Multiplication: More Than Repeated Addition and Times Tables

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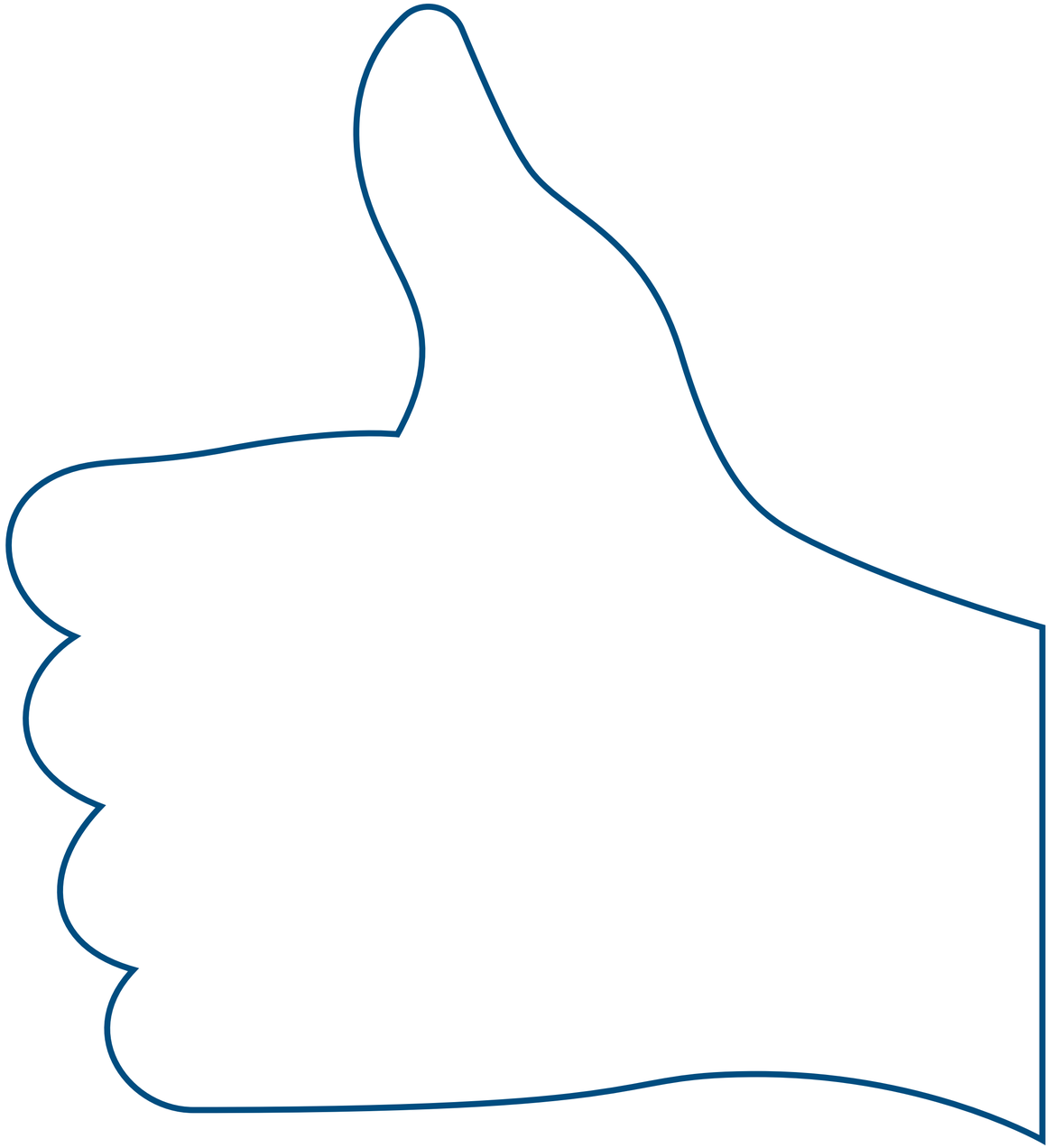
BCAMT

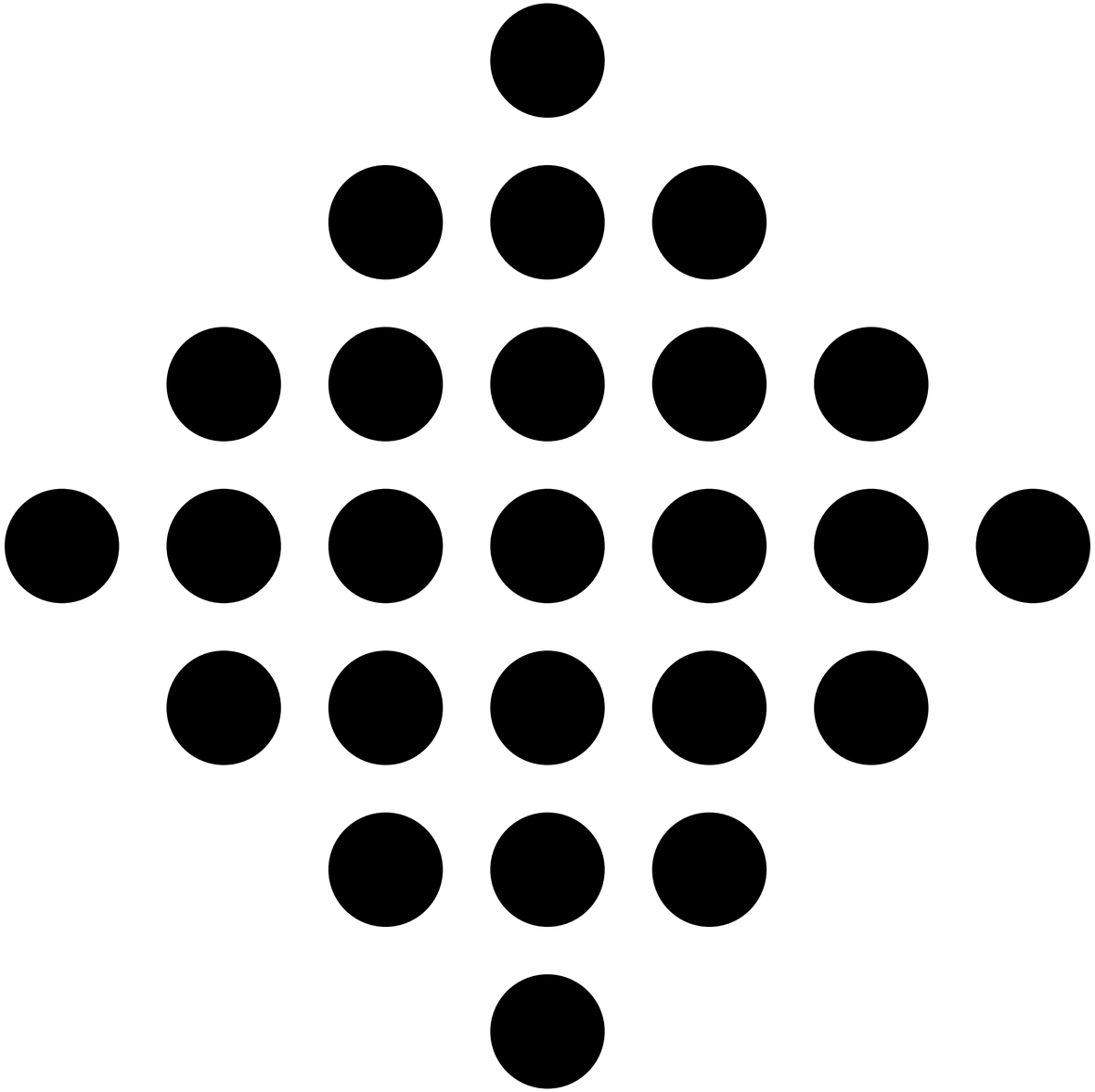
British Columbia Association of Mathematics Teachers

