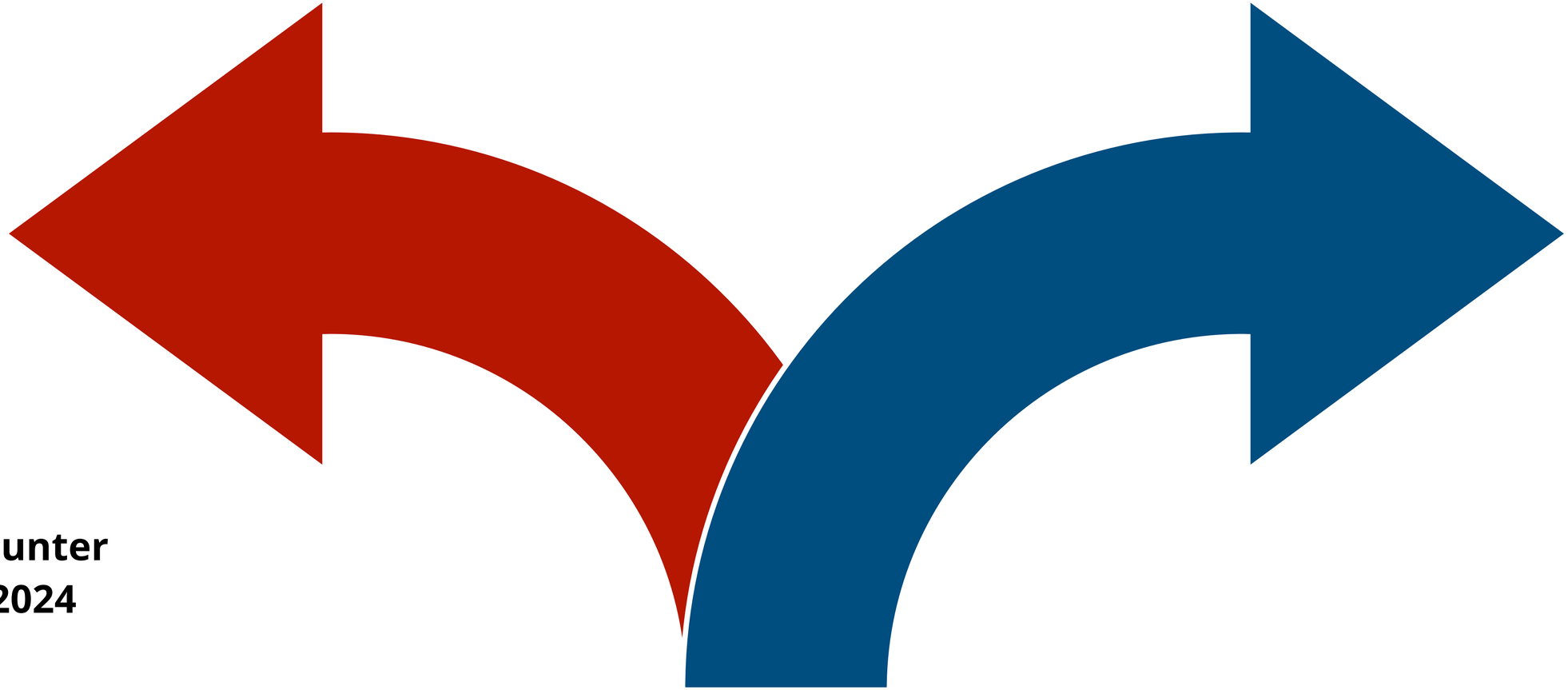
Making and Justifying Mathematical Decisions



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Explain and justify mathematical ideas and decisions

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Explain mathematical ideas

justify mathematical decisions

Explain mathematical ideas
justify mathematical decisions

Explain mathematical ideas
justify mathematical decisions

Explain mathematical ideas

justify mathematical decisions

The Mathematical Processes: Reasoning and Proving

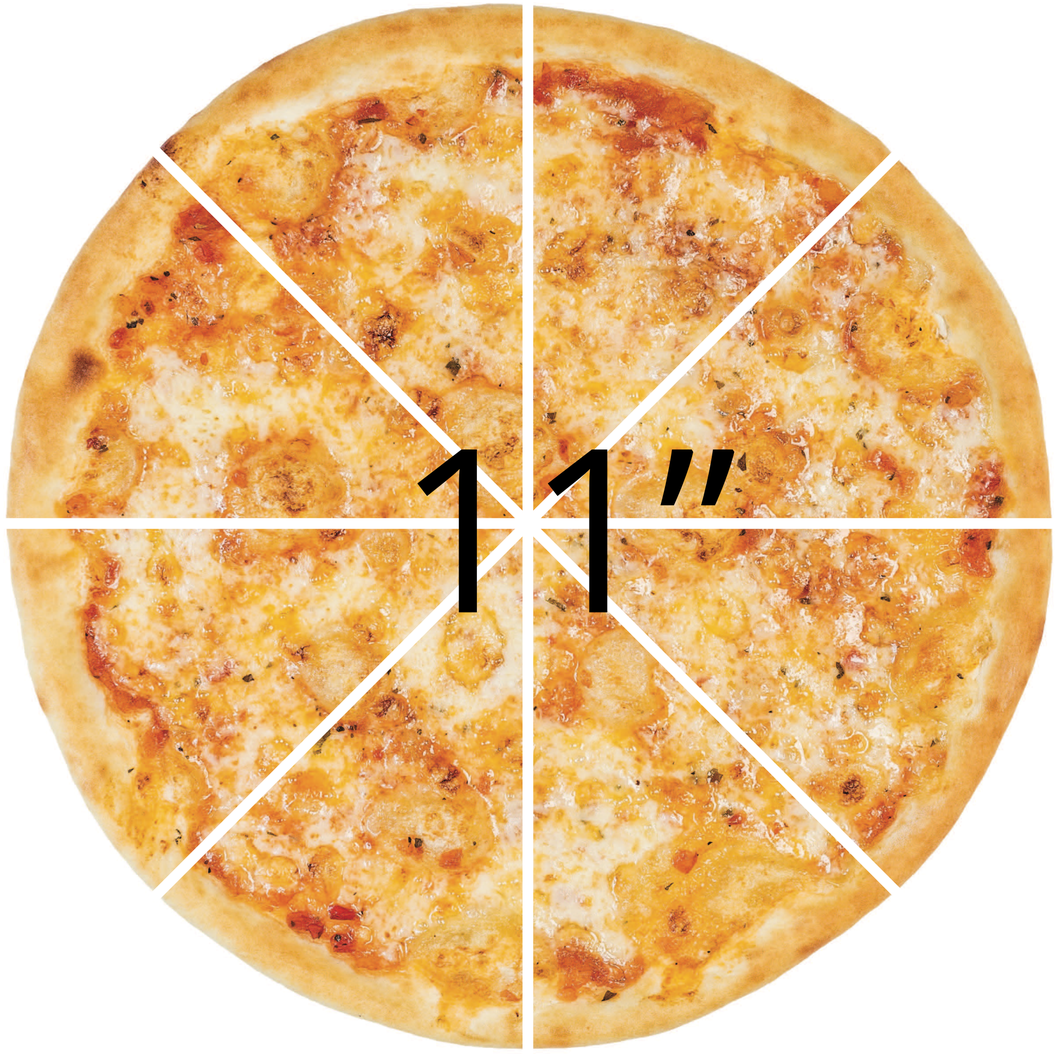
Reasoning and proving are a mainstay of mathematics and involves students using their understanding of mathematical knowledge, concepts, and skills to **justify** their thinking. Proportional reasoning, algebraic reasoning, spatial reasoning, statistical reasoning, and probabilistic reasoning are all forms of mathematical reasoning. Students also use their understanding of numbers and operations, geometric properties, and measurement relationships to reason through solutions to problems. Teachers can provide all students with learning opportunities where they must form mathematical conjectures and then test or prove them to see if they hold true. Initially, students may rely on the viewpoints of others to **justify** a choice or an approach to a solution. As they develop their own reasoning skills, they will begin to **justify** or prove their solutions by providing evidence.

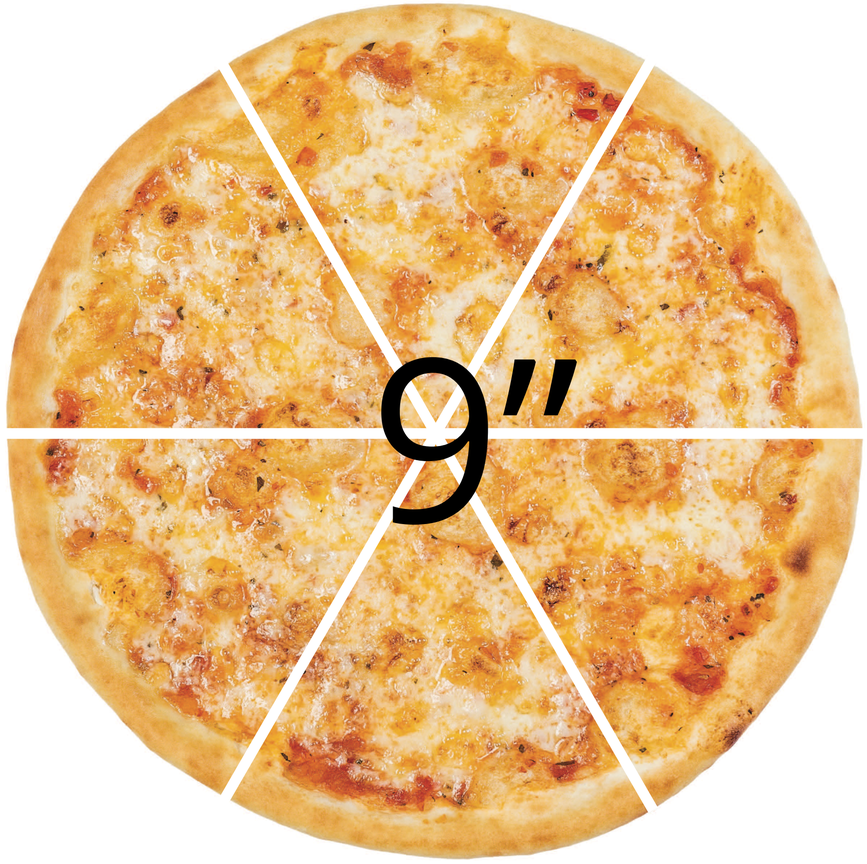
3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They **justify** their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Think back to the last time that you observed a student ***make***—a necessary precursor to ***justify***—a mathematical decision.

Would you rather...

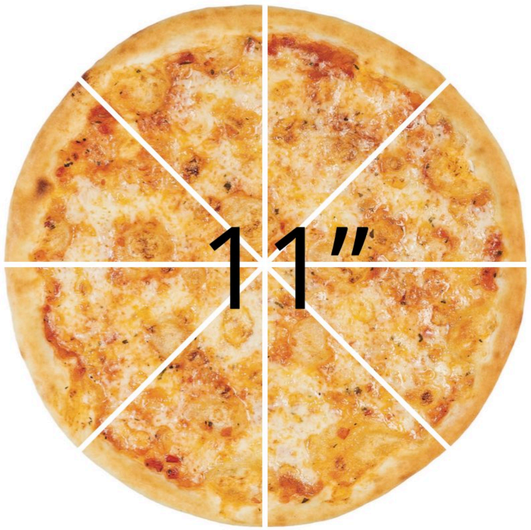

11”

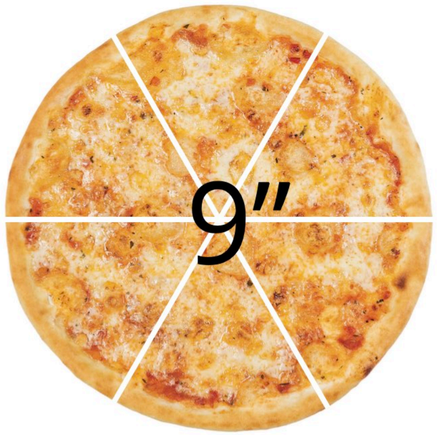

9”

... calculate by **adding whole numbers and fractions separately** or **expressing mixed numbers as improper fractions**?

$$2\frac{5}{6} + 3\frac{1}{2}$$

What is the **same**?





… calculate by **adding whole numbers and fractions separately** or **expressing mixed numbers as improper fractions**?

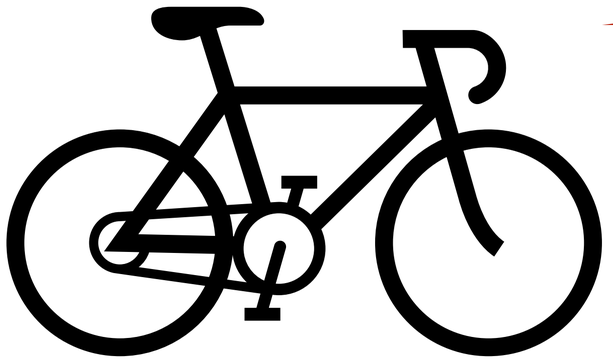
$$2\frac{5}{6} + 3\frac{1}{2}$$

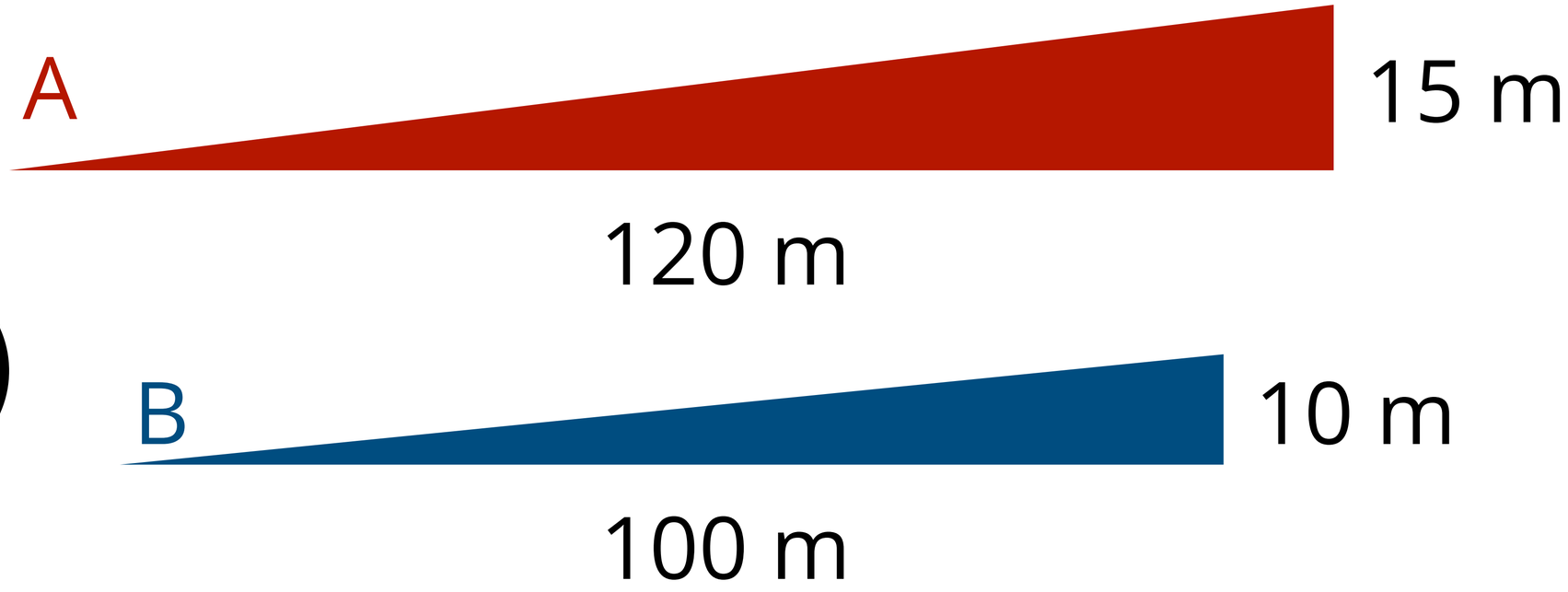
What’s **different**?

decisions about “real-world” situations

... sign up for a **monthly** or **annual** subscription?

