hline 3 & 2 \\ \hline \end{array} < \frac{1}{2} < \begin{array}{|c|c|c|} \hline & & \\ \hline & & \\ \hline \end{array}$$

0

5

6

7

8

Place the digits from 0 to 9 in the boxes below to make the statement true.

$$\begin{array}{|c|} \hline 1 \\ \hline 4 \\ \hline \end{array} < \begin{array}{|c|c|} \hline 9 \\ \hline 3 & 2 \\ \hline \end{array} < \frac{1}{2} < \begin{array}{|c|c|c|} \hline & & \\ \hline 0 & & \\ \hline \end{array}$$

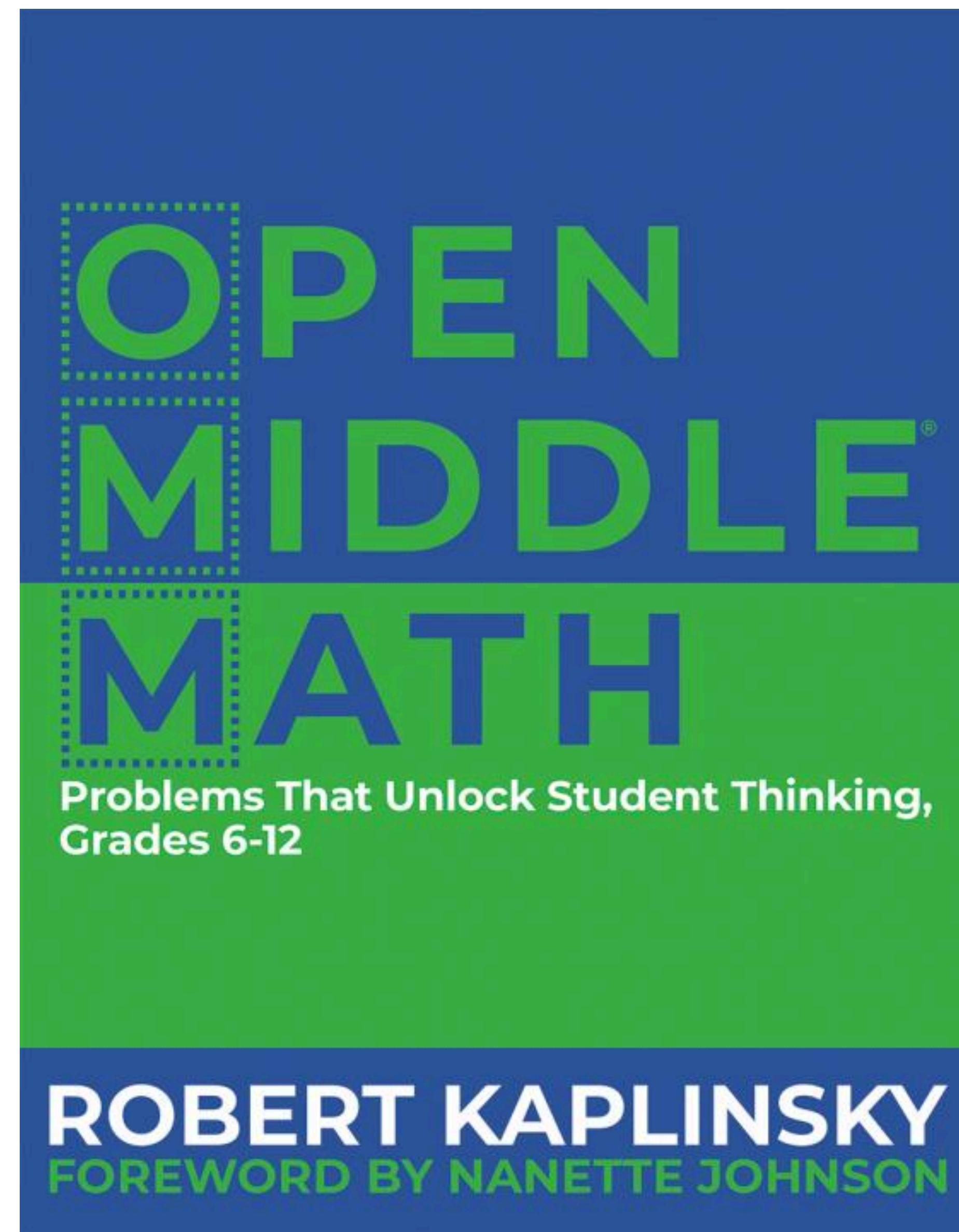
5 6 7 8

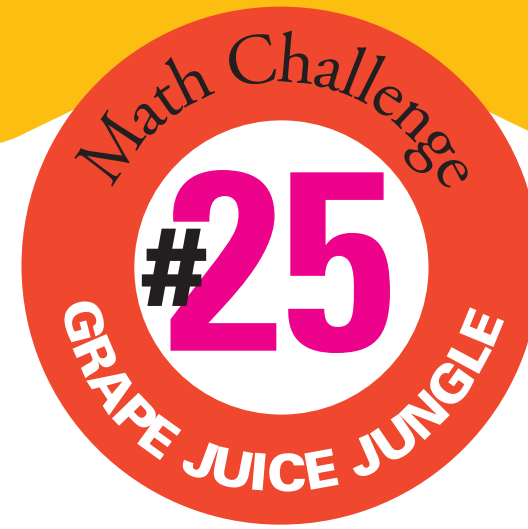
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$$\begin{array}{|c|} \hline 1 \\ \hline 4 \\ \hline \end{array} < \begin{array}{|c|c|} \hline 9 \\ \hline 3 & 2 \\ \hline \end{array} < \frac{1}{2} < \begin{array}{|c|c|c|} \hline 5 & 6 \\ \hline 0 & 7 & 8 \\ \hline \end{array}$$