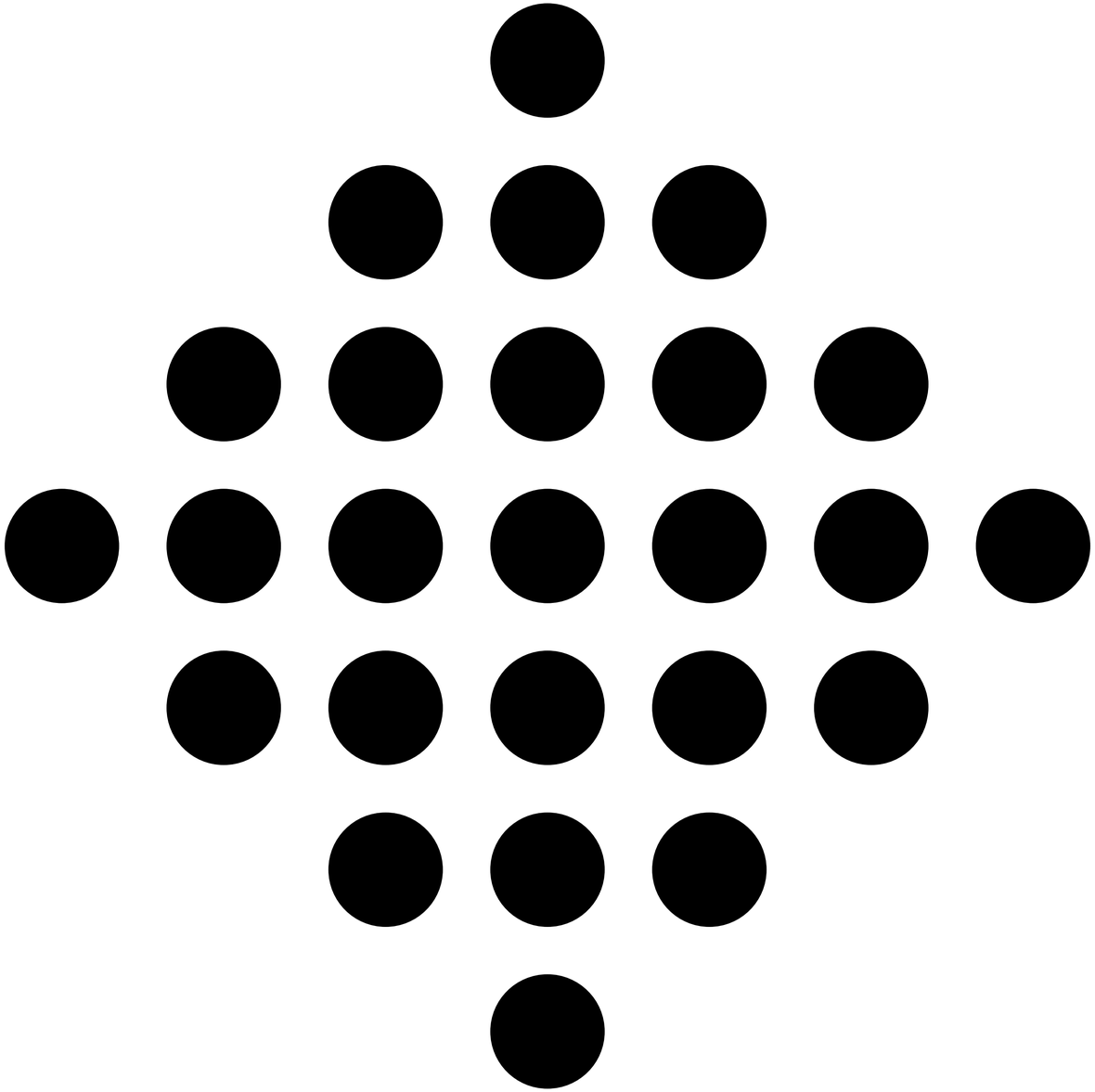
More Than Repeated Addition

*How many do you **see**?*

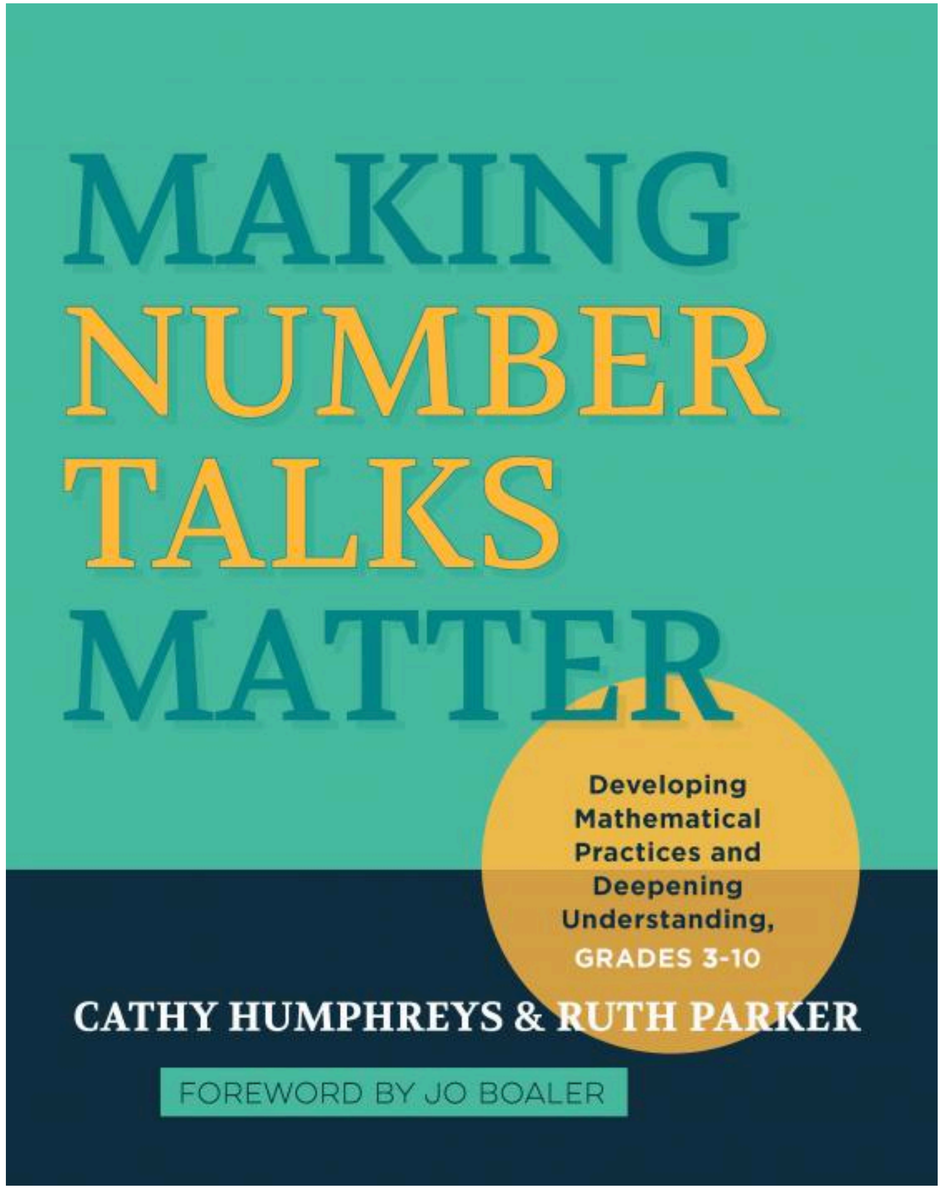
*How do **you** see them?*







"No matter what grade you teach, even high school, so-called 'dot cards' (which may or may not have dots) are a great way to start your students on the path to mathematical reasoning. We say this because, from experience, we have realized that with dot cards, students only need to describe what they see—and ***people have many different ways of seeing!****"*



NUMBER TALK IMAGES

En vedette / Featured...