… climb hill **A** or **B**?





Explain mathematical ideas

justify mathematical decisions?


INCLUSIVE WITH
PEOPLE
AND
TRAPEZOIDS


INCLUSIVE WITH
PEOPLE
EXCLUSIVE WITH
TRAPEZOIDS

decisions about mathematical methods

procedures

… solve by **substitution** or **elimination**?

$$4x - y - 3 = 0$$

$$6x - 2y - 5 = 0$$

… graph by **determining *x*- and *y*-intercepts** or **writing the equation in slope-intercept form**?

$$2x + 3y + 12 = 0$$

strategies

… determine

20% of 75 or

75% of 20?

Jerry's Juice

2 : 3

Grapeade

5 : 8

Good Grape

3 : 4

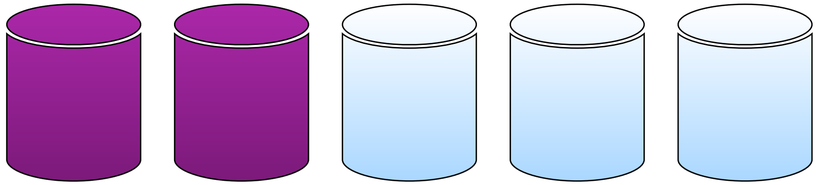
Jane's Juice

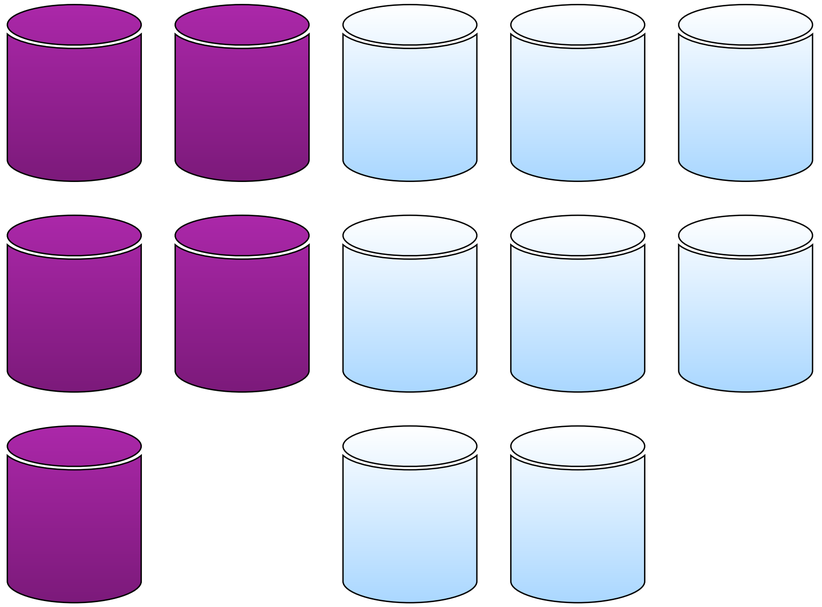
4 : 7

figurethis.nctm.org/challenges/c25/challenge.htm

Jerry's Juice

2 : 3



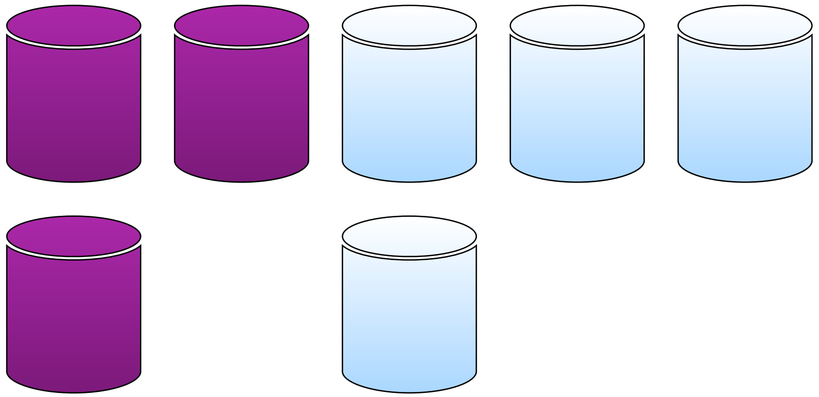


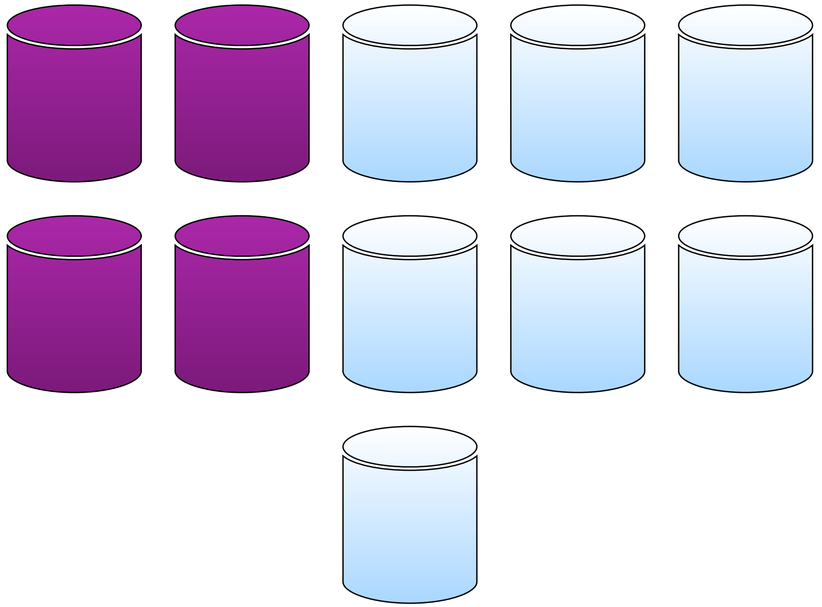
Grapeade

5 : 8

Good Grape

3 : 4





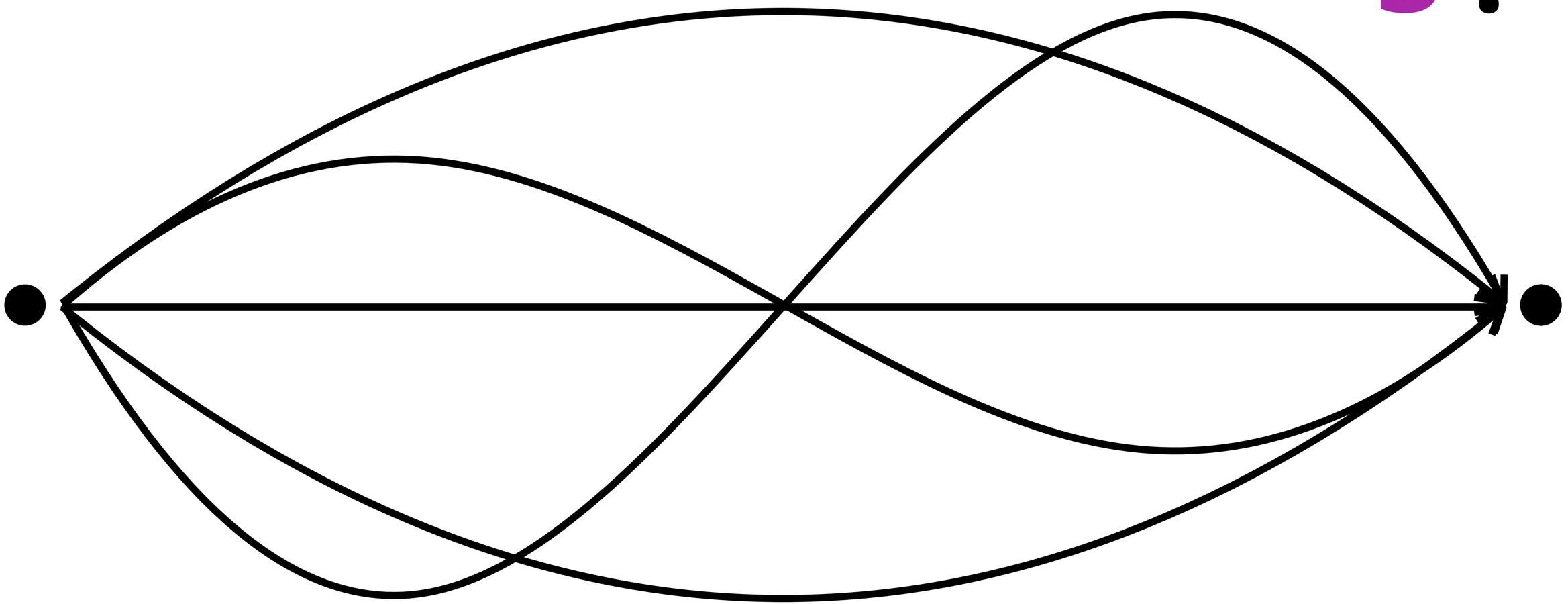
Jane's Juice

4 : 7

Which tastes the **juiciest**?

Good Grape

3 : 4



beginning

middle

open

@ddmeyer

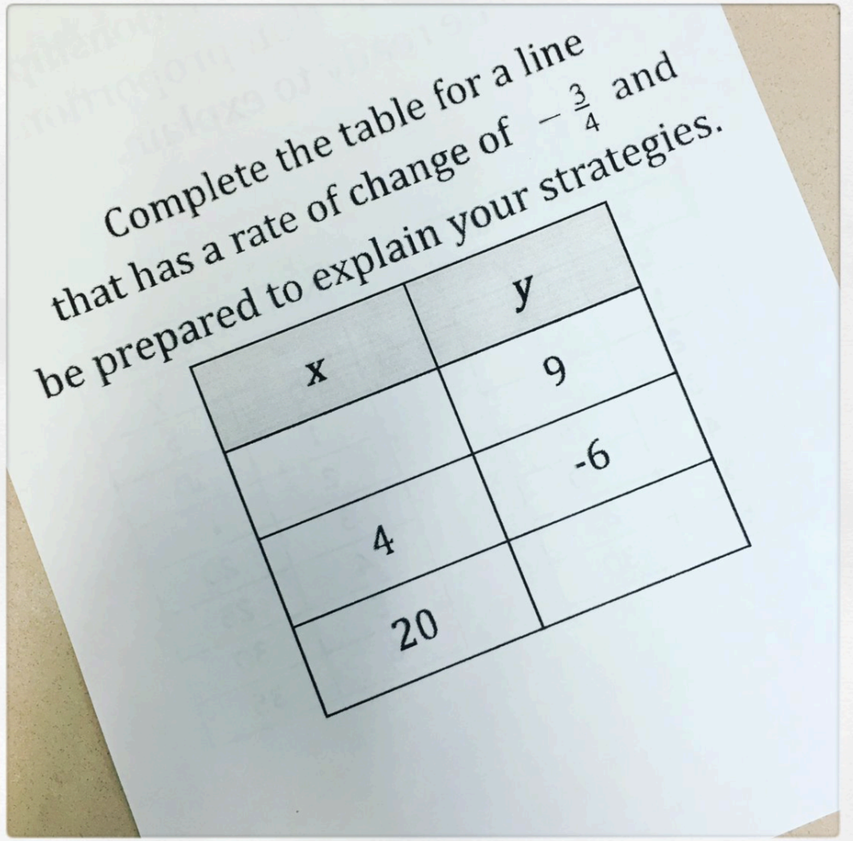
end

closed

Cathy Yenca
@mathycathy

Following

I don't remember who shared this, but thank you! Made one large paper copy for each group, asked them to collaborate. So many rich conversations! #iteachmath #MTBoS



Complete the table for a line that has a rate of change of $-\frac{3}{4}$ and be prepared to explain your strategies.

x	y
	9
	-6
4	
20	

Complete the table for a line that has a rate of change of $-\frac{3}{4}$ and be prepared to explain your strategies.

x	y
	9
	-6
4	
20	

decisions about mathematical representations

Split Time

threeacts.mrmeyer.com/splittime/

Split Time

What's the first question that comes to your mind?