Open Middle Math

*[M]ost math problems begin with everyone having the same problem and working toward the same answer. As a result, the beginning and ending are closed. What varies is the middle. Sometimes a problem's instructions tell students to complete a problem using a specific method (a closed middle). Other times, **there are possibly many ways to solve the problem** (an open middle). **Problems with open middles tend to be much more interesting and lead to richer conversations.***



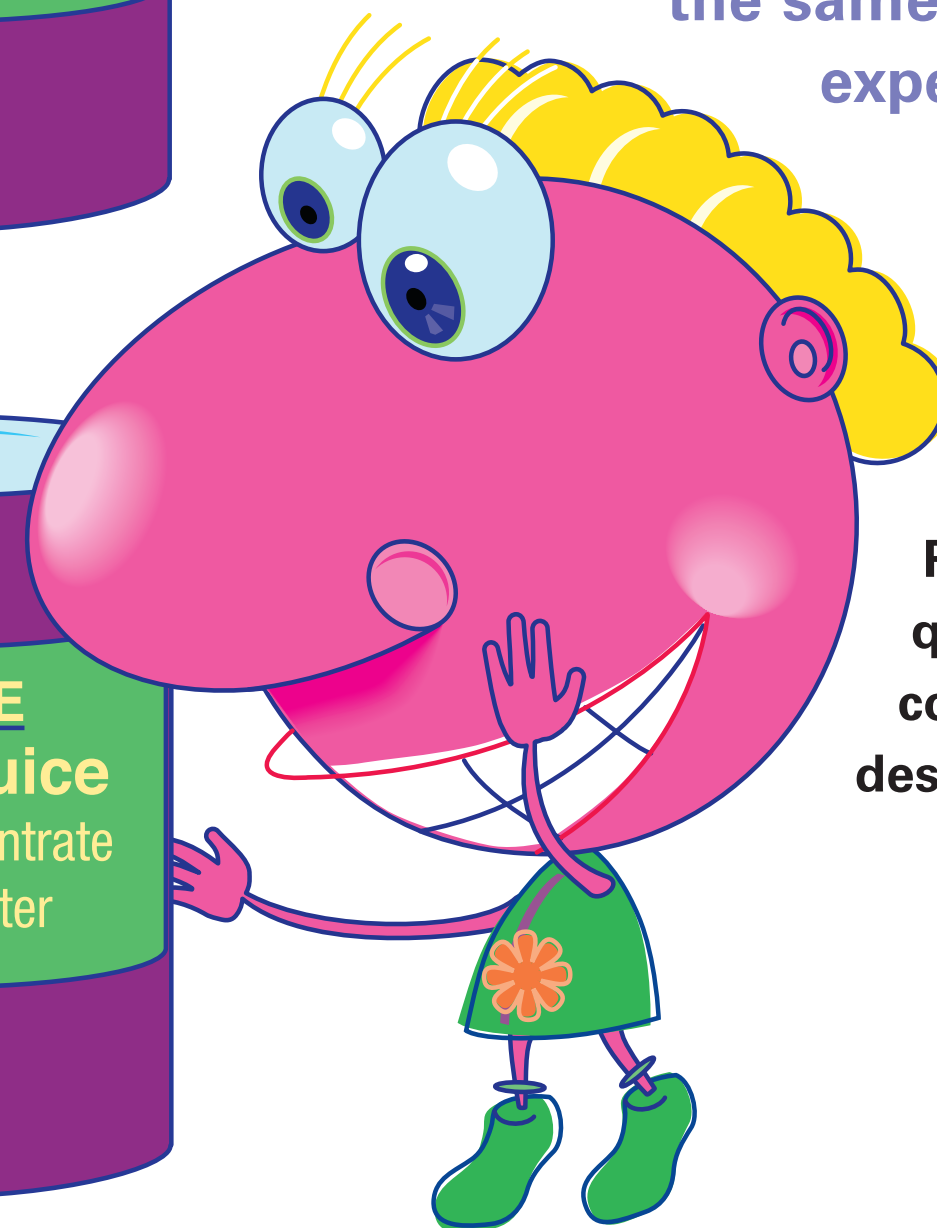


FigureThis!
Math Challenges for Families

Which tastes

JUICIER?