Custom Number Talk Images

Créer vos propres images! Voici une ressource de Berkeley Everett. Offert en format PowerPoint ou Google Slides, vous n'avez qu'à modifier les diapositives afin de produire des images sur mesure.
You can create your own Number Talk Images thanks to Berkeley Everett! Using his shared PowerPoint or Google Slides templates, you simply edit/re-arrange images of kumquats, blueberries, nuts and toaster pastries.

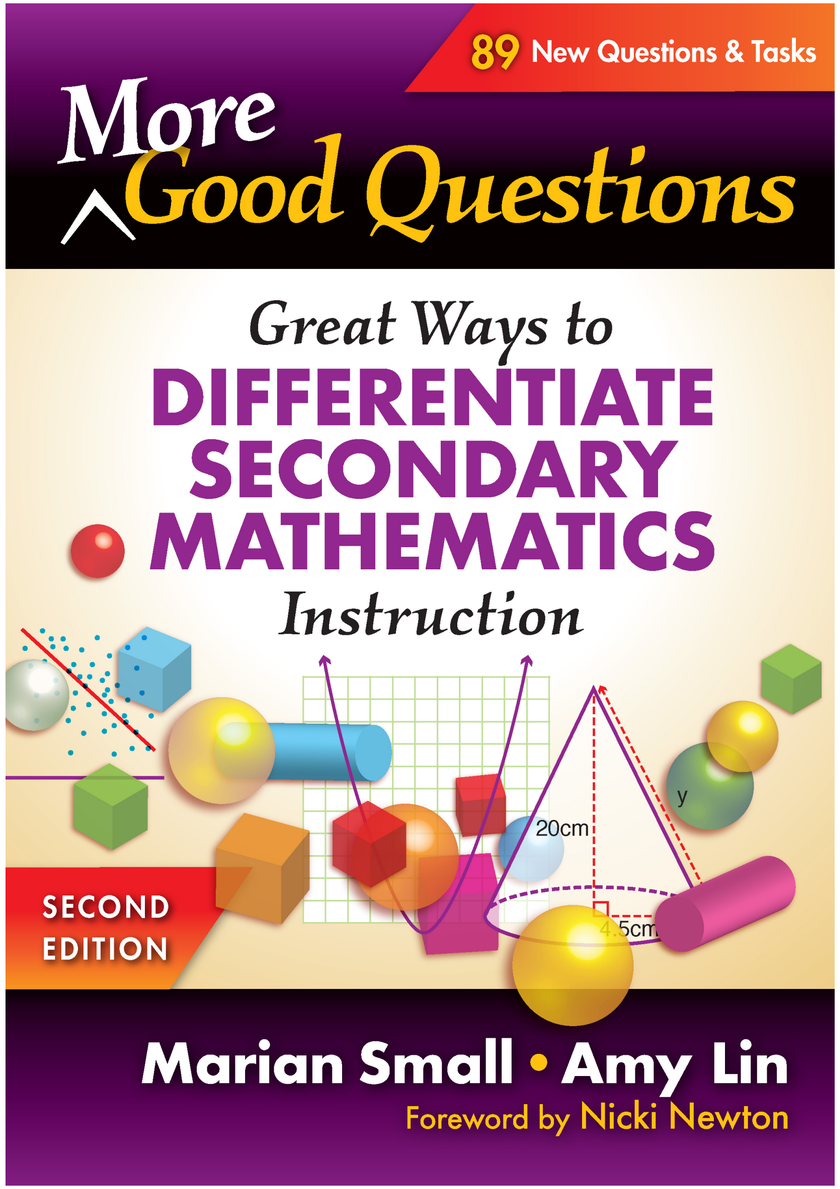
Ca-Lishea served at the Houston Food Bank by putting together food boxes that go to senior citizens. Each box contains 26 meals. The boxes are loaded onto a pallet that holds 45 boxes. How many meals are there on the pallet?

71

“When you’re a grown-up, nobody says
‘Please multiply these numbers.’
You have to know ***when*** *to do it.*
That’s all that actually matters when you’re a big person.”

Marian Small OAME2024

*"The operations of addition, subtraction, multiplication, and division hold the **same fundamental meanings** no matter the domain in which they are applied."*