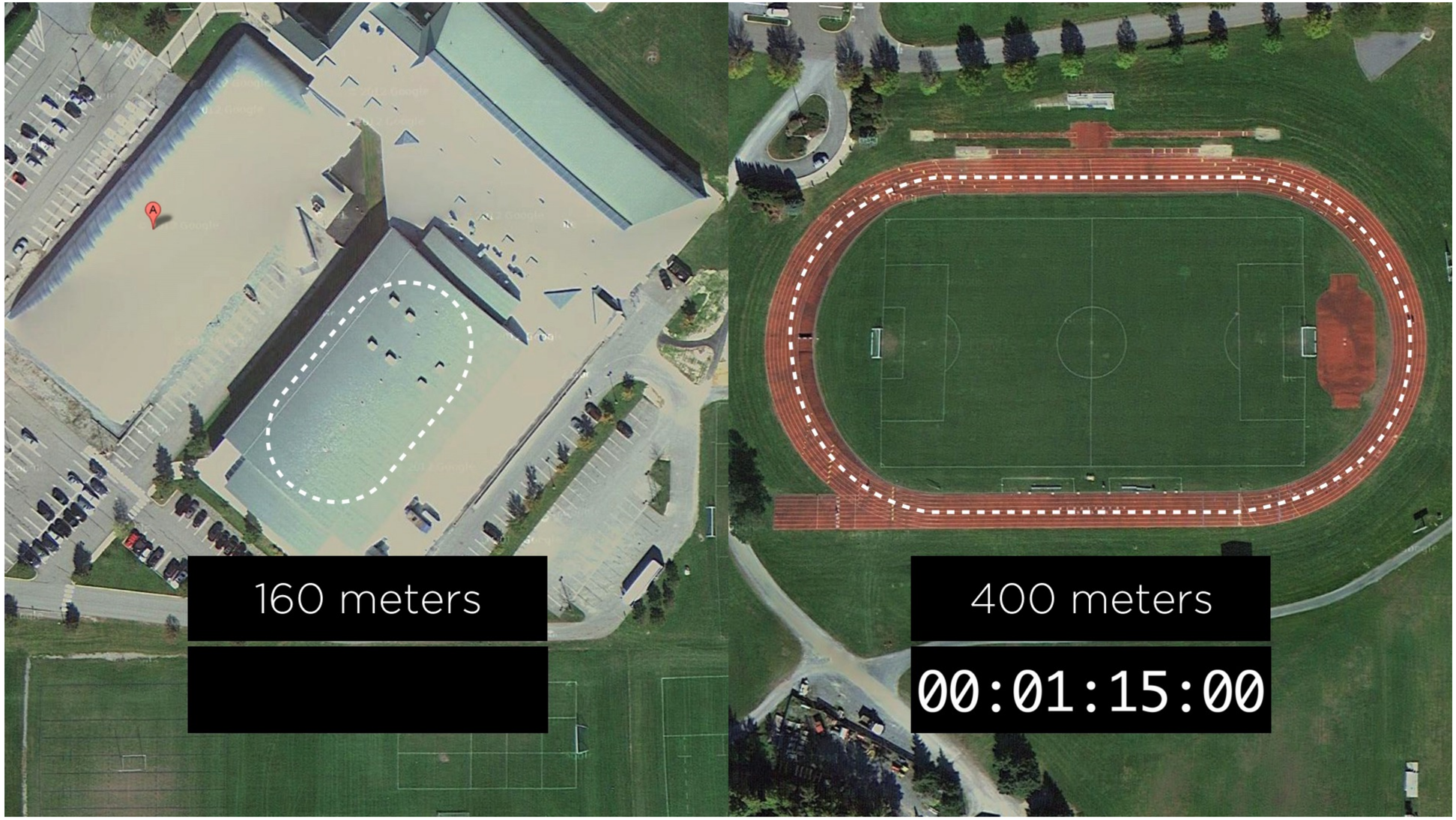
What's a guess that's too *low*?

What's a guess that's too *high*?

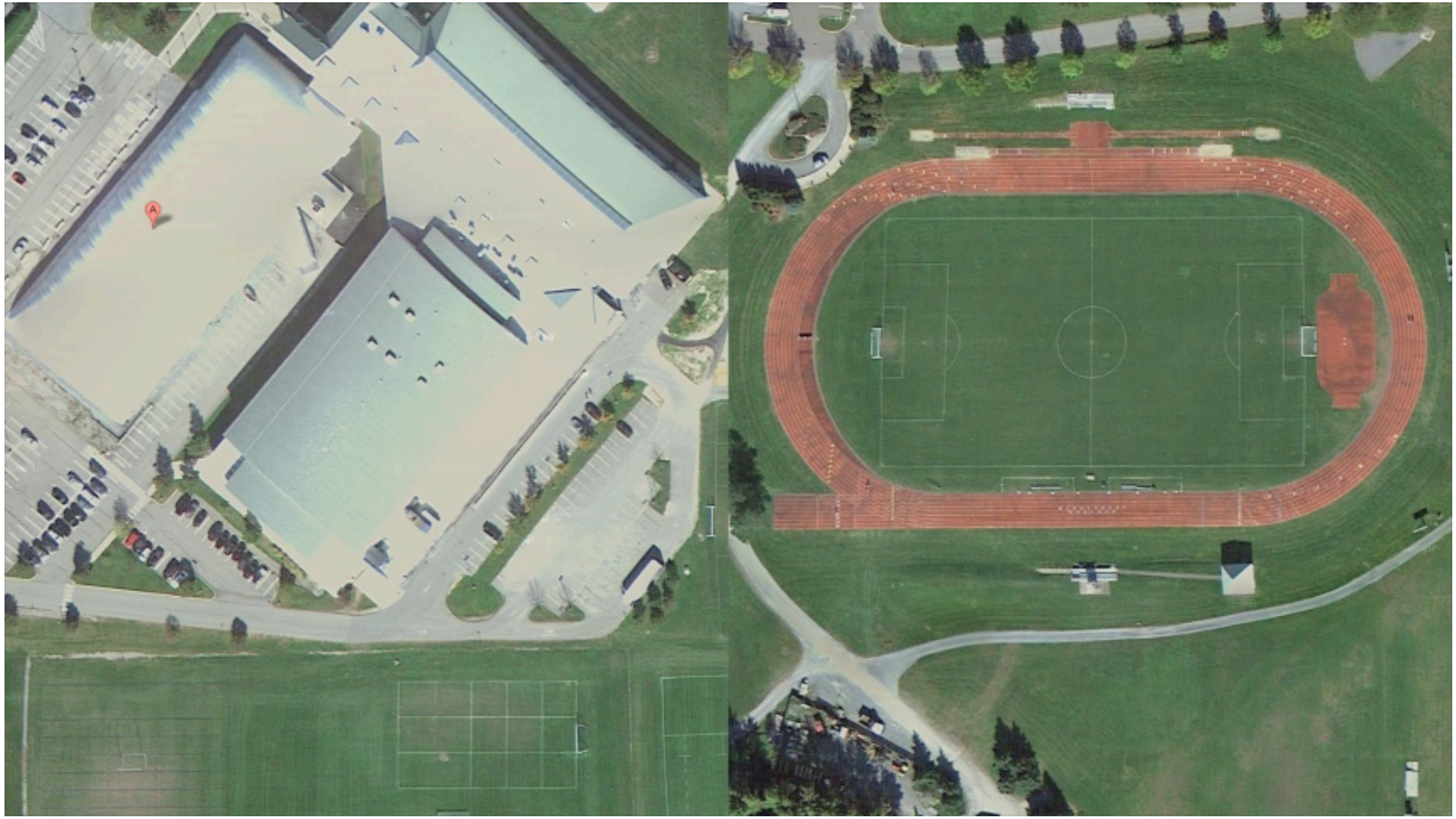
Write down your estimate.

What information would be helpful to know here?