Figure This! If all grape juice concentrates are the same strength, which recipe would you expect to have the strongest grape taste?



Ratios are fractions that compare two or more quantities. Shoppers use ratios to compare prices; cooks use them to adjust recipes. Architects and designers use ratios to create scale drawings.

Jerry's Juice

2 : 3

Grapeade

5 : 8

Good Grape

3 : 4

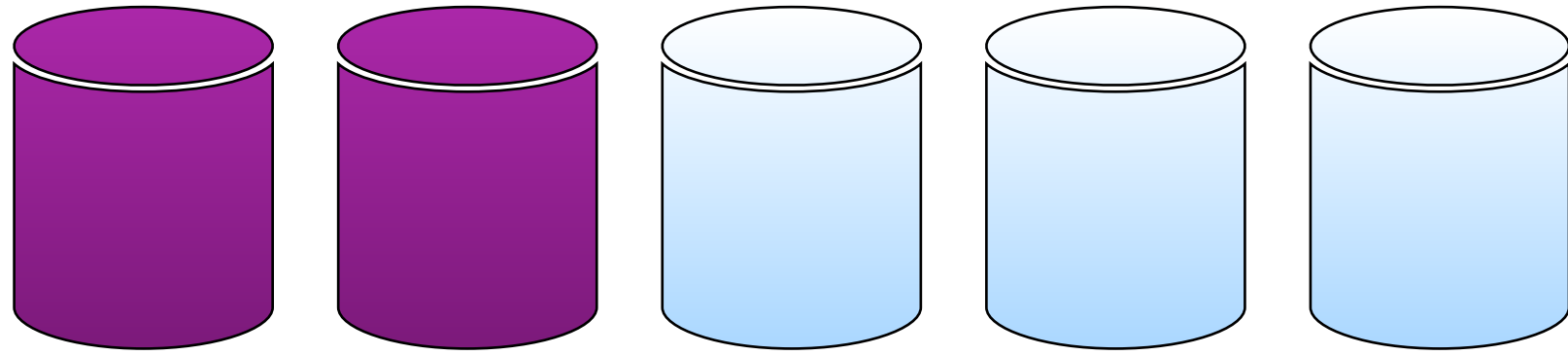
Jane's Juice

4 : 7

Jerry's Juice

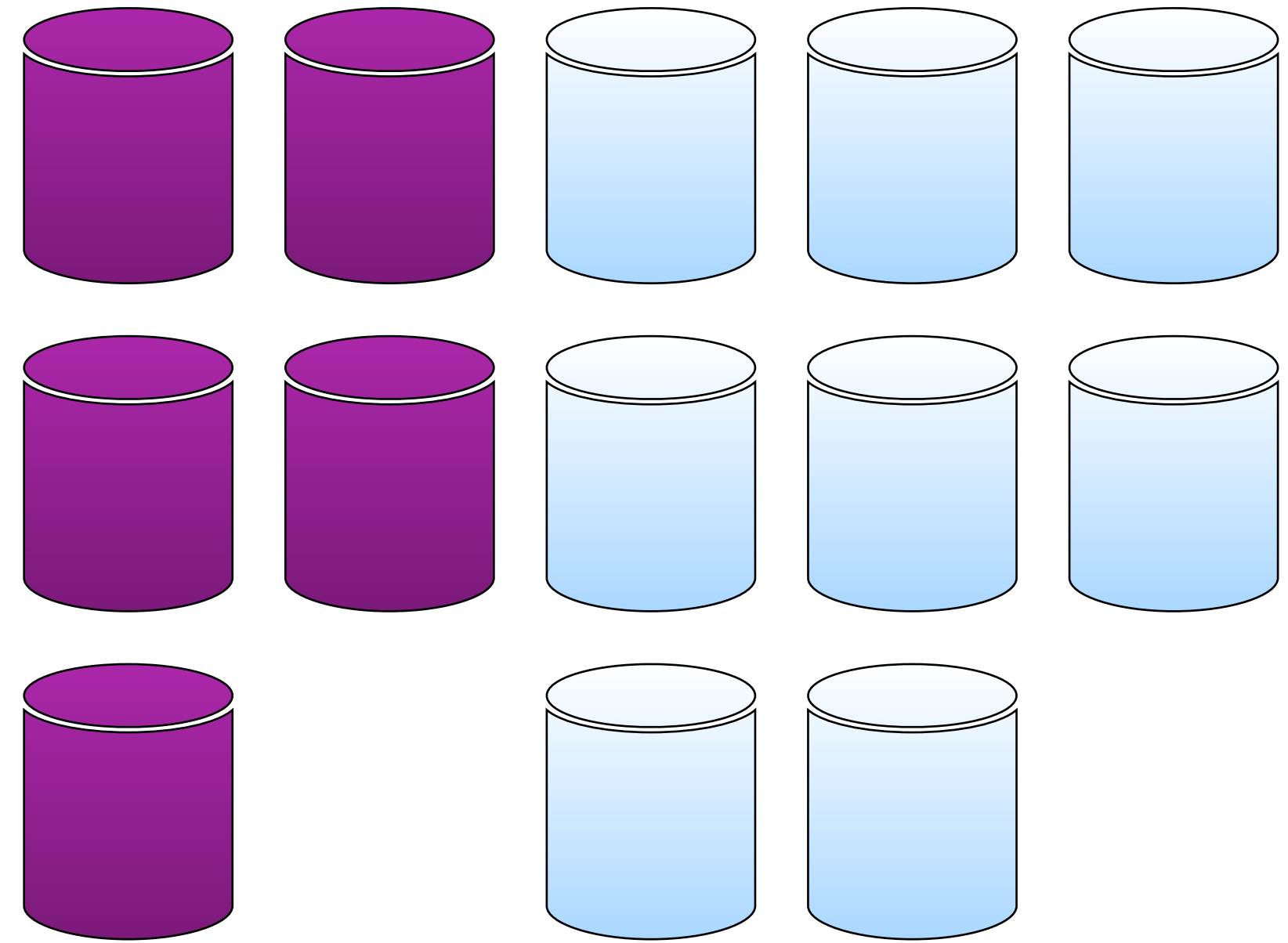
2 : 3

$$\frac{2}{2 + 3}$$



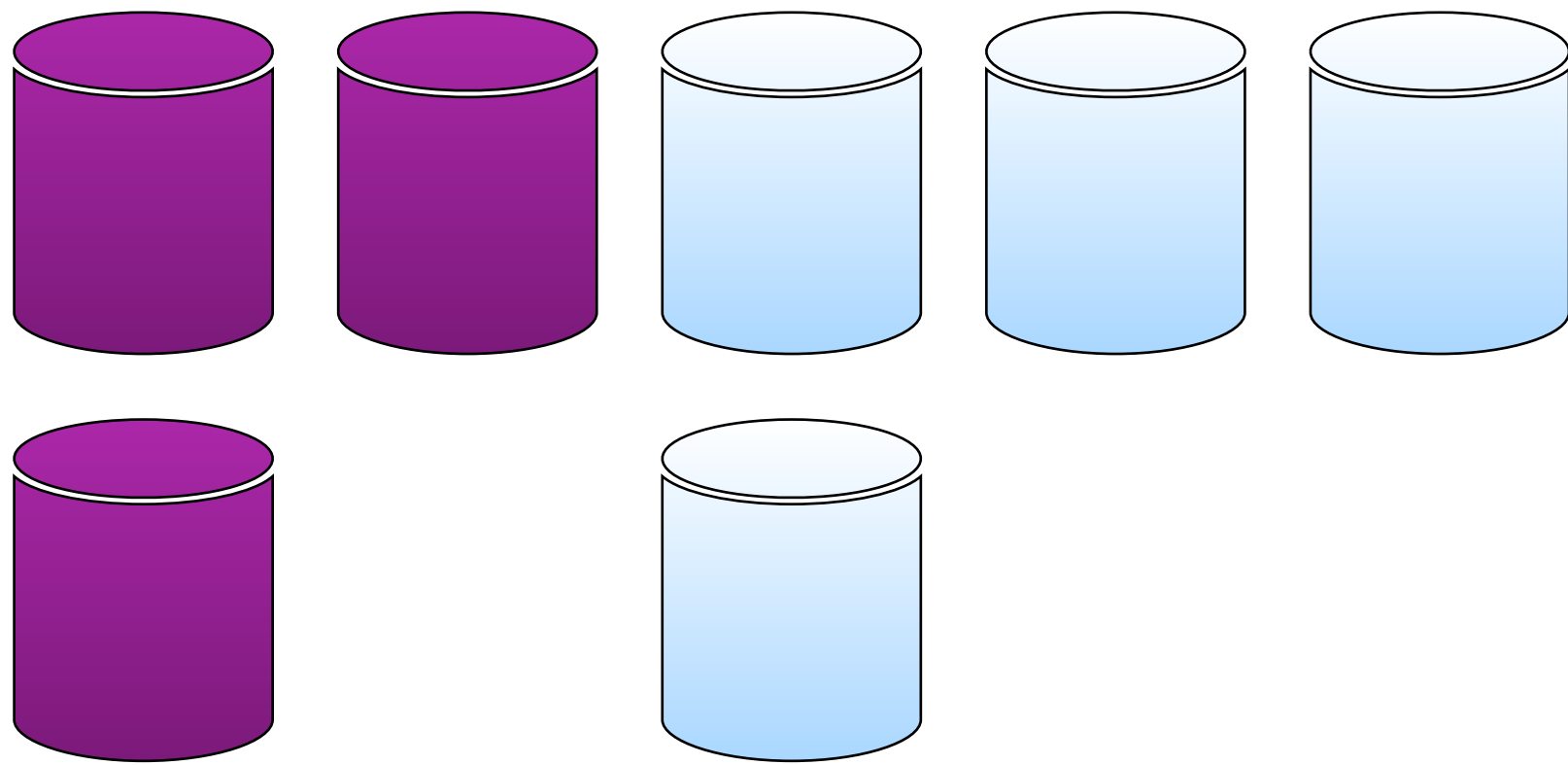