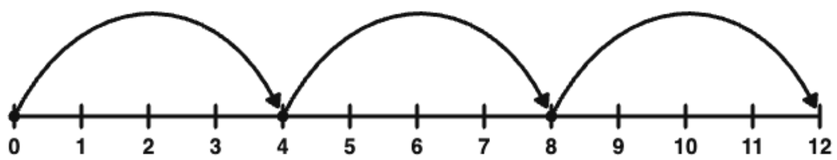


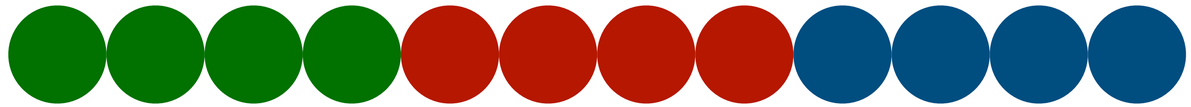
*What does multiplication **mean**?*

What does 3×4 ***look** like?*

Meanings

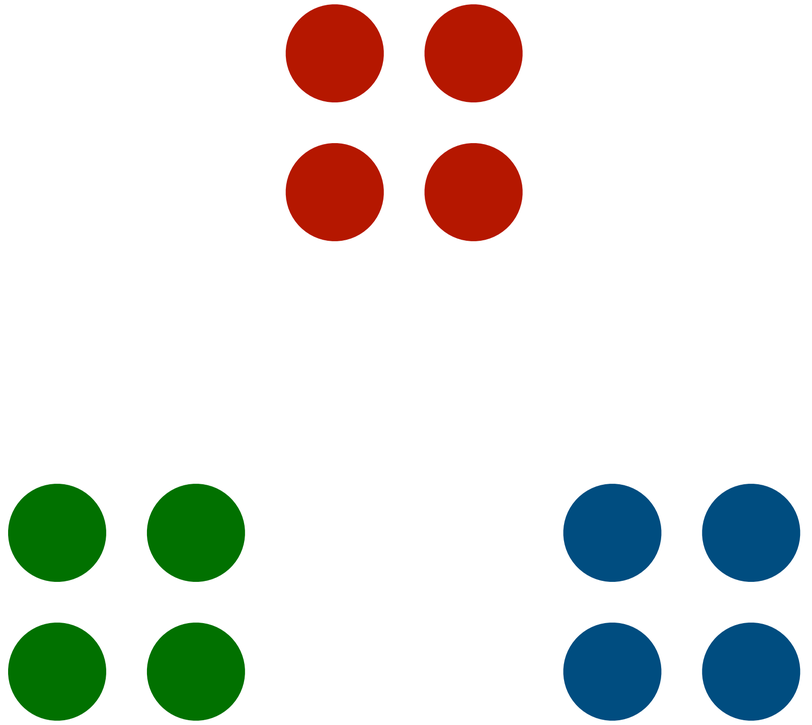
- Repeated Addition





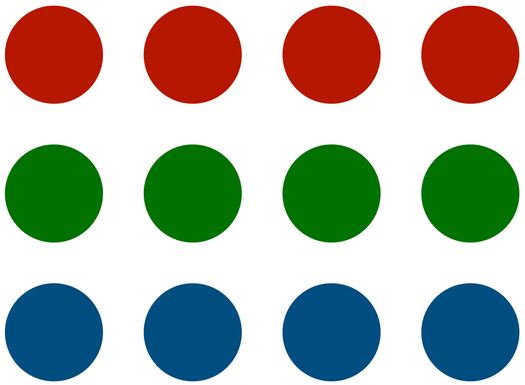
Meanings

- Repeated Addition
- Equal Groups or Sets



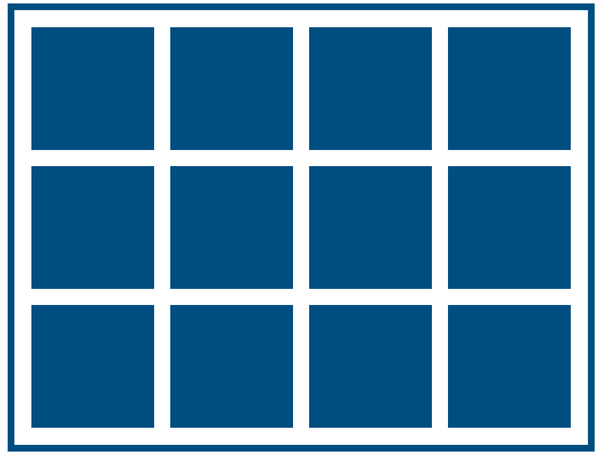
Meanings

- Repeated Addition
- Equal Groups or Sets
- Array



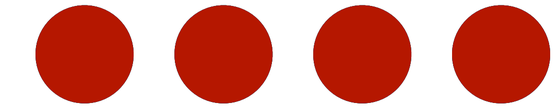
Meanings

- Repeated Addition
- Equal Groups or Sets
- Array
- Area of a Rectangle



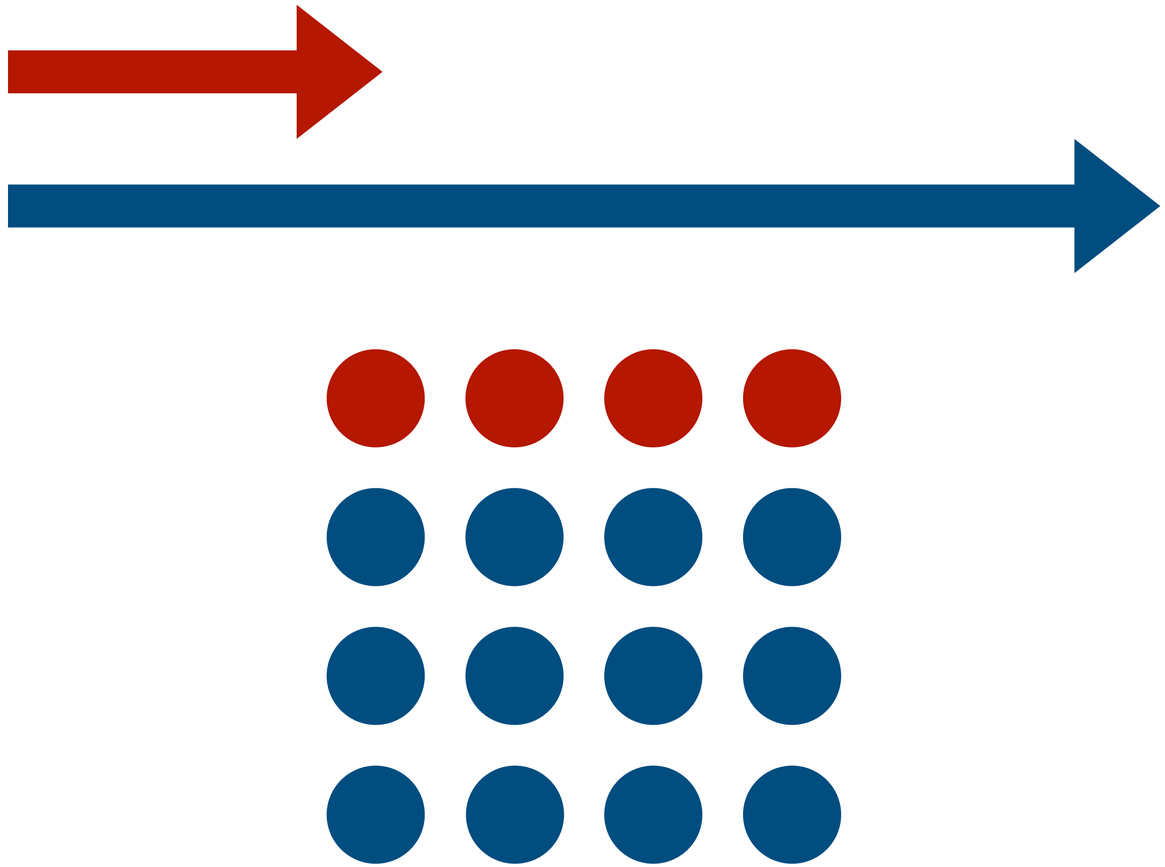
Meanings

- Repeated Addition
- Equal Groups or Sets
- Array
- Area of a Rectangle
- Comparison



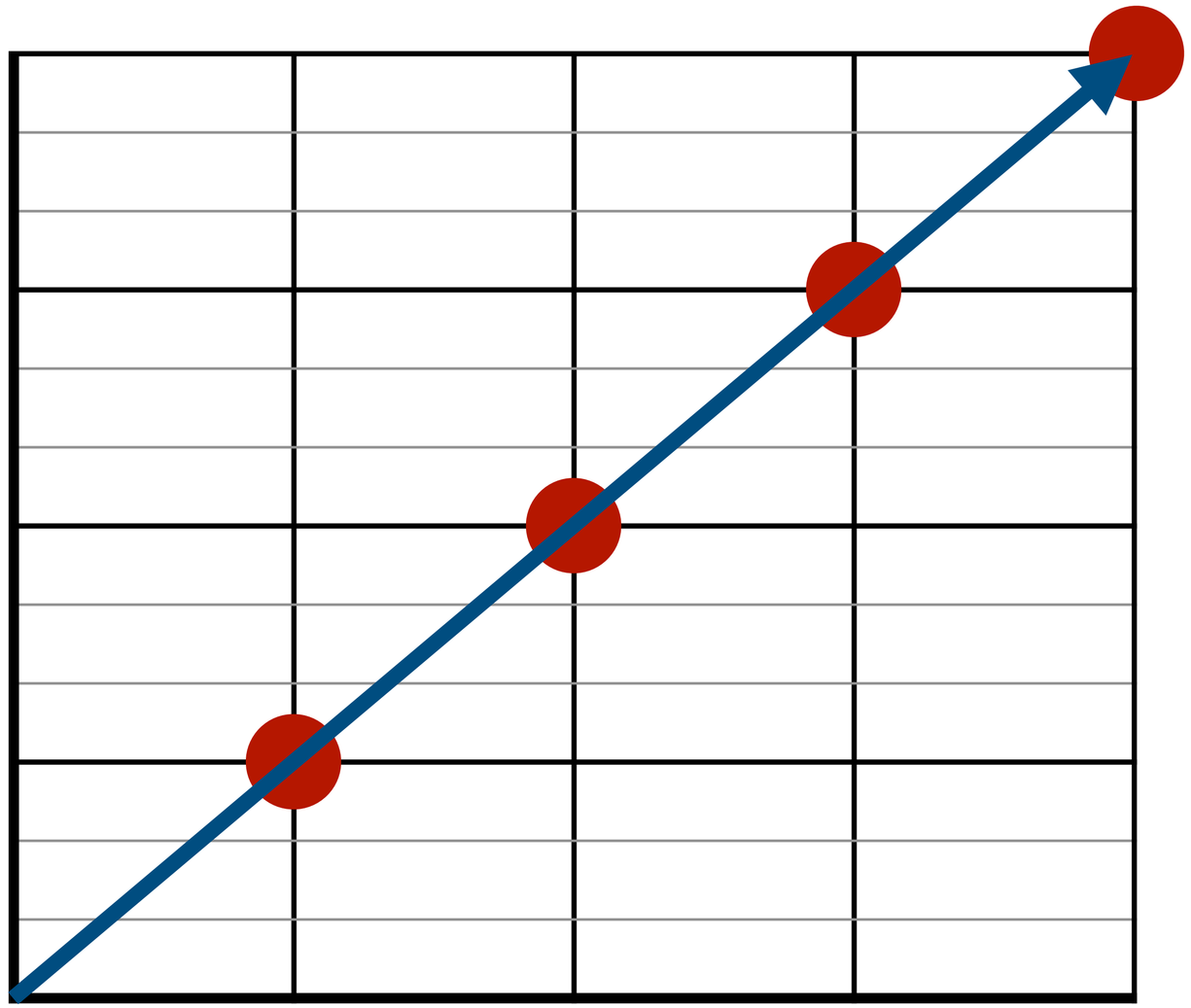
Meanings

- Repeated Addition
- Equal Groups or Sets
- Array
- Area of a Rectangle
- Comparison