Split Time



Split Time



$$\frac{160}{x} = \frac{400}{75}$$

Representation: Ratio Table

metres	400			
seconds	75			

Representation: Ratio Table

metres	400	40		
seconds	75	7.5		

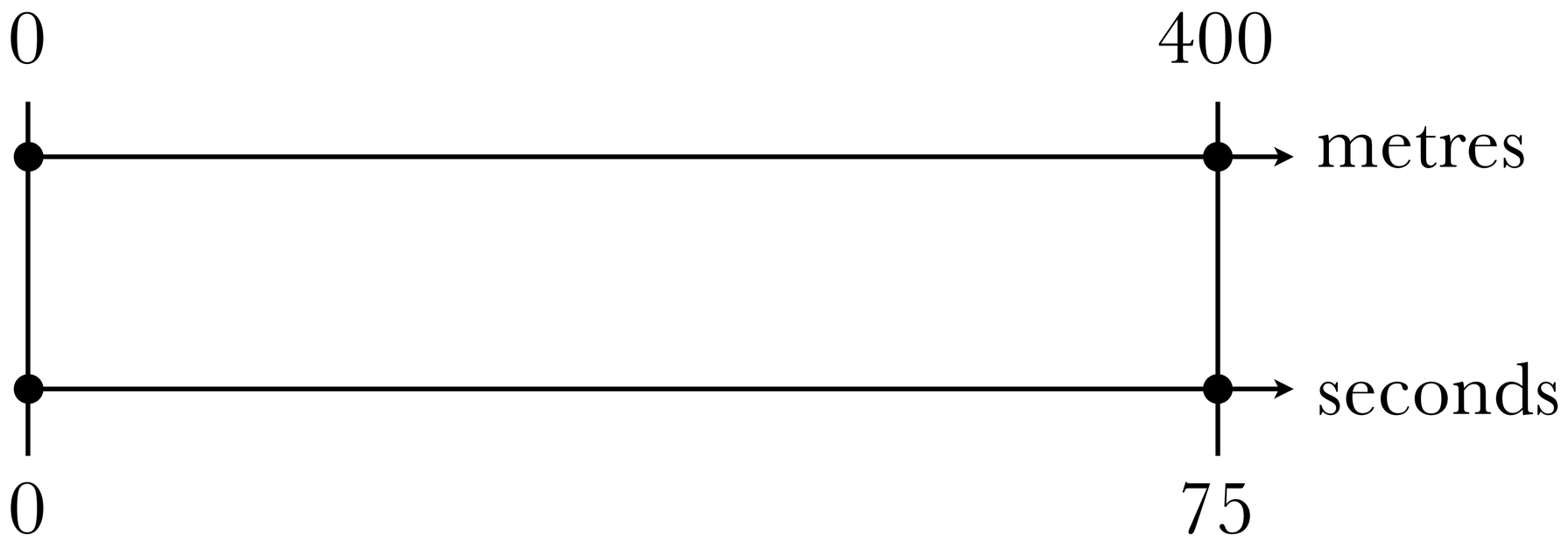
Representation: Ratio Table

metres	400	40	80	
seconds	75	7.5	15	

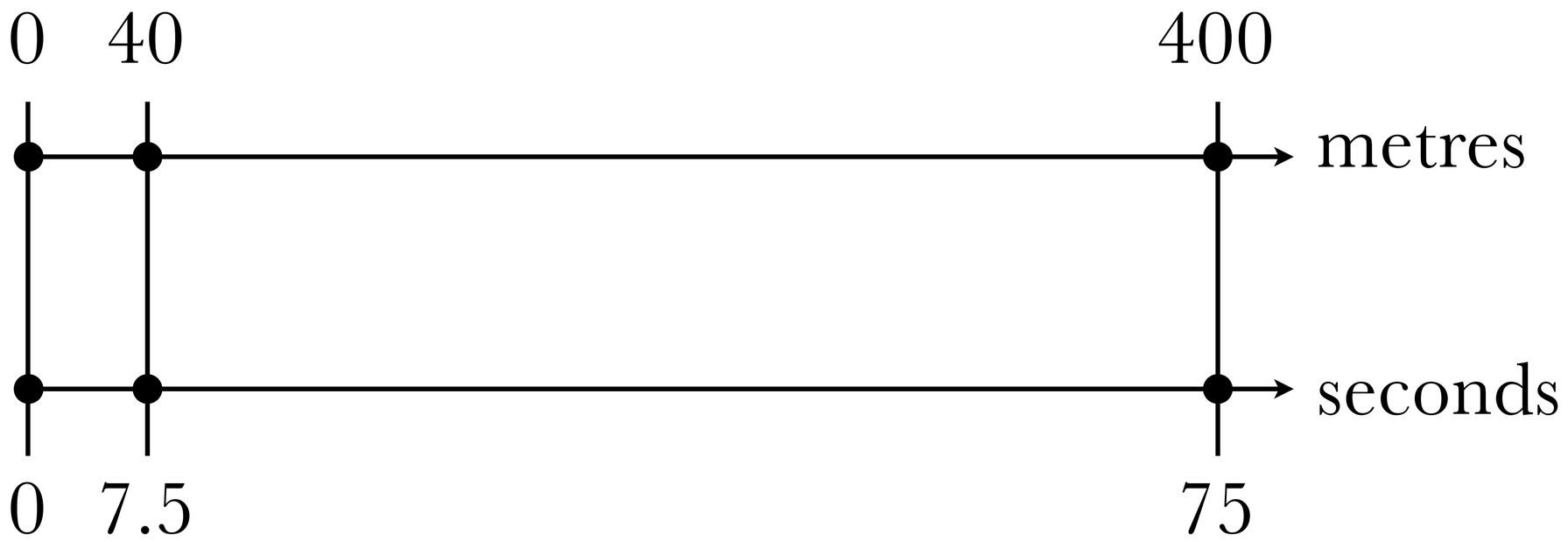
Representation: Ratio Table