Grapeade

5 : 8



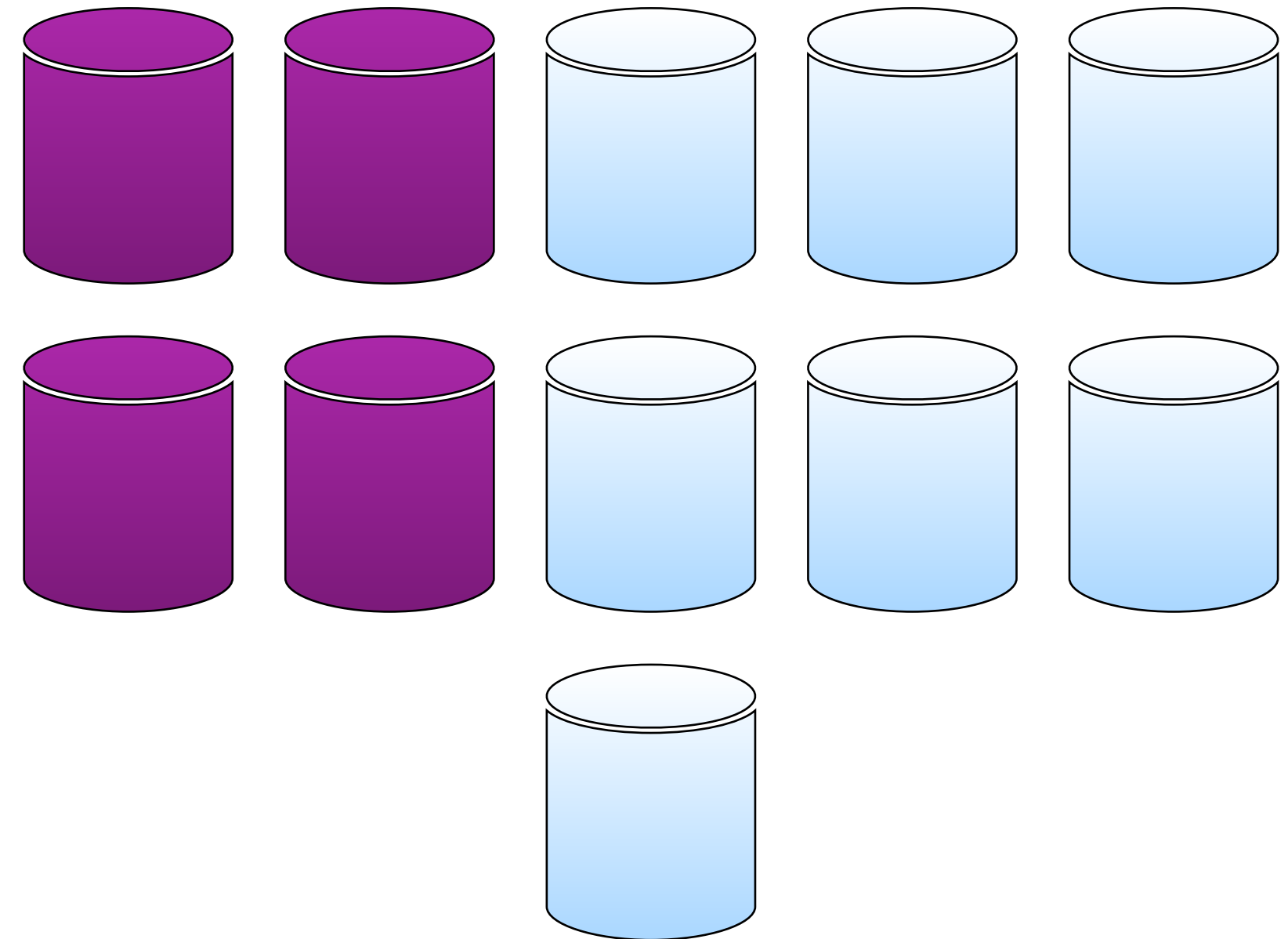
Good Grape

3 : 4



Jane's Juice

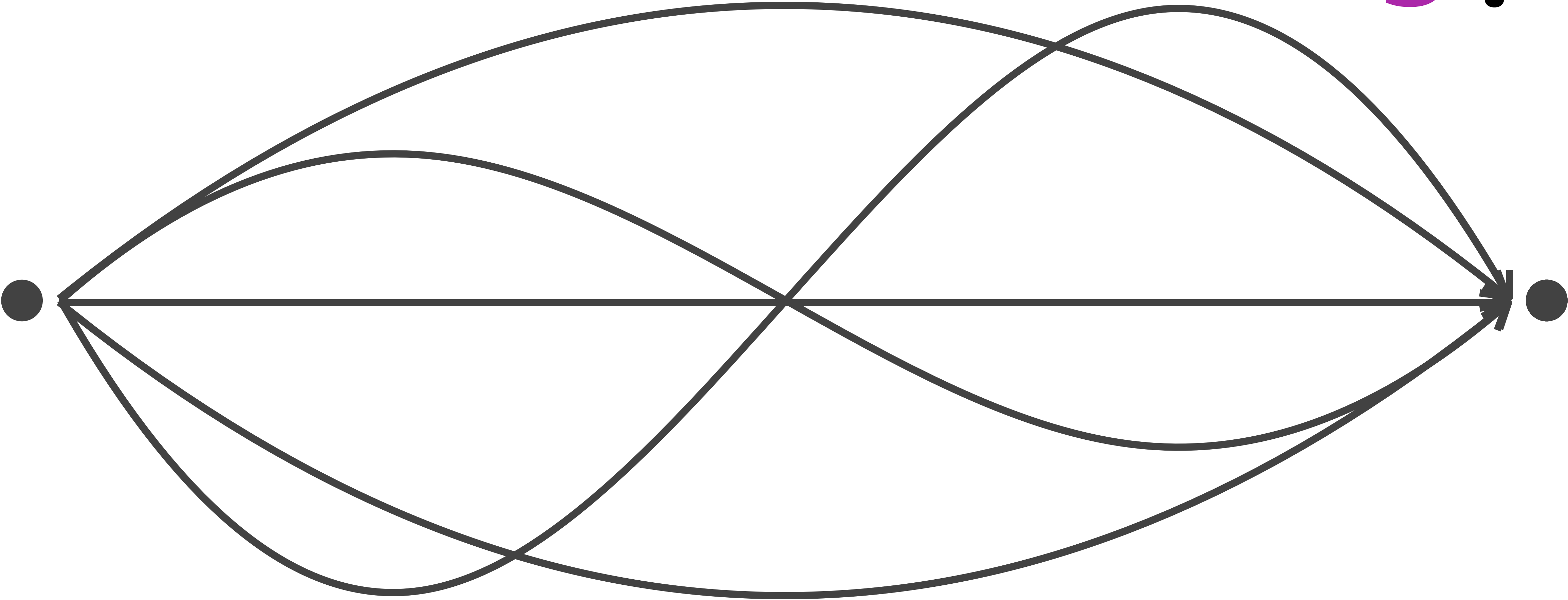
4 : 7



Which tastes the **juiciest**?