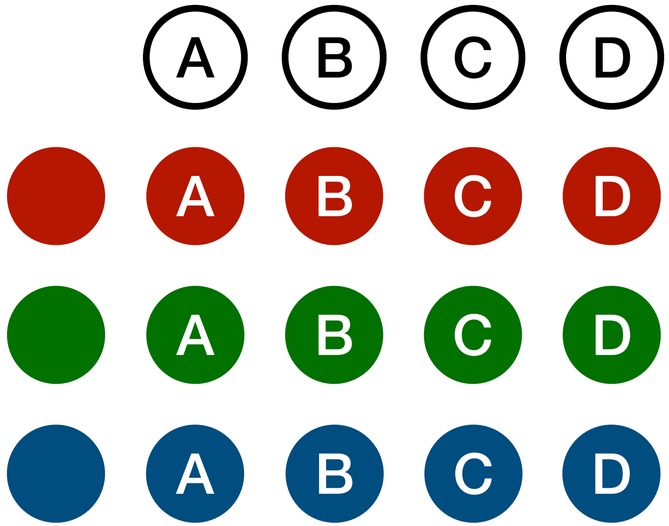
Meanings

- Repeated Addition
- Equal Groups or Sets
- Array
- Area of a Rectangle
- Comparison
- Rate



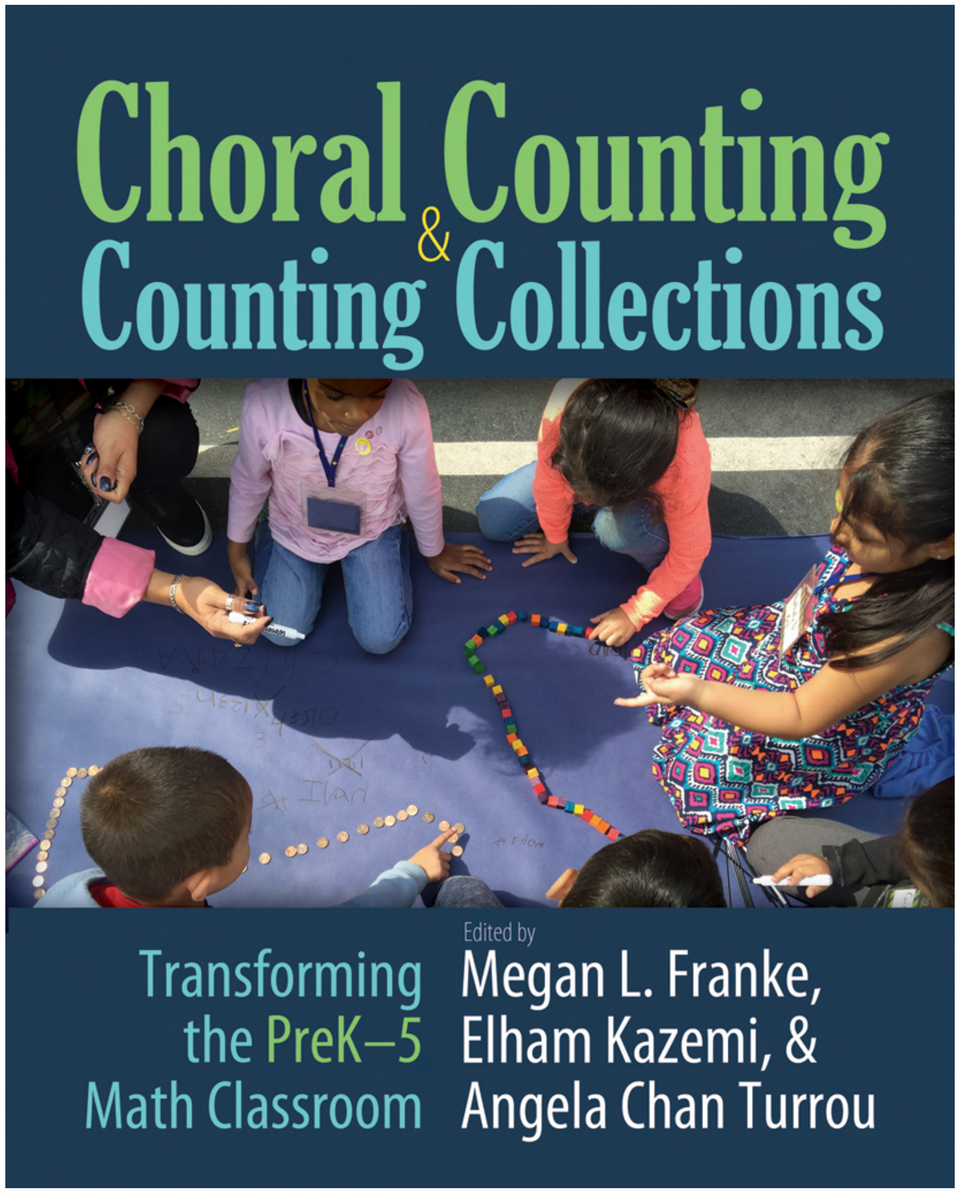
Meanings

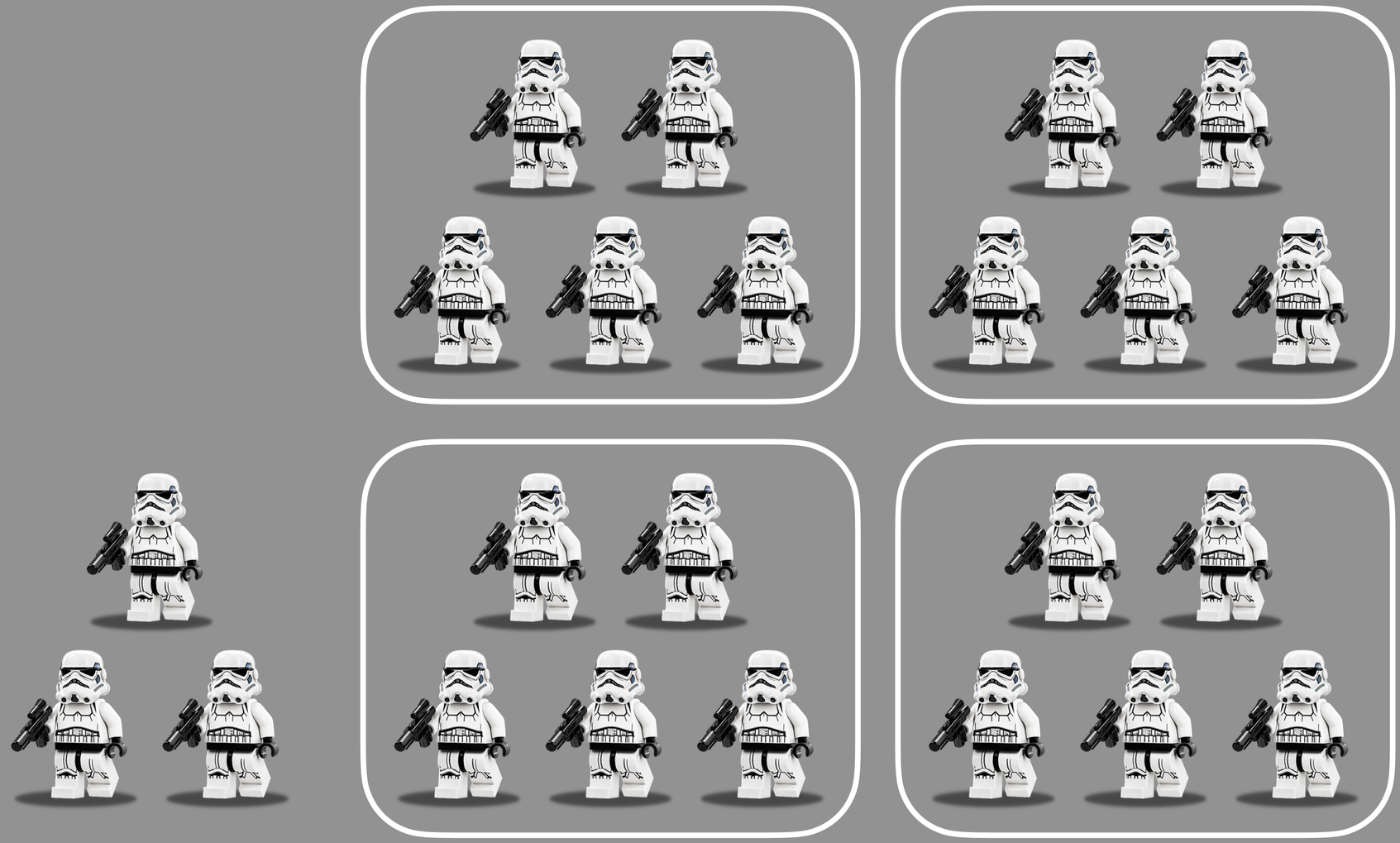
- Repeated Addition
- Equal Groups or Sets
- Array
- Area of a Rectangle
- Comparison
- Rate
- Combinations