metres	400	40	80	160
seconds	75	7.5	15	30

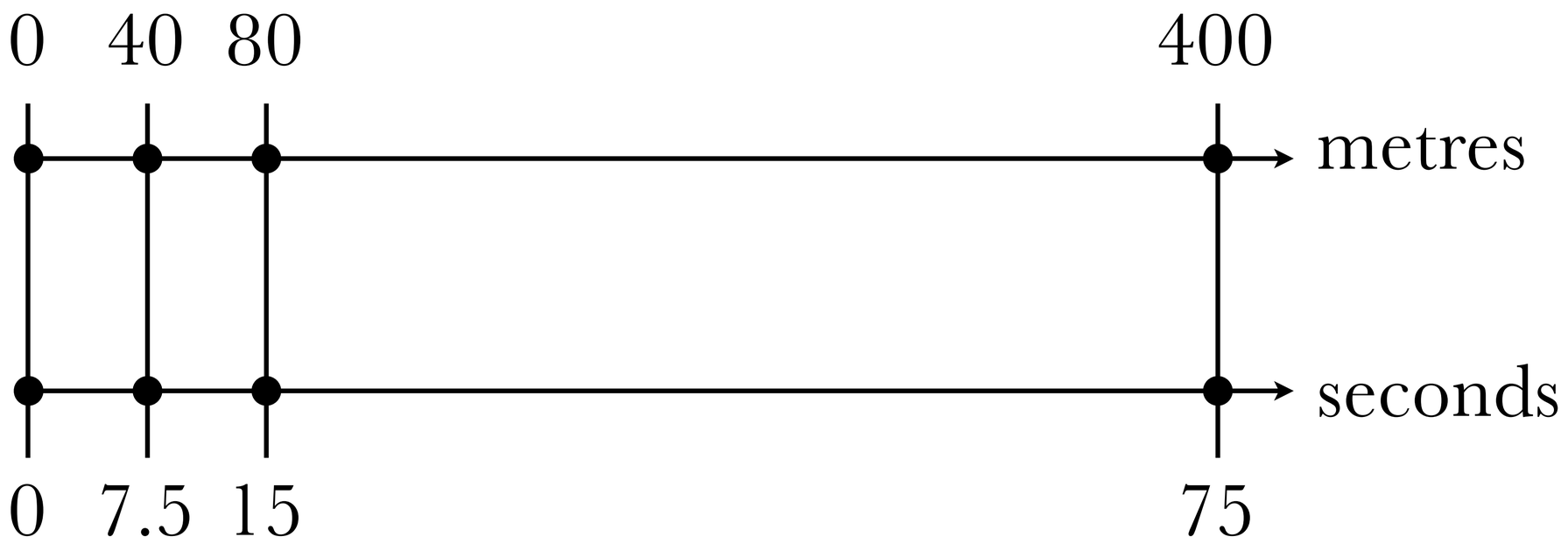
Representation: Double Number Line



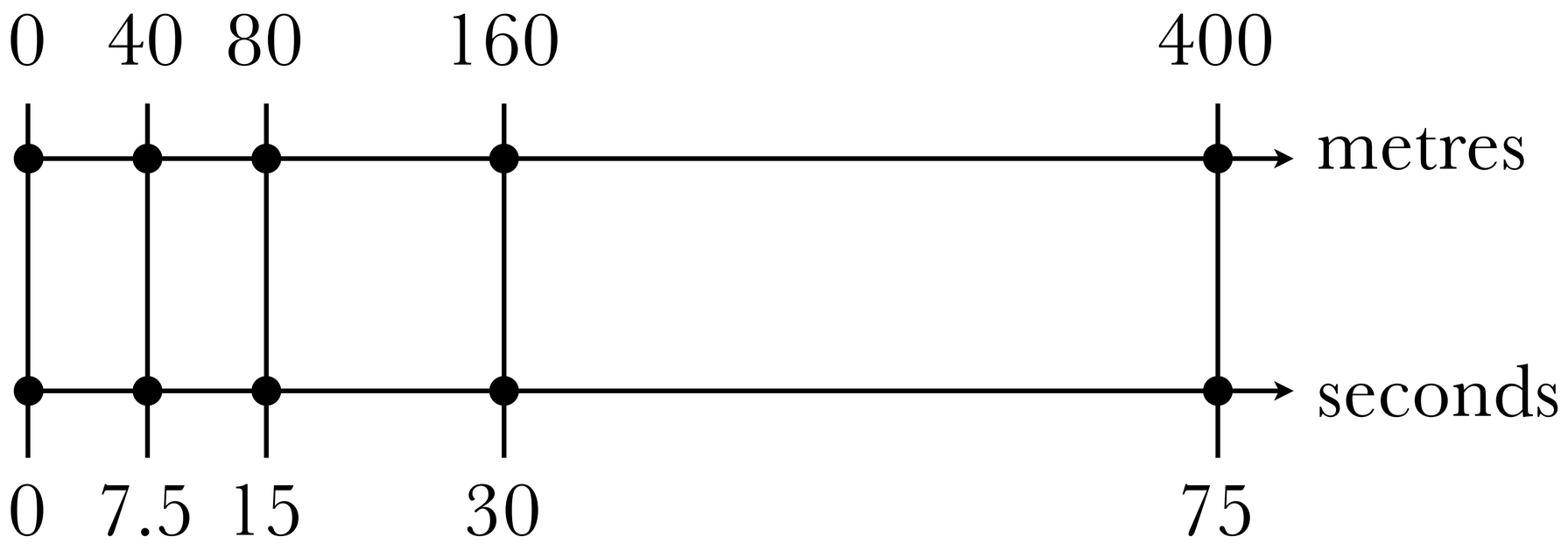
Representation: Double Number Line

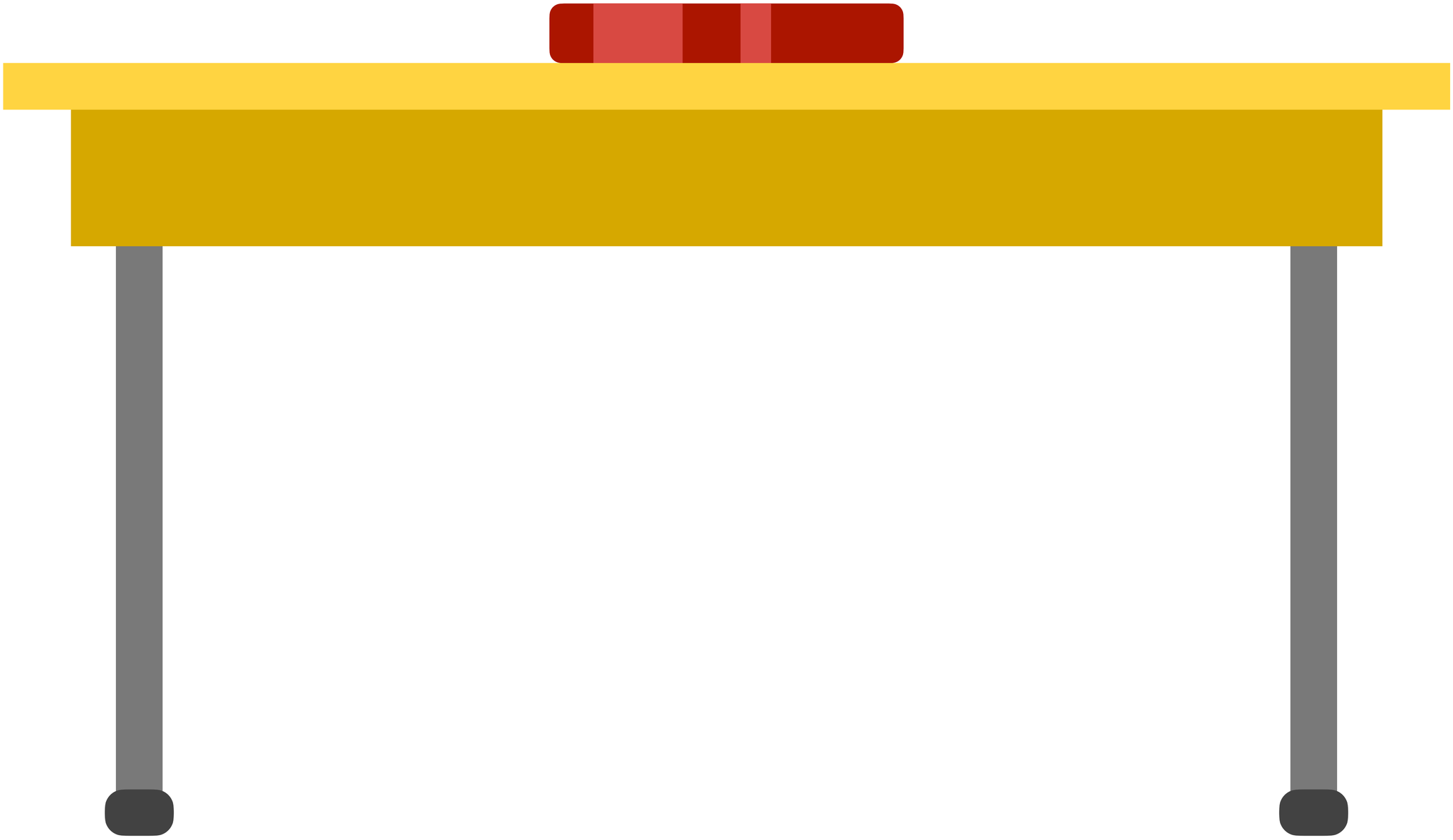


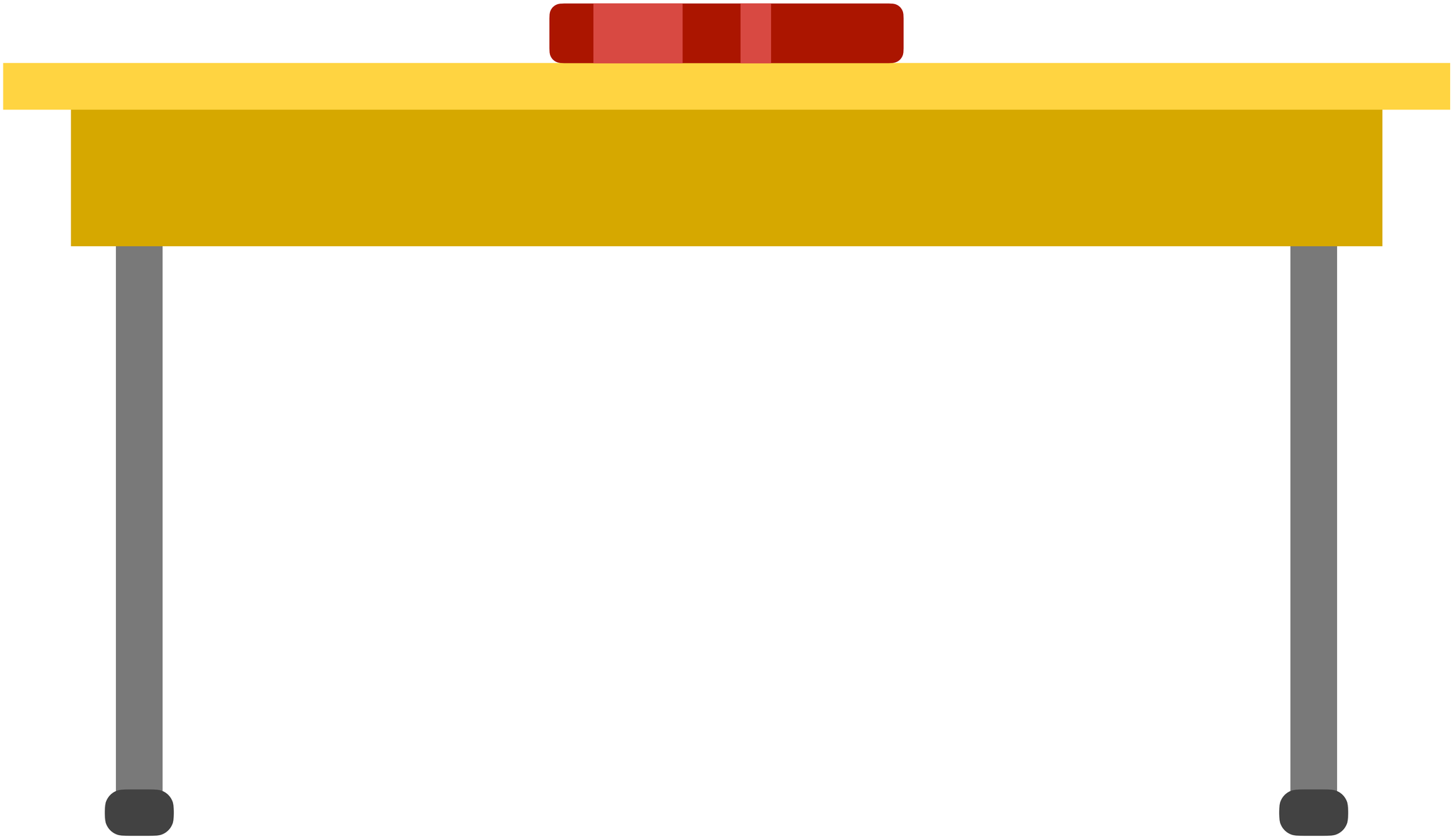
Representation: Double Number Line

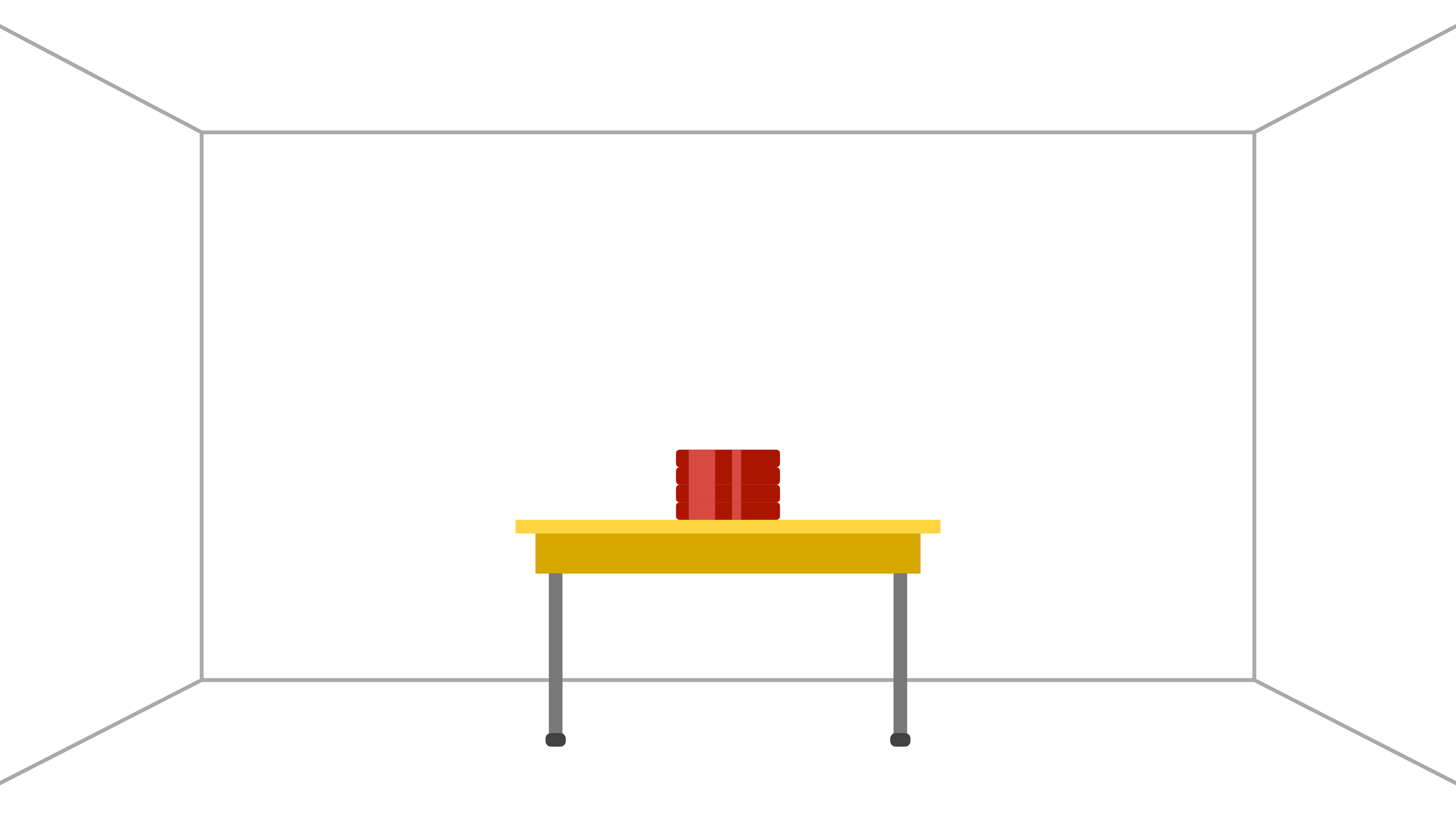