Good Grape

3 : **4**



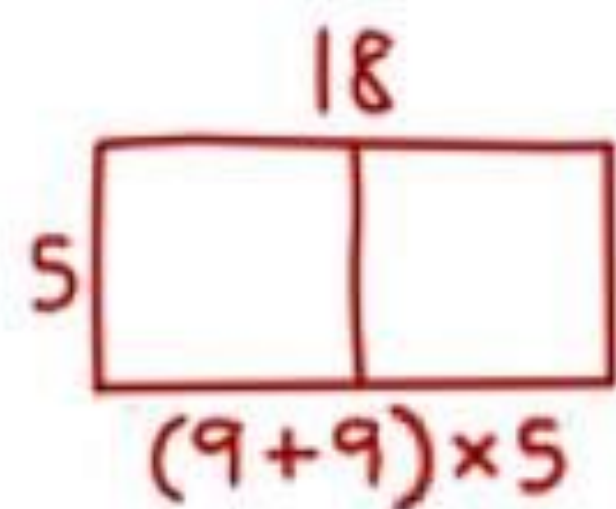
beginning

middle
open

end
closed

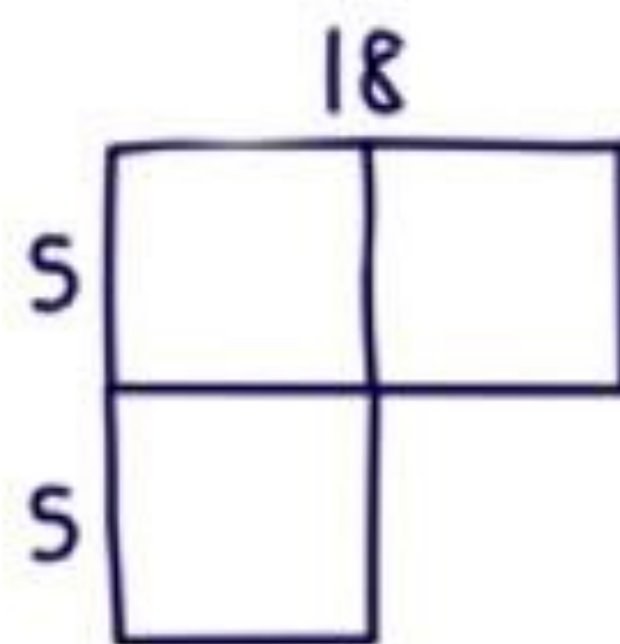
$$18 \times 5$$

Neil



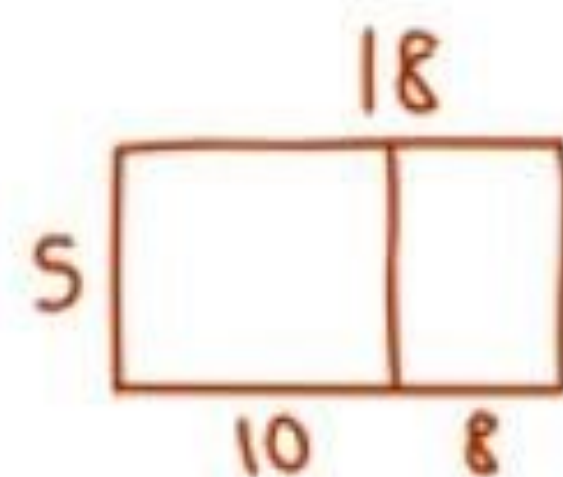
$$45 + 45 = 90$$

Ricardo



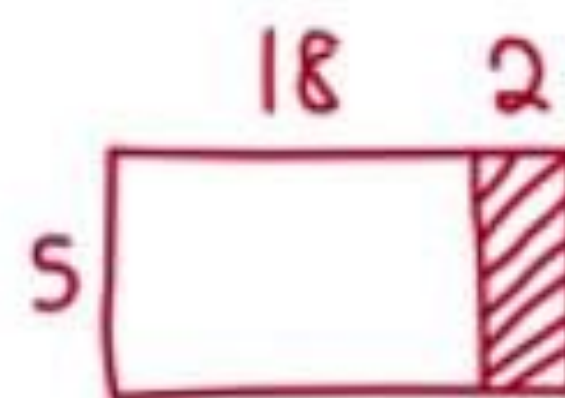
$$18 \times 5 = 9 \times 10$$

Sammi



$$50 + 40 = 90$$

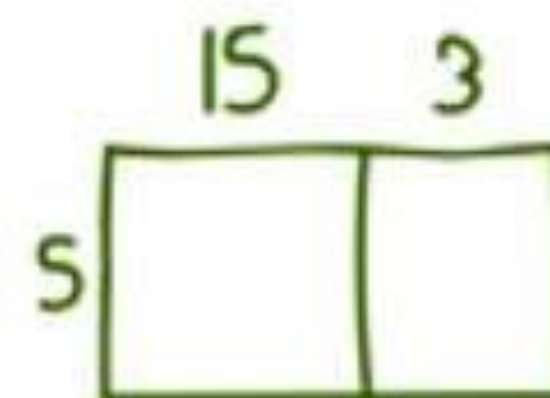
Jaime



$$2 \times 5 = 10$$

$$100 - 10 = 90$$

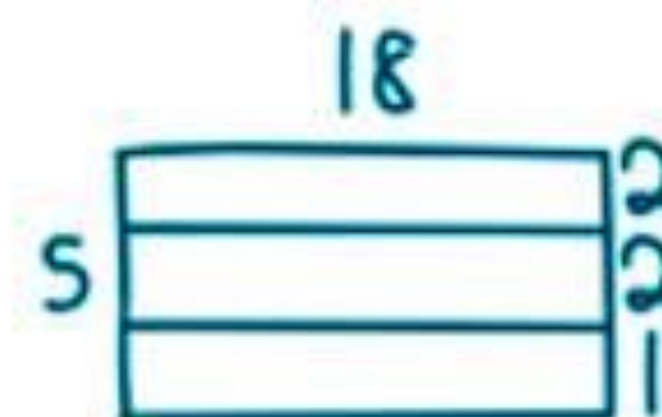