ORIGINAL
EggSquisites

*"Today, **you and your partner** are going to **count a collection**, figure out **how many** you have in your collection, and **show me a picture** of how you counted."*





$$3 + 4 \times 5$$

$$3 + 4 \times 5$$

$$5 + 3 \times 6$$

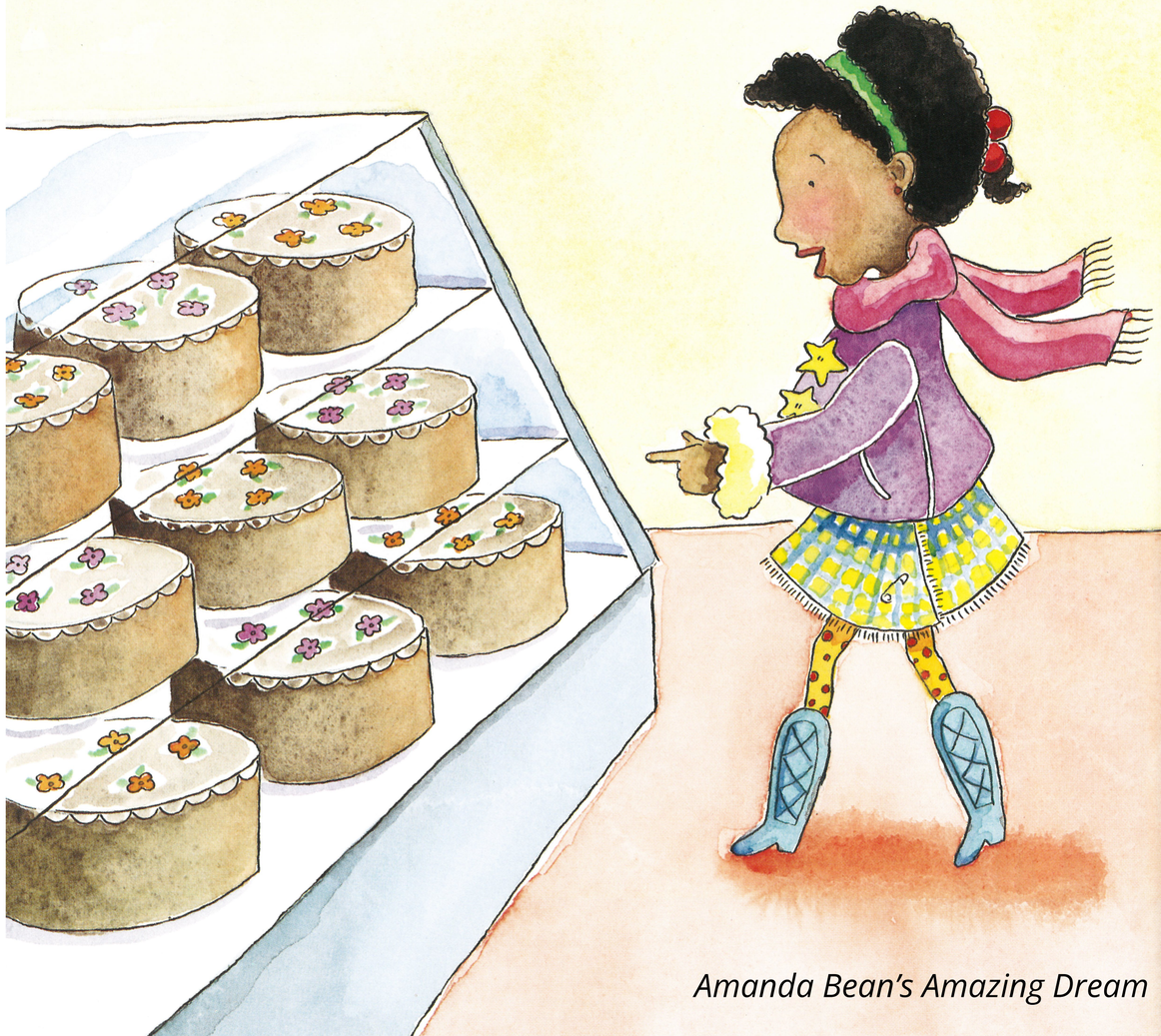
$$4 \times 6 - 1$$

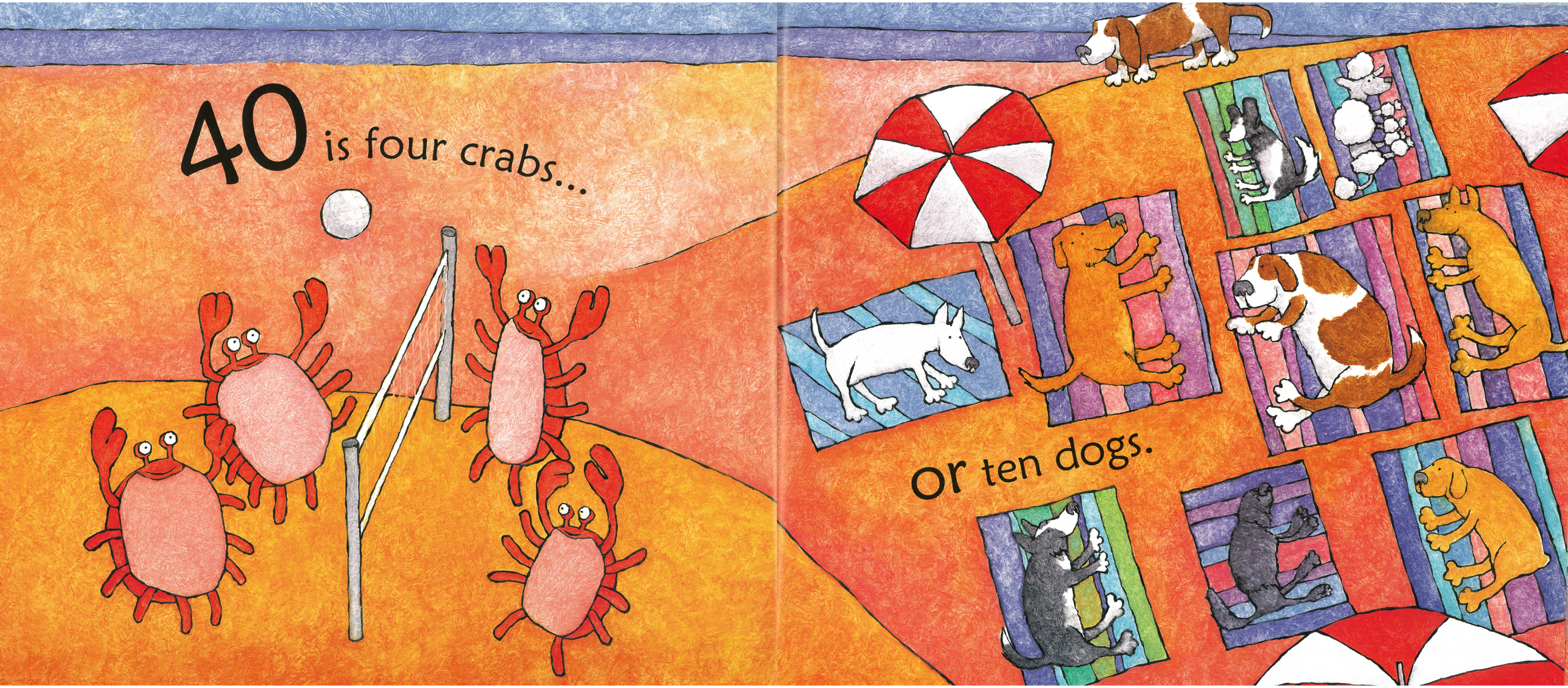
Now we are learning about multiplying. I understand many things about multiplying. I know that it is like adding lots of things quickly.