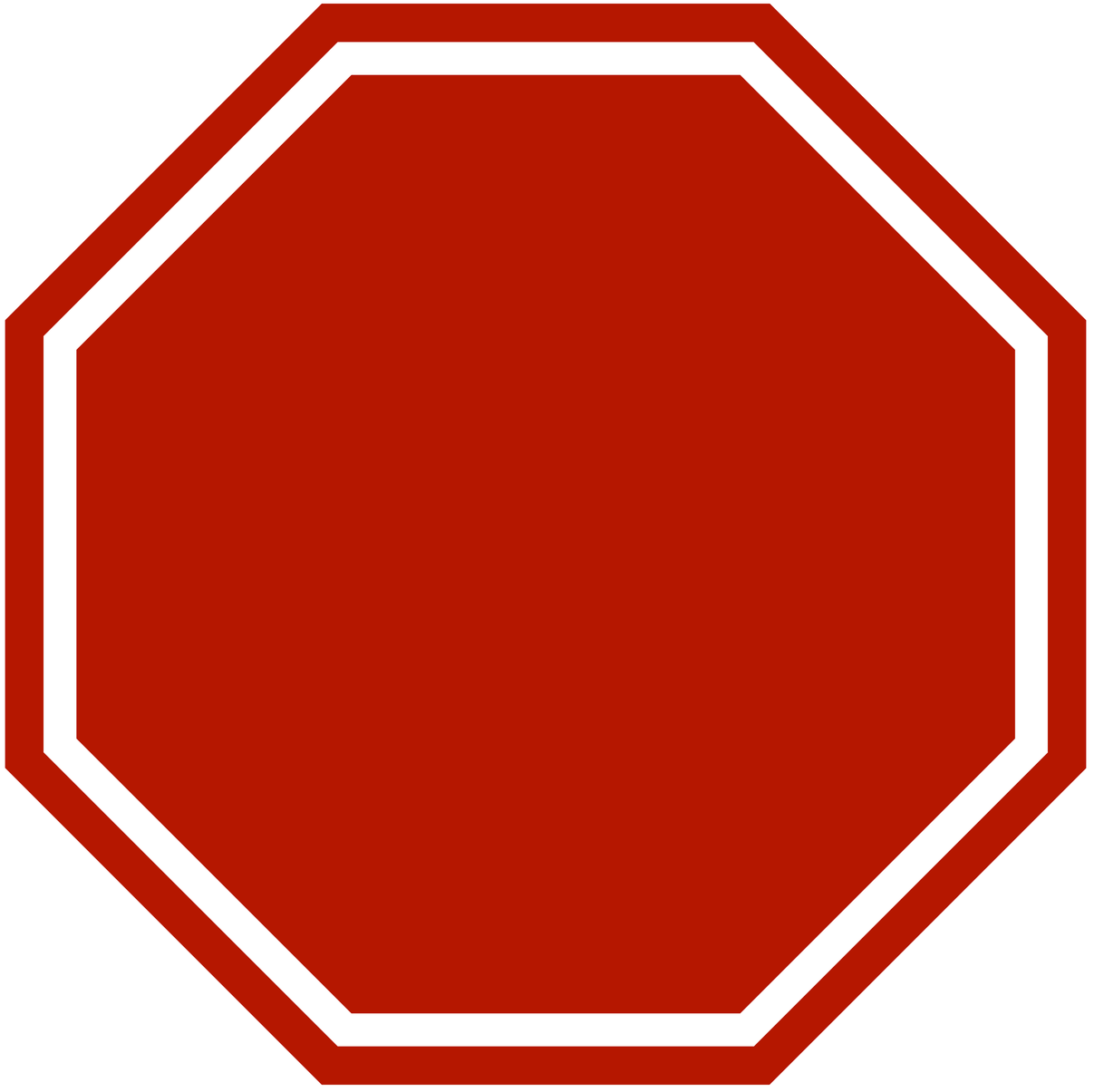
Representation: Double Number Line

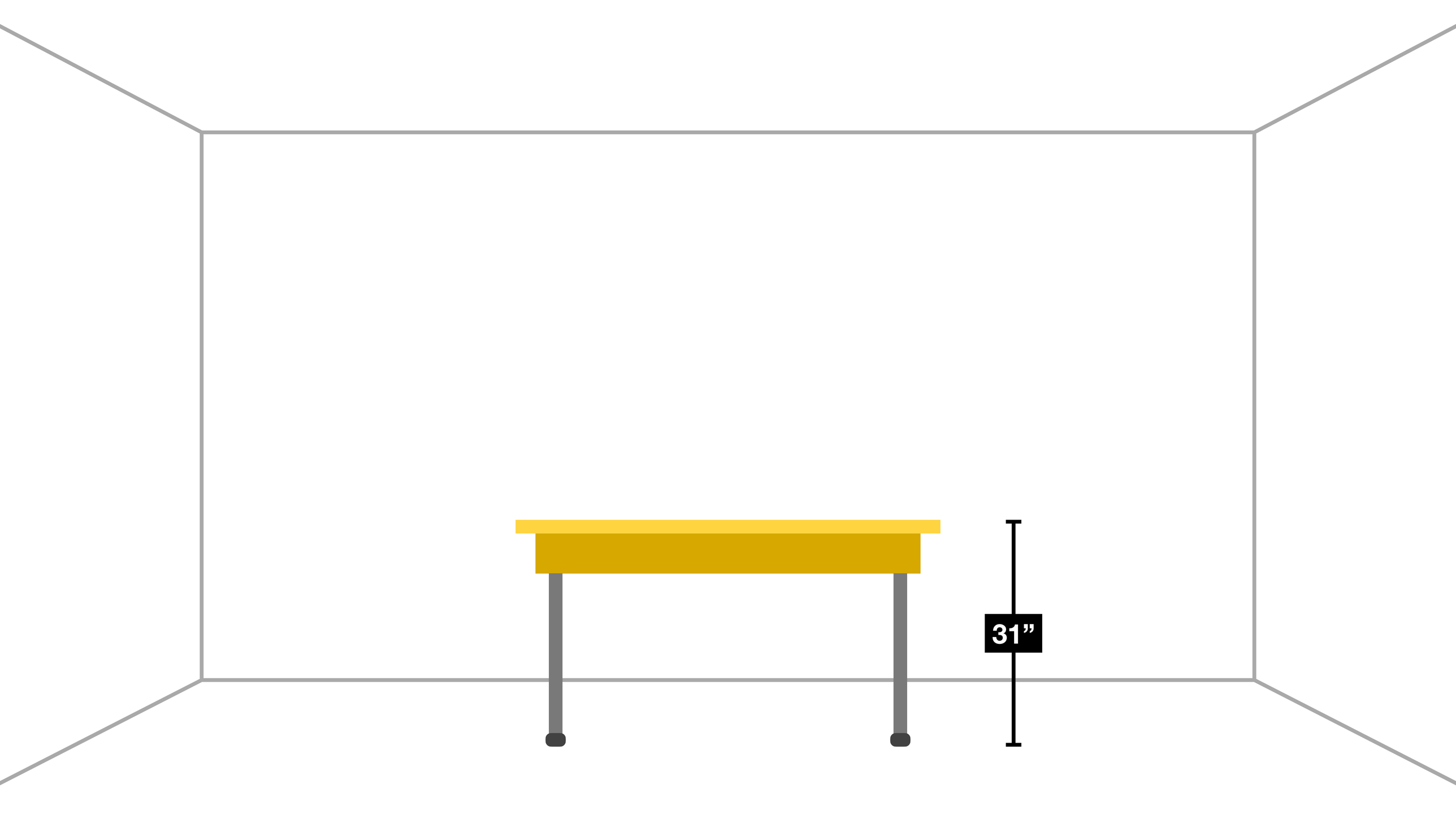


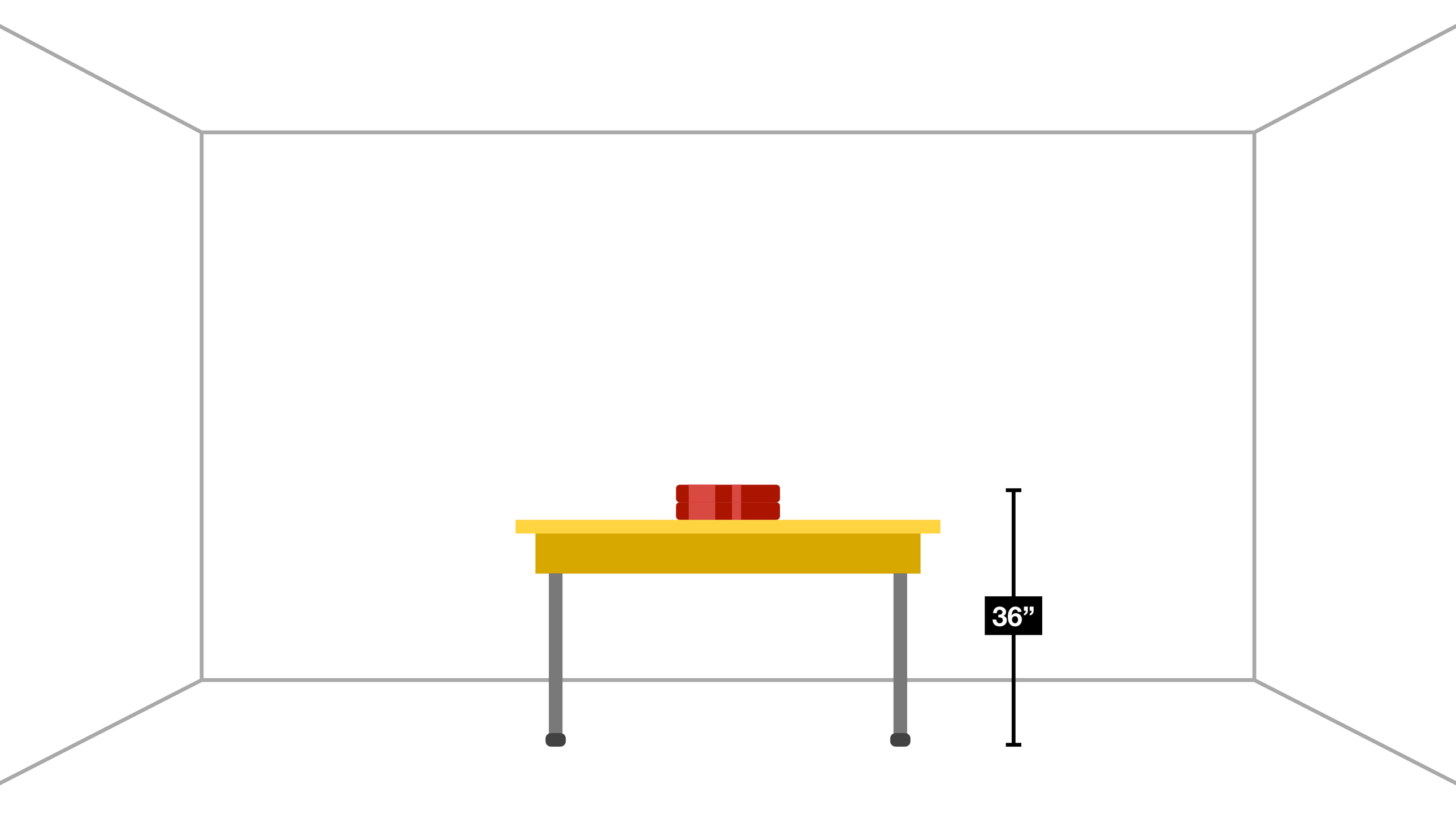


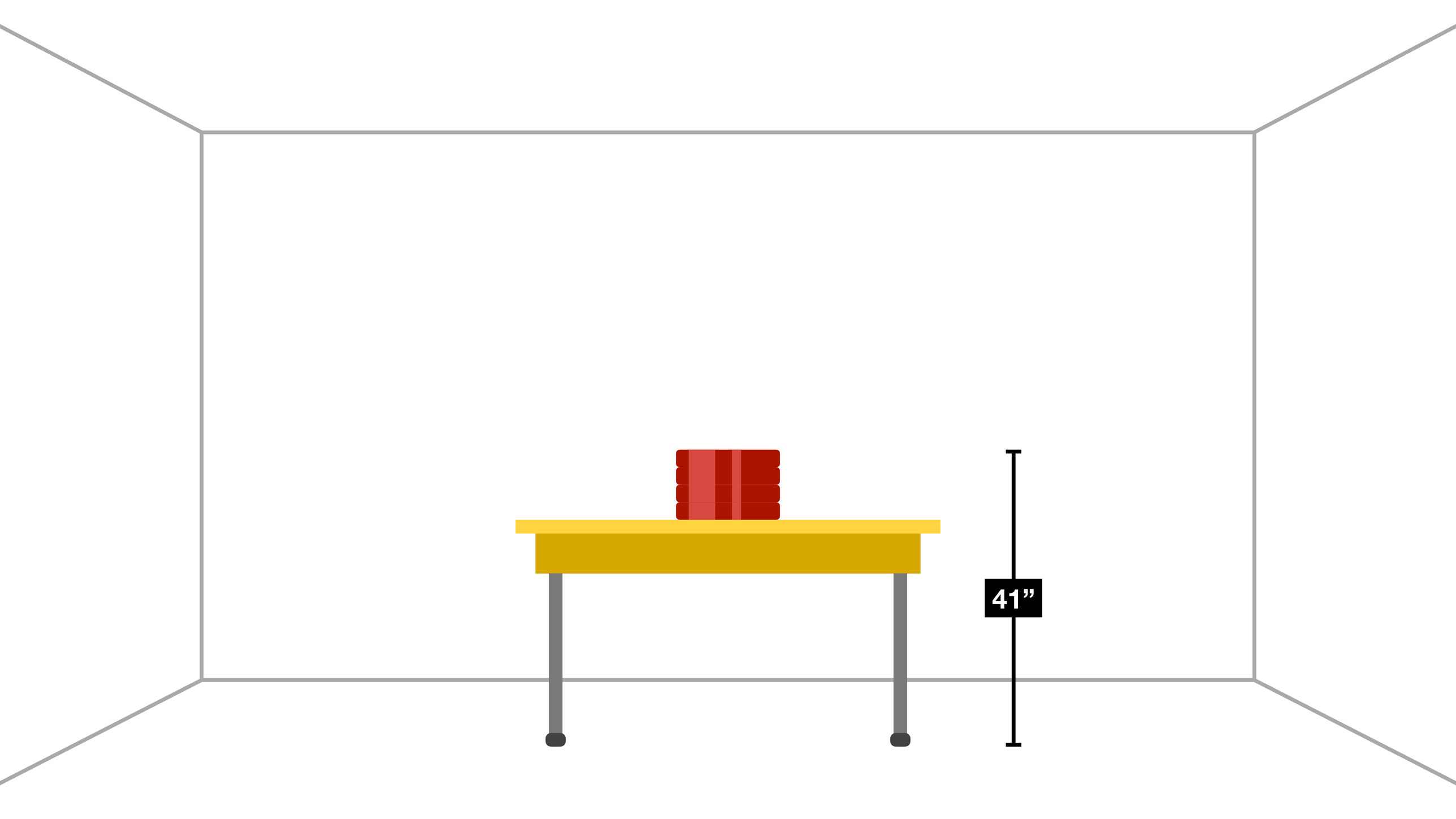


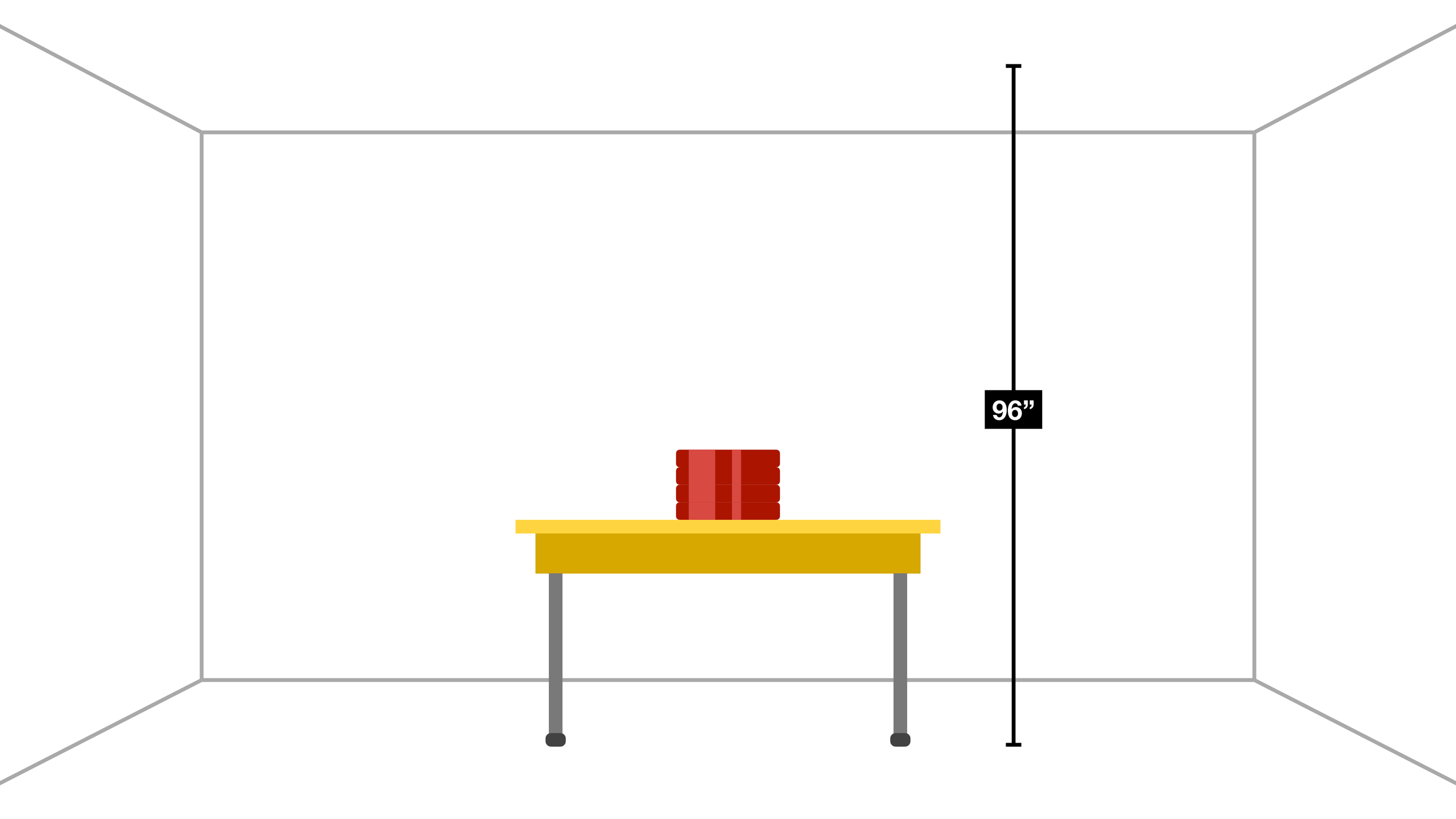


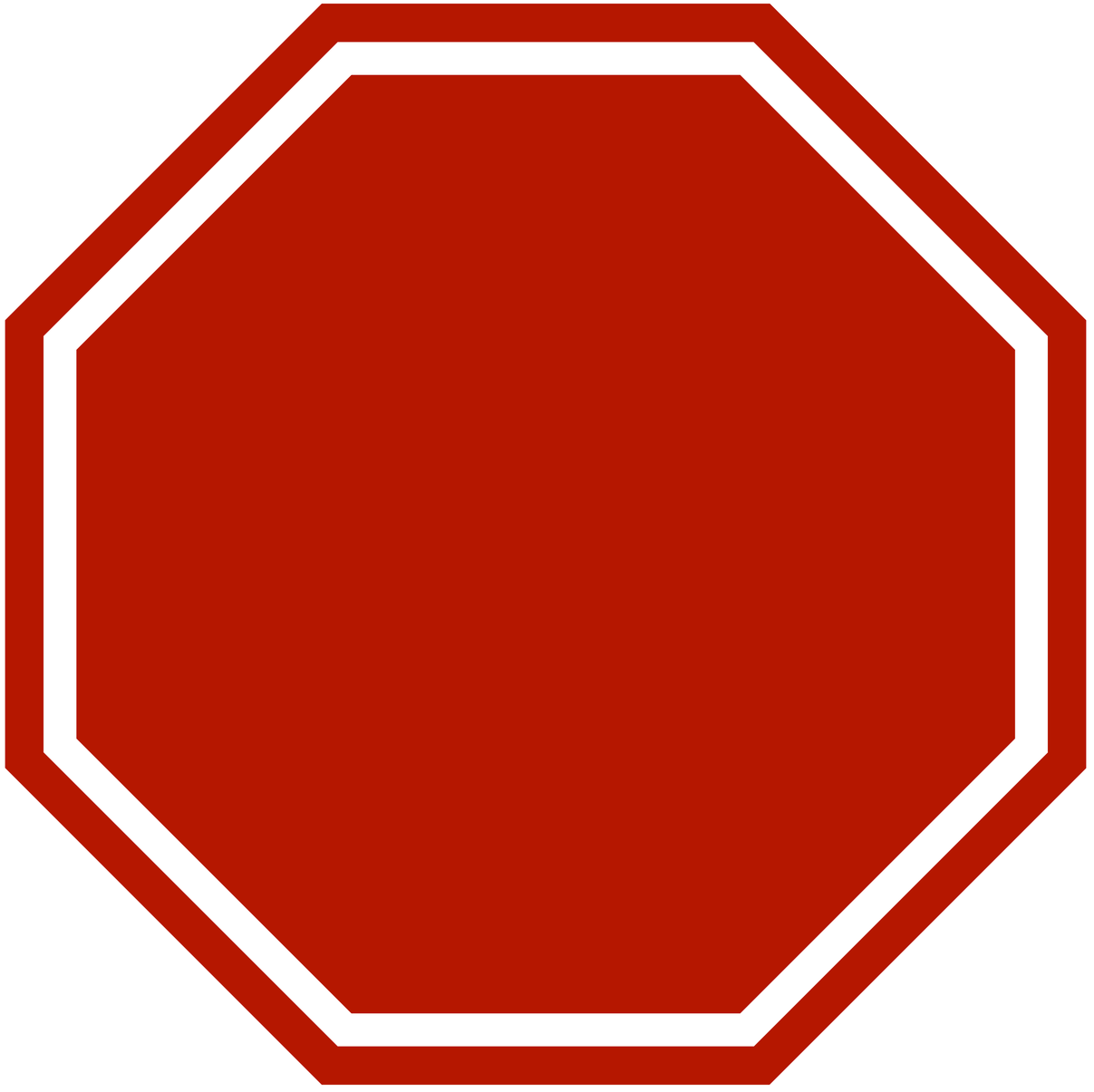


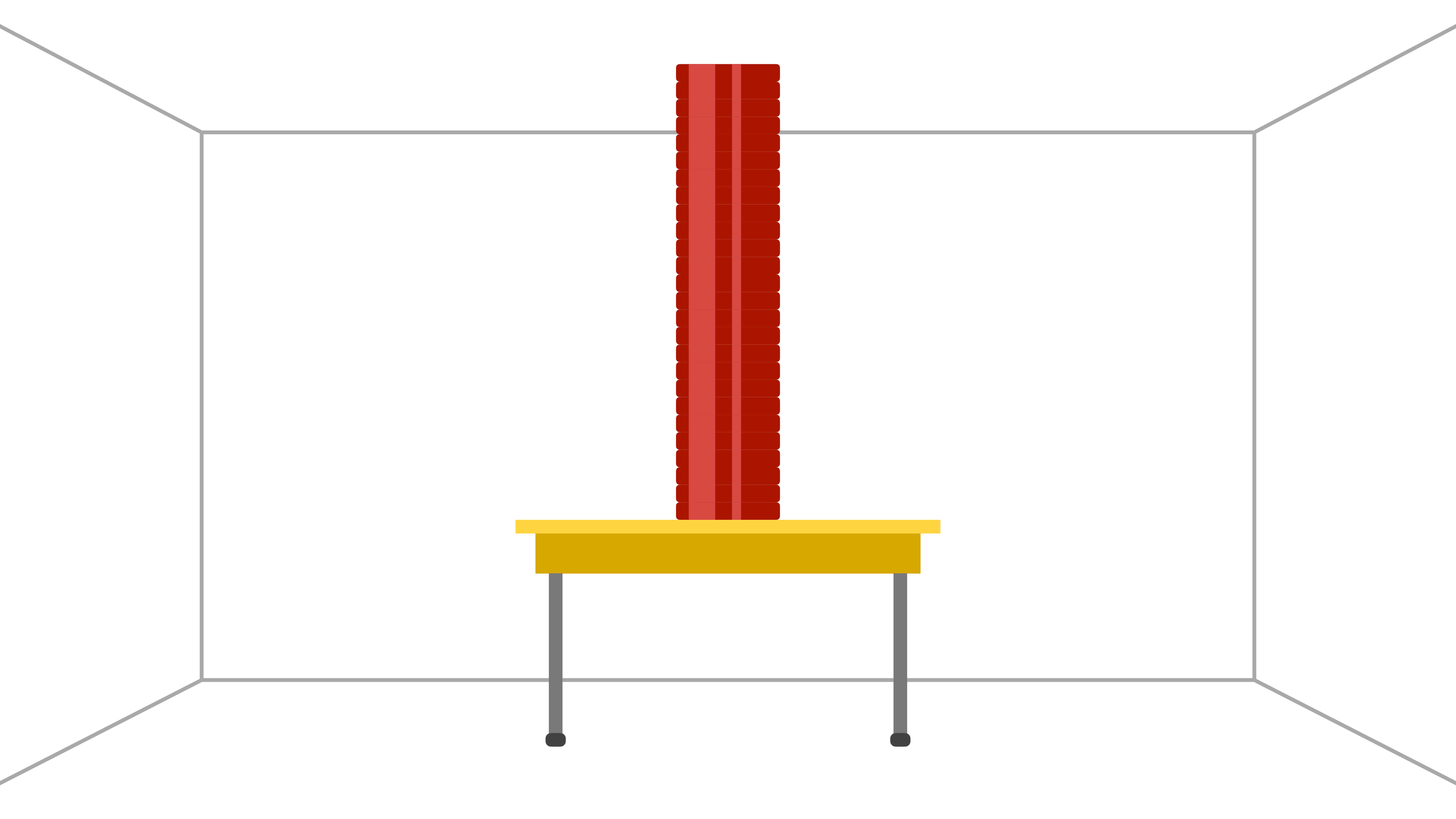

31”

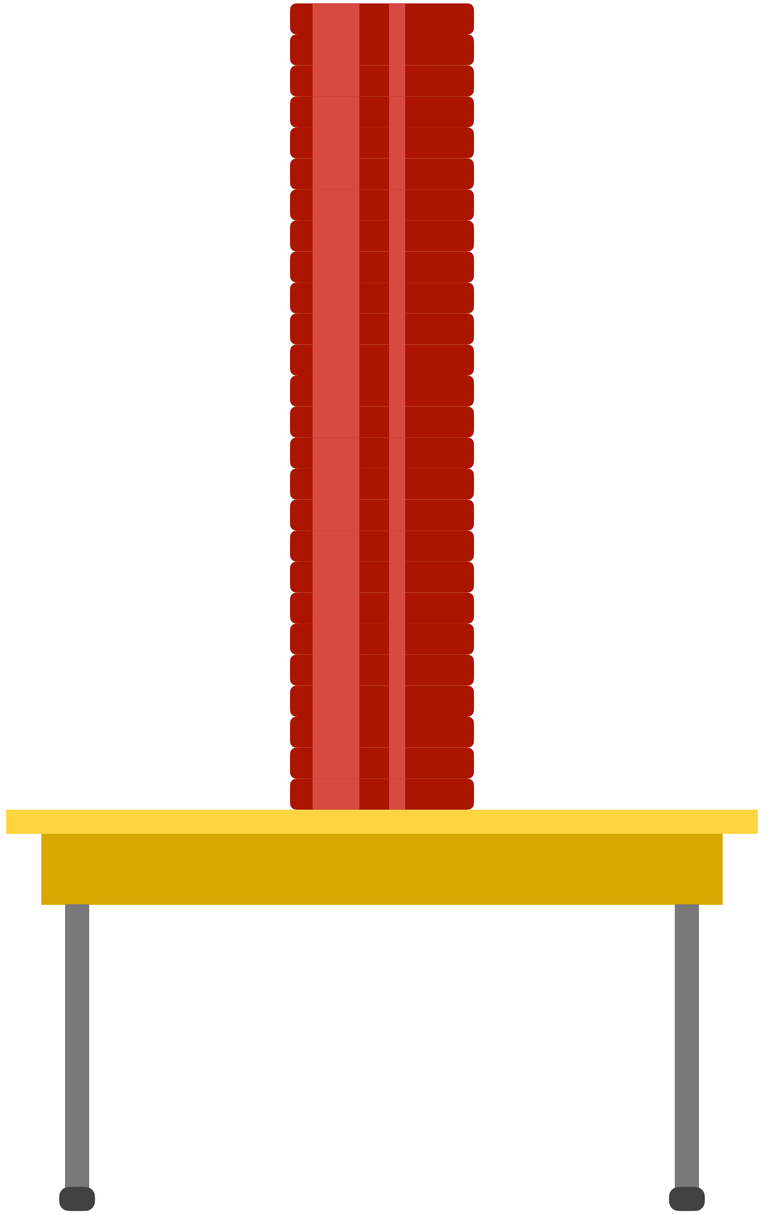

36”