Ariane



$$3 \times 5 = 15$$

$$75 + 15 = 90$$

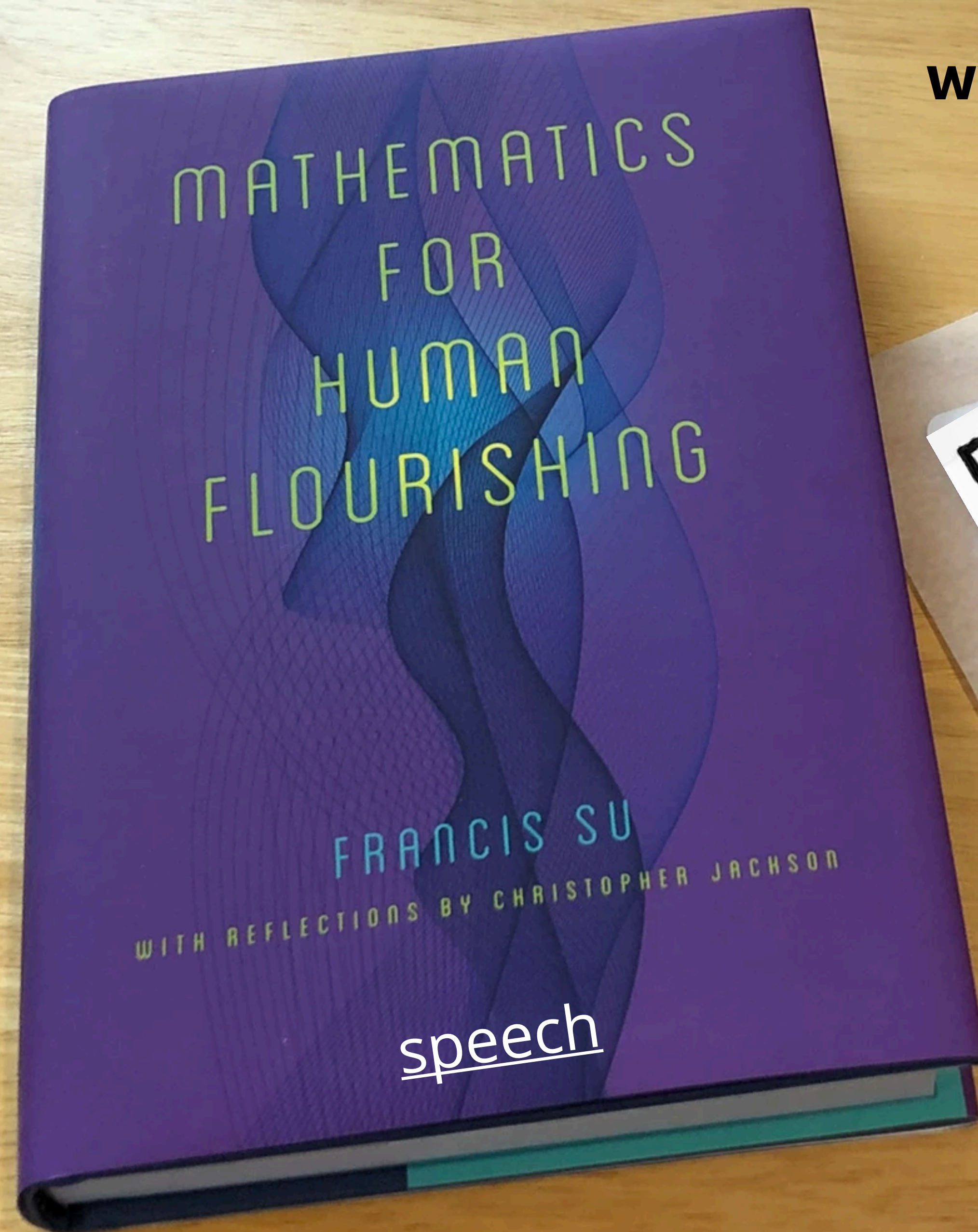
Bryan



$$36 + 36 + 18 = 90$$



www.bcamt.ca/nw2021



justice

Links

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- www.openmiddle.com
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Chris Hunter

K-12 Numeracy Helping Teacher

email: hunter_c@surreyschools.ca

Twitter: [@ChrisHunter36](https://twitter.com/ChrisHunter36)

blog: chrishunter.ca