This is good.

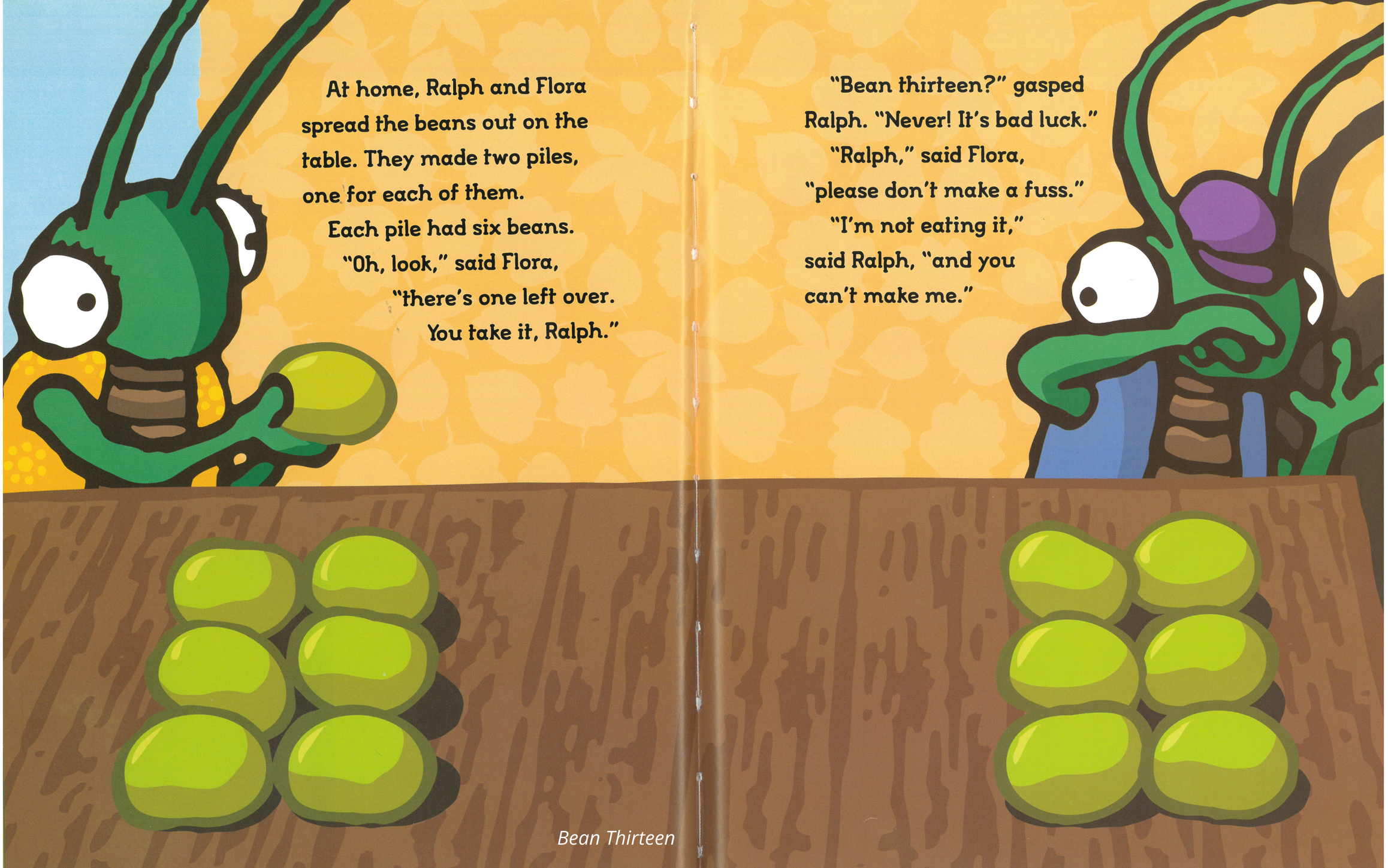
I know about the multiplication sign, ×. It means that things can come in groups, or rows, or columns.

This is also good.





One Is a Snail, Ten Is a Crab



At home, Ralph and Flora spread the beans out on the table. They made two piles, one for each of them.

Each pile had six beans.

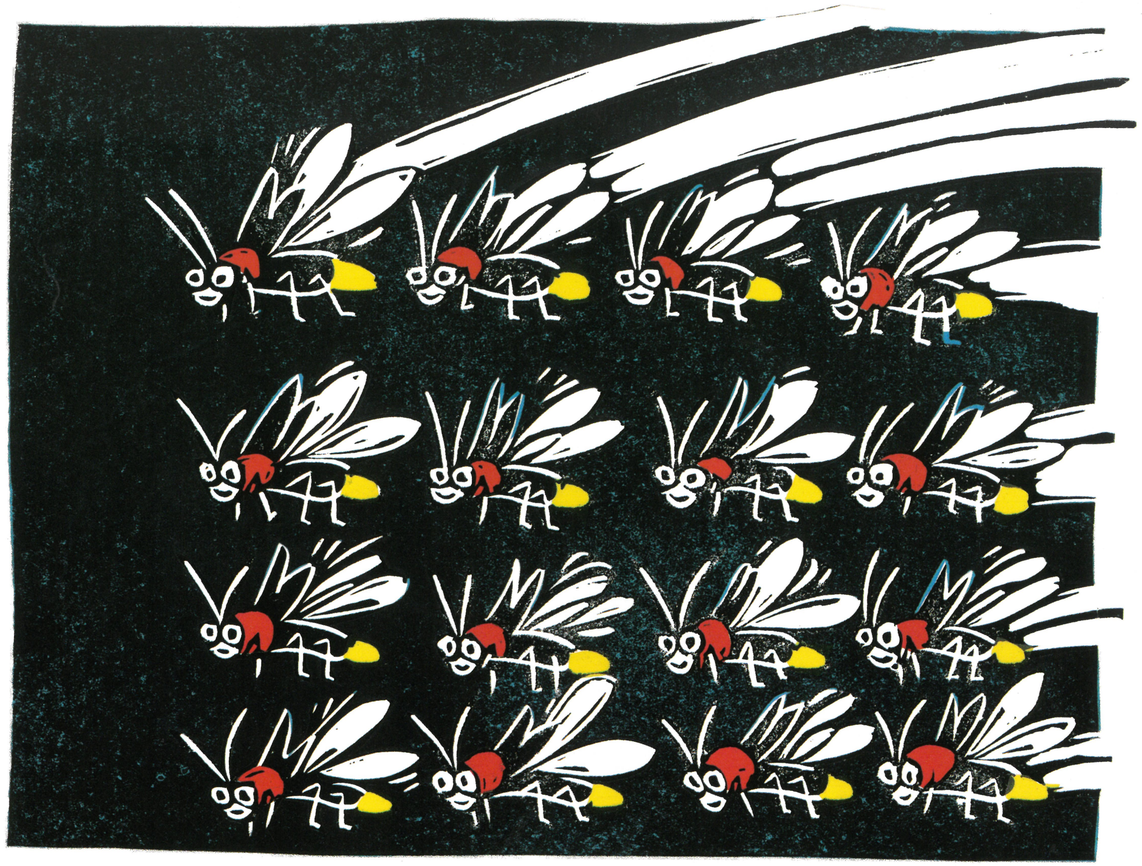
"Oh, look," said Flora, "there's one left over. You take it, Ralph."