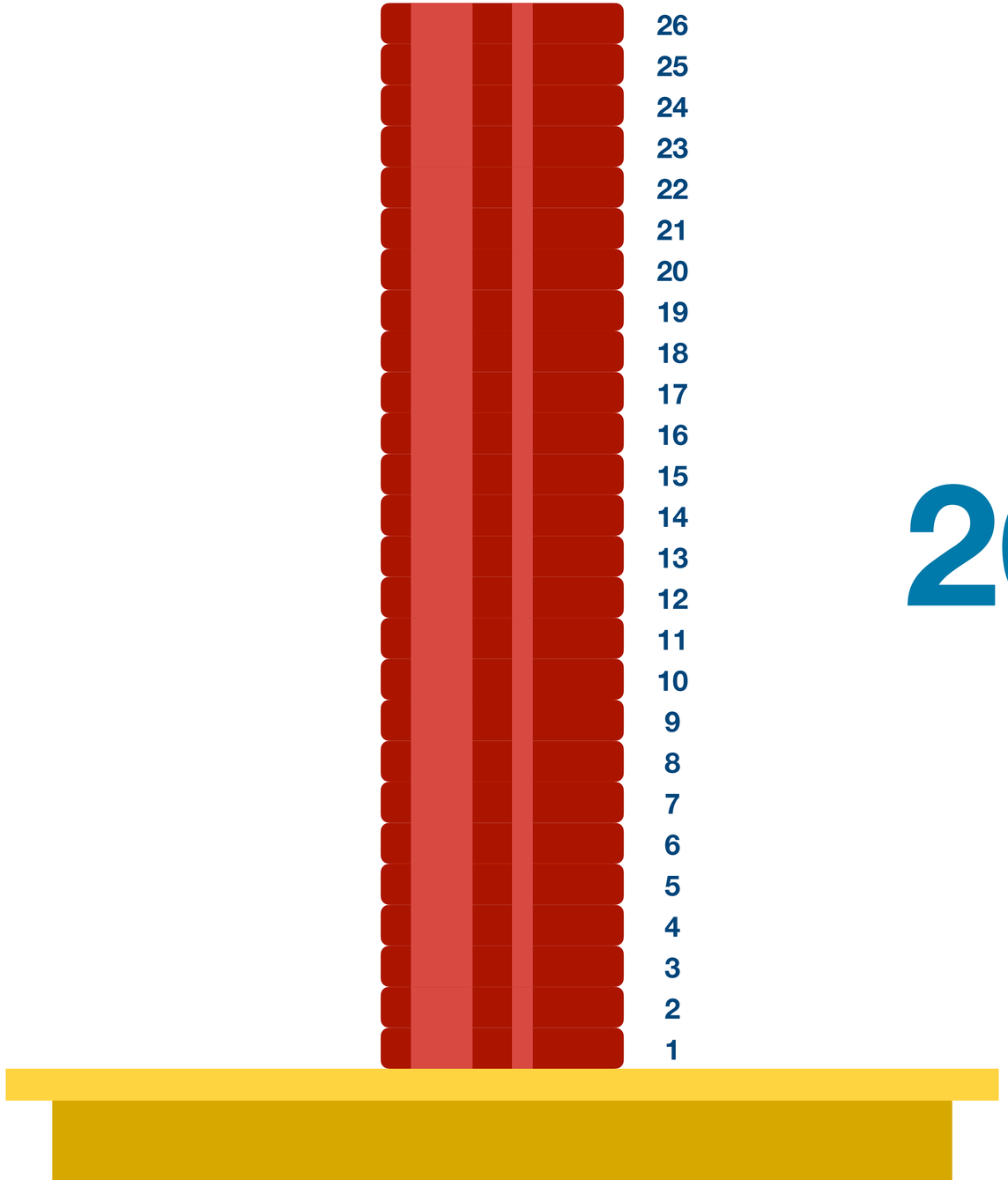

41”


96”







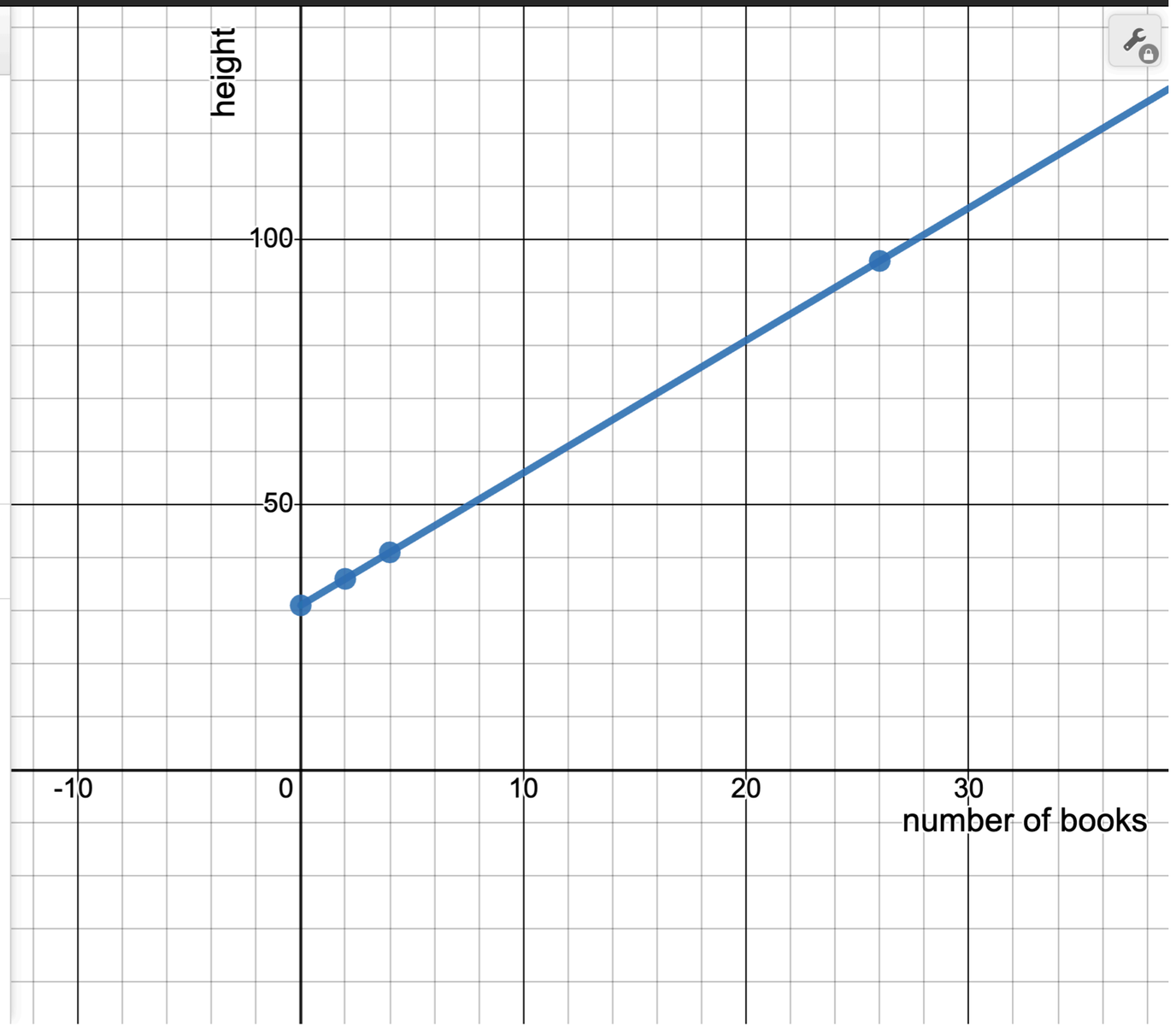


26

Stacks Save desmos Chris

x_1	y_1
0	31
2	36
4	41
------	------
26	96

$y = 2.5x + 31 \ \{x \geq 0\}$



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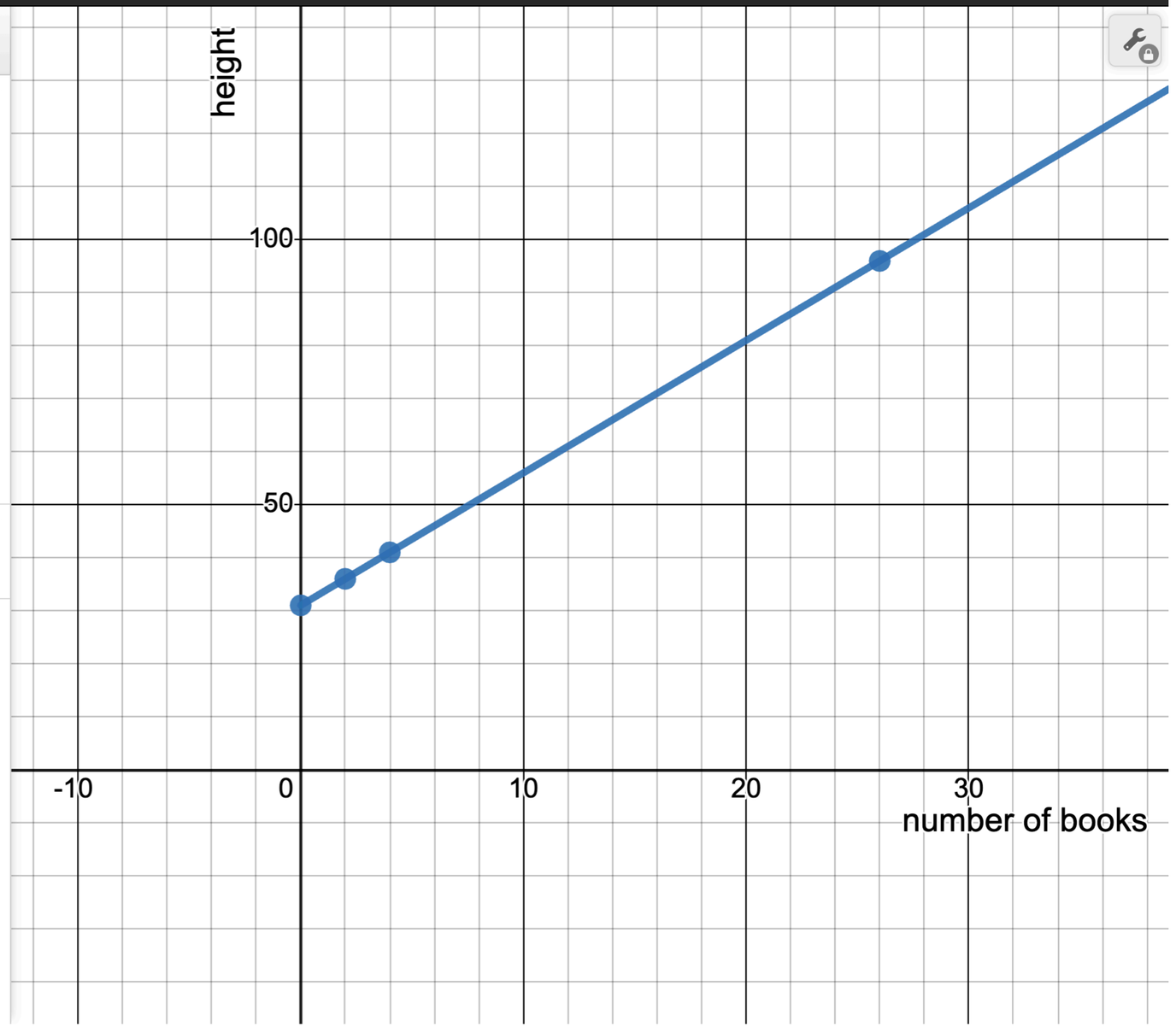
Which representation is the *best*?

Stacks Save desmos Chris

x_1	y_1
0	31
2	36
4	41
------	------
26	96

$y = 2.5x + 31 \ \{x \geq 0\}$



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Which representation is the *best*?

decisions about mathematical objects

Menu Math

A.	Is even	B.	Is a multiple of 3
C.	Is a perfect cube	D.	Is prime
E.	Is a factor of 72	F.	Is a perfect square
G.	Has exactly 4 four factors	H.	Is odd
I.	Is composite	J.	Is divisible by 12

natbanting.com/menu-math/

Menu Math

A.	Two negative x-intercepts	B.	Vertex in quadrant II
C.	Never enters quadrant III	D.	Vertex on the y-axis
E.	Positive y-intercept	F.	No x-intercepts
G.	Never enters quadrant I	H.	Has a minimum value
I.	Horizontally stretched	J.	Line of symmetry enters quadrant IV

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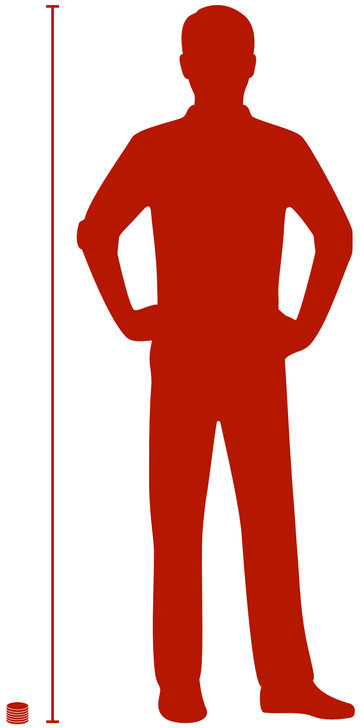
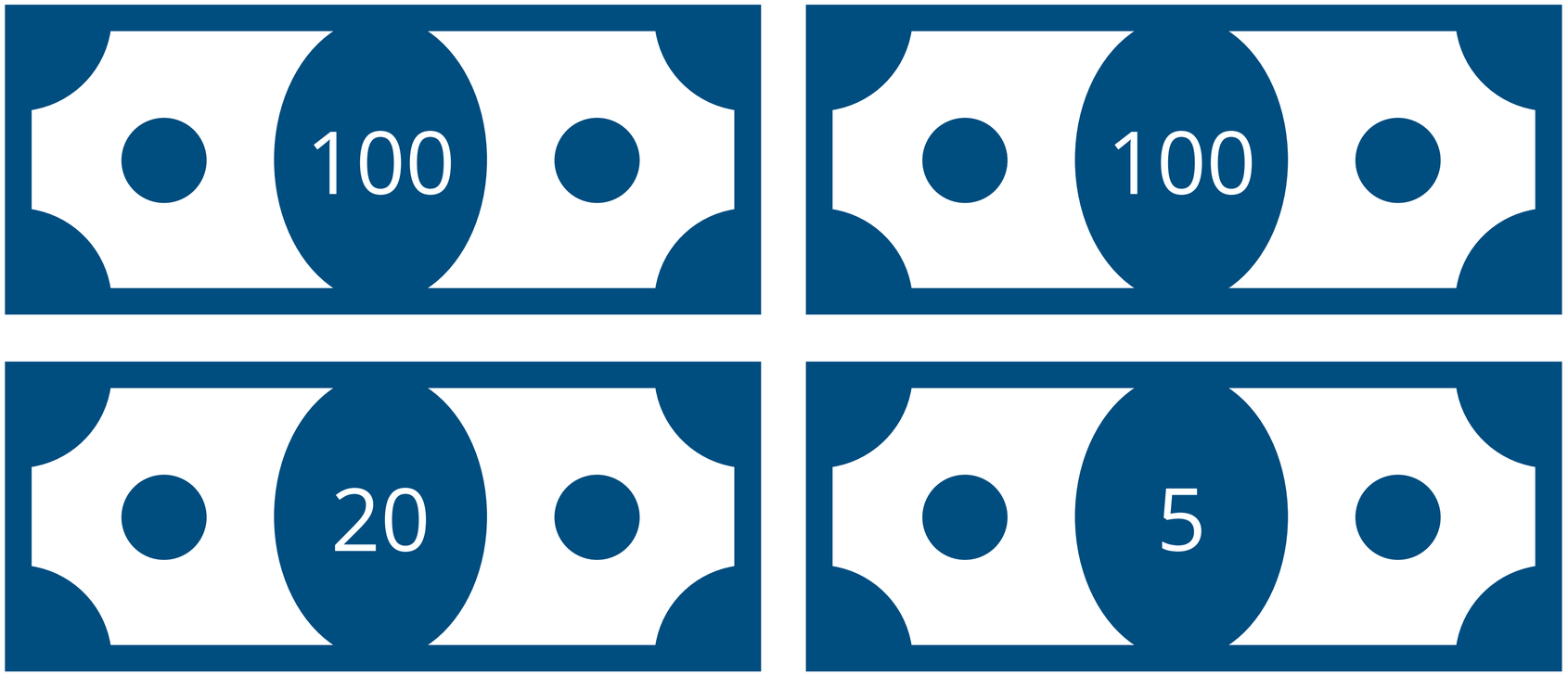
decisions, decisions

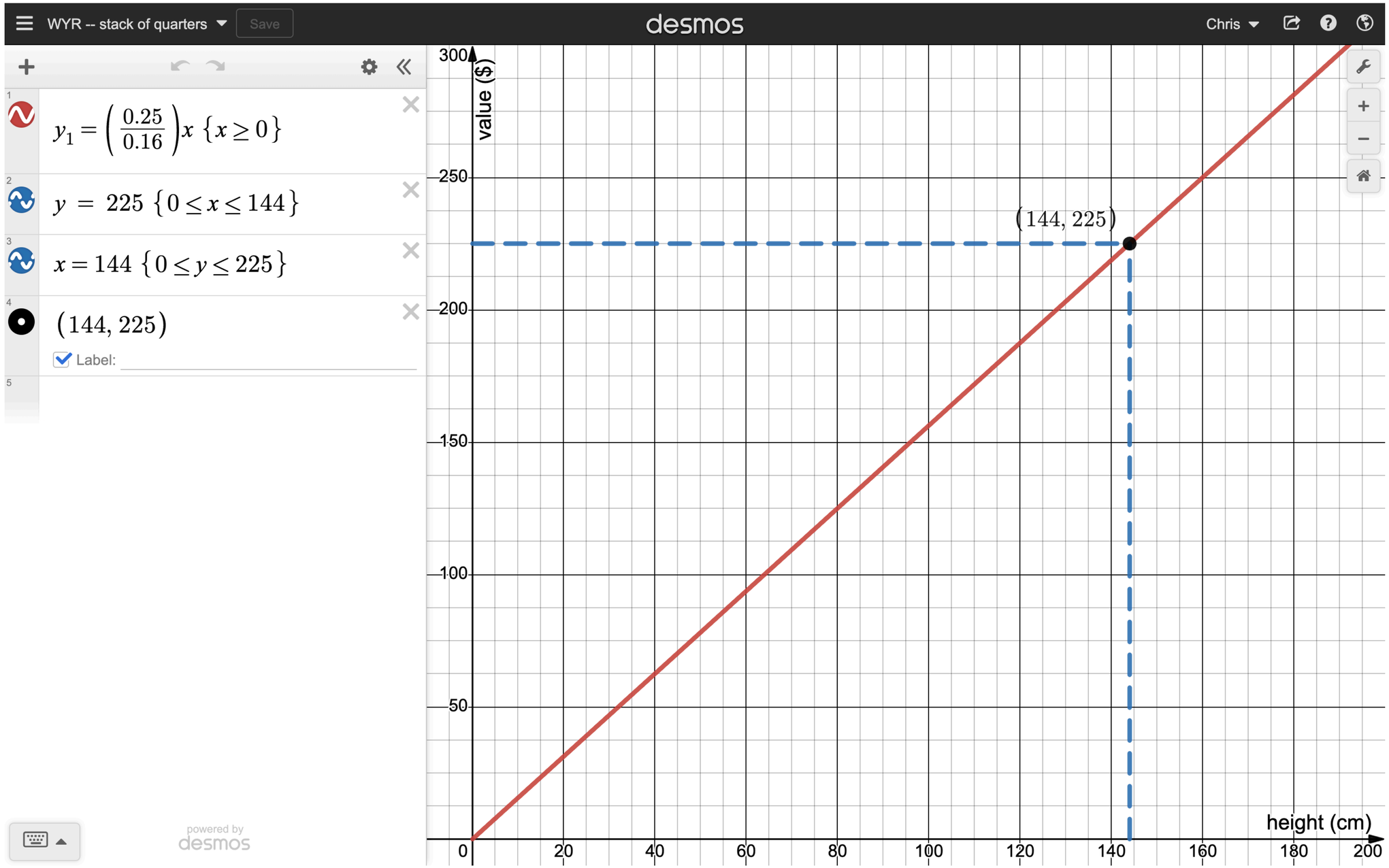
… have a stack of quarters from the floor to the top of your head or $225?





25¢			23.88 mm	1.58 mm

wouldyourathermath.com

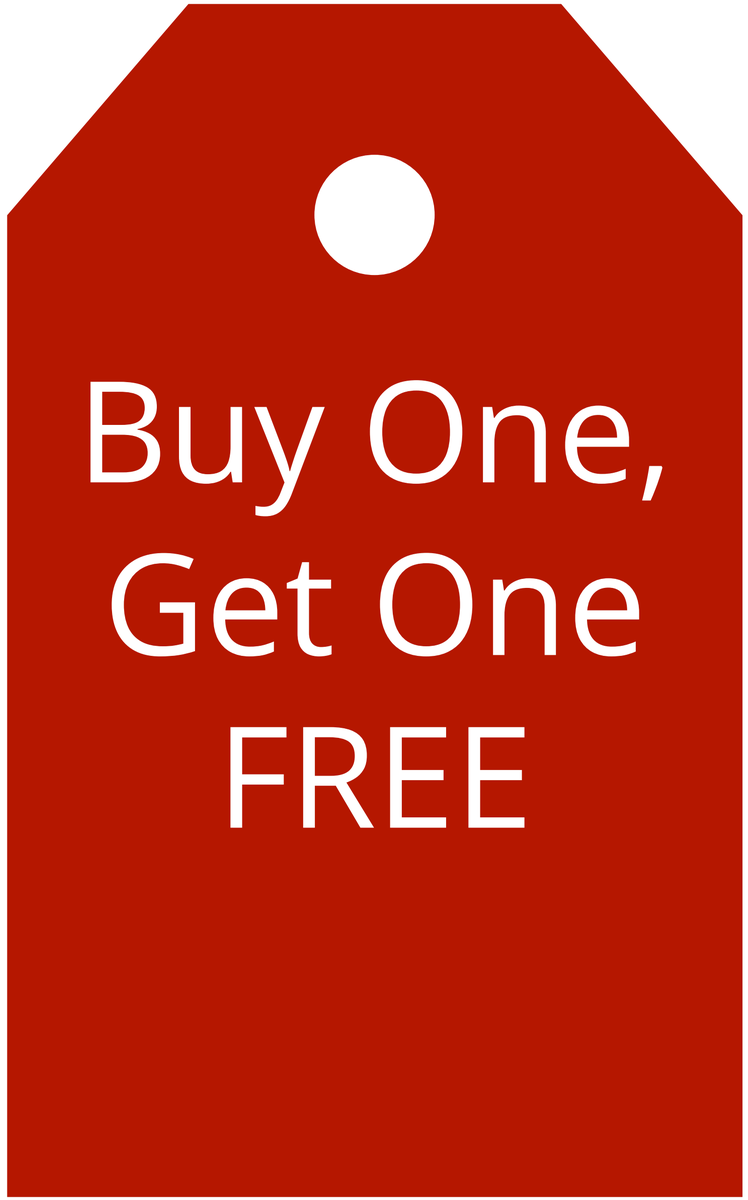

WYR -- stack of quarters
Save
desmos
Chris
$y_1 = \left(\frac{0.25}{0.16}\right)x \ \{x \geq 0\}$
$y = 225 \ \{0 \leq x \leq 144\}$
$x = 144 \ \{0 \leq y \leq 225\}$
$(144, 225)$
Label:
value ($)
height (cm)
(144, 225)
300
250
200
150
100
50
0
20
40
60
80
100
120
140
160
180
200
powered by
desmos

robertkaplinsky.com/work/ticket-option/

Student A		
Number of Tickets	**Price**	**Unit Price**
1	$0.50	$0.50
10	$5.00	$0.50
20	$10.00	$0.50
50	$25.00	$0.50
100	$50.00	$0.50

Student B		
Number of Tickets	**Price**	**Unit Price**
1	$0.75	$0.75
12	$8.00	$0.67
25	$15.00	$0.60
50	$25.00	$0.50
120	$48.00	$0.40

assumptions


Buy One,
Get One
FREE

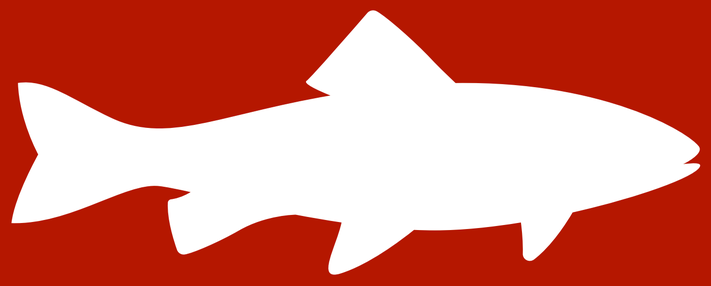

40%
OFF

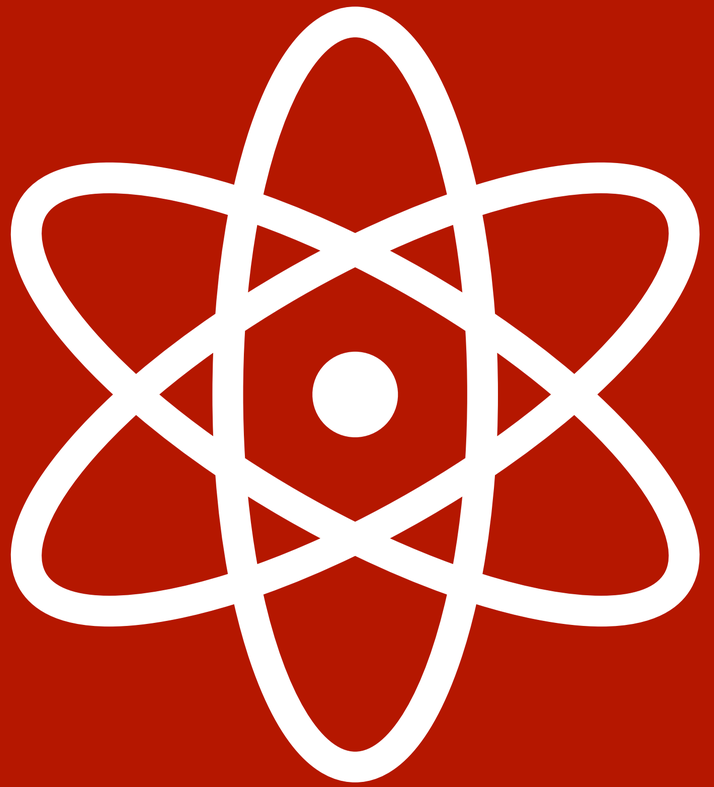

Buy One,
Get One
50% OFF


20%
OFF

… be a server at restaurant A or B?

inspired by wouldyourathermath.com





	1st Choice	2nd Choice
Aquarium	12	5
Planetarium	8	14
Science World	10	11

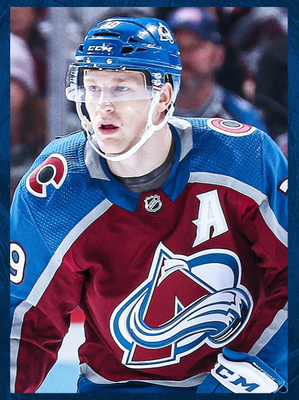
	14	7	5	26
	10	13	7	30
	9	15	13	37

	9	15	13	37
	10	13	7	30
	14	7	5	26

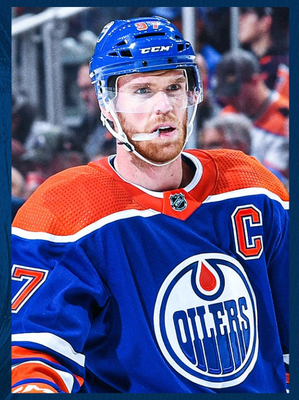
	14	7	5	26
	10	13	7	30
	9	15	13	37

1st	10 pts
2nd	7 pts
3rd	5 pts
4th	3 pts
5th	1 pt

WHO IS YOUR HART TROPHY FAVOURITE?

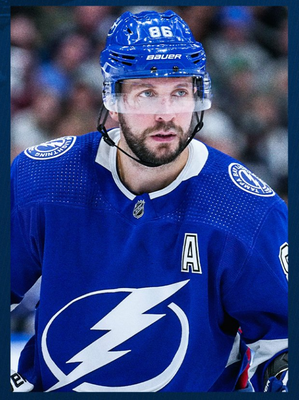


NATHAN MacKINNON



CONNOR McDAVID

AUSTON MATTHEWS



NIKITA KUCHEROV

OTHER

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