"Bean thirteen?" gasped Ralph. "Never! It's bad luck."

"Ralph," said Flora, "please don't make a fuss."

"I'm not eating it," said Ralph, "and you can't make me."

"Too small, once again," he said, wrinkling his nose.
"Could you come a bit closer or add to your rows?"



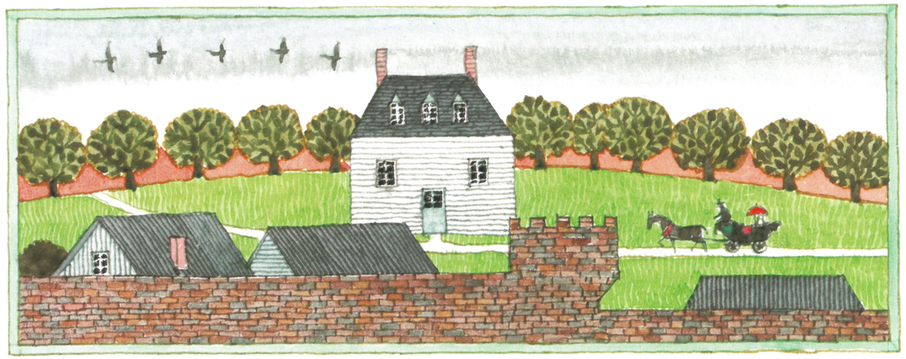
The fireflies figured they'd rather add more,
and lined up above him in four rows of four.

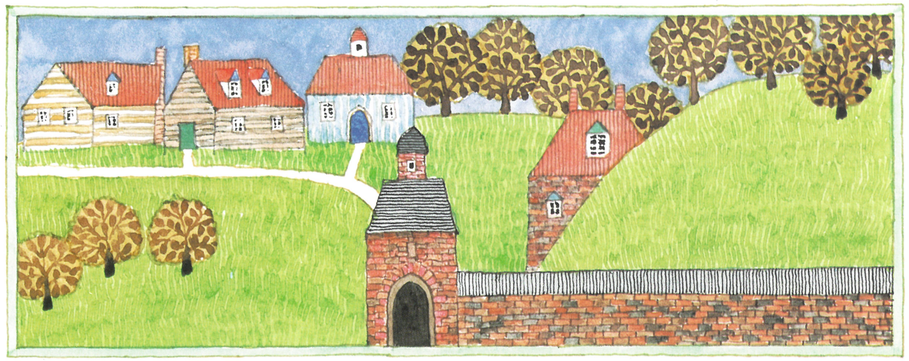
Mrs. Haktak smiled. "I know how we can get anything we want," she said. She put their ten coins into one purse, then threw it into the pot. She pulled out two purses with ten coins in each.

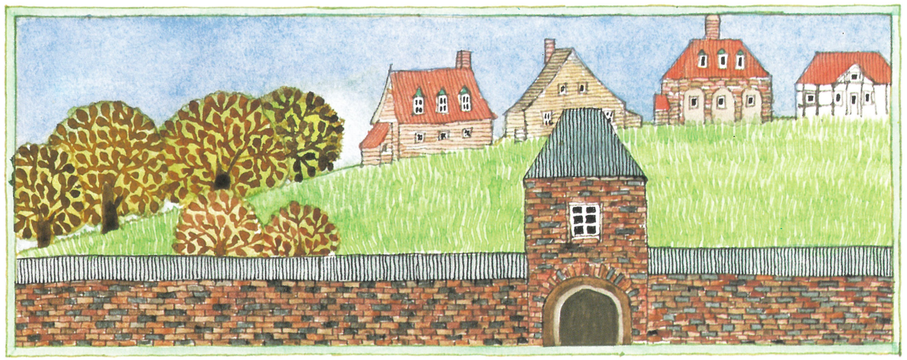
"What a clever wife I have!" cried Mr. Haktak. "Each time we do this we will have twice as much money as before!"

The Haktaks worked late into the night, filling and emptying the pot until the floor was covered with coins.

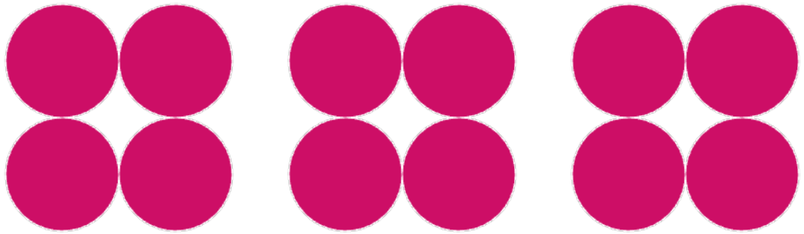
Within each walled kingdom there were 5 villages.

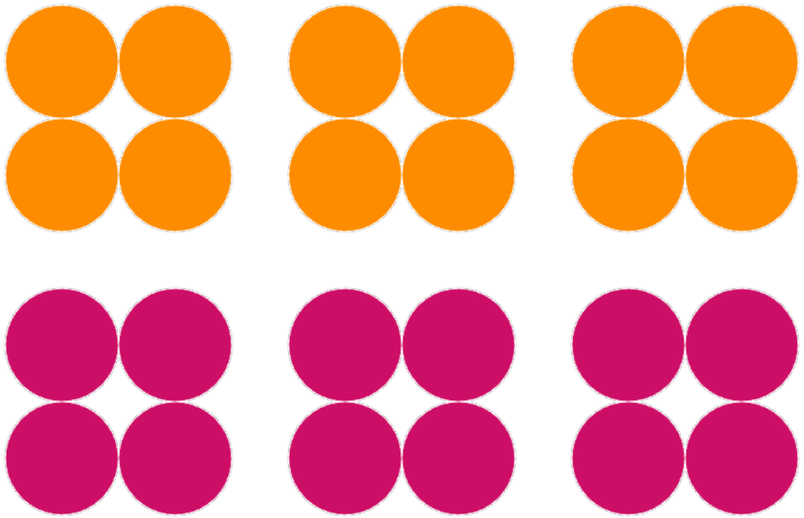


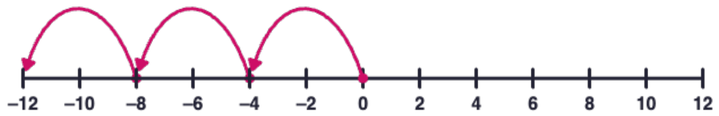


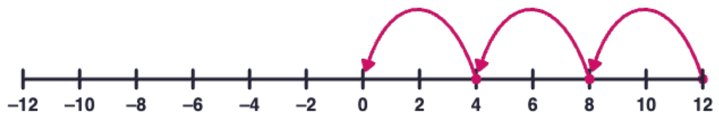


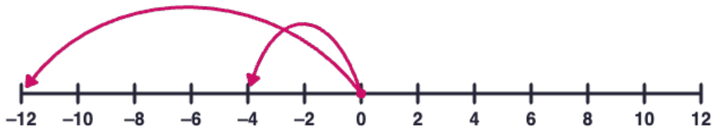
$(+3) \times (-4)$ and $(-3) \times (-4)$

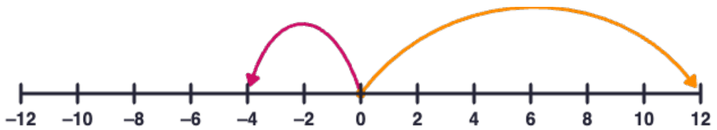












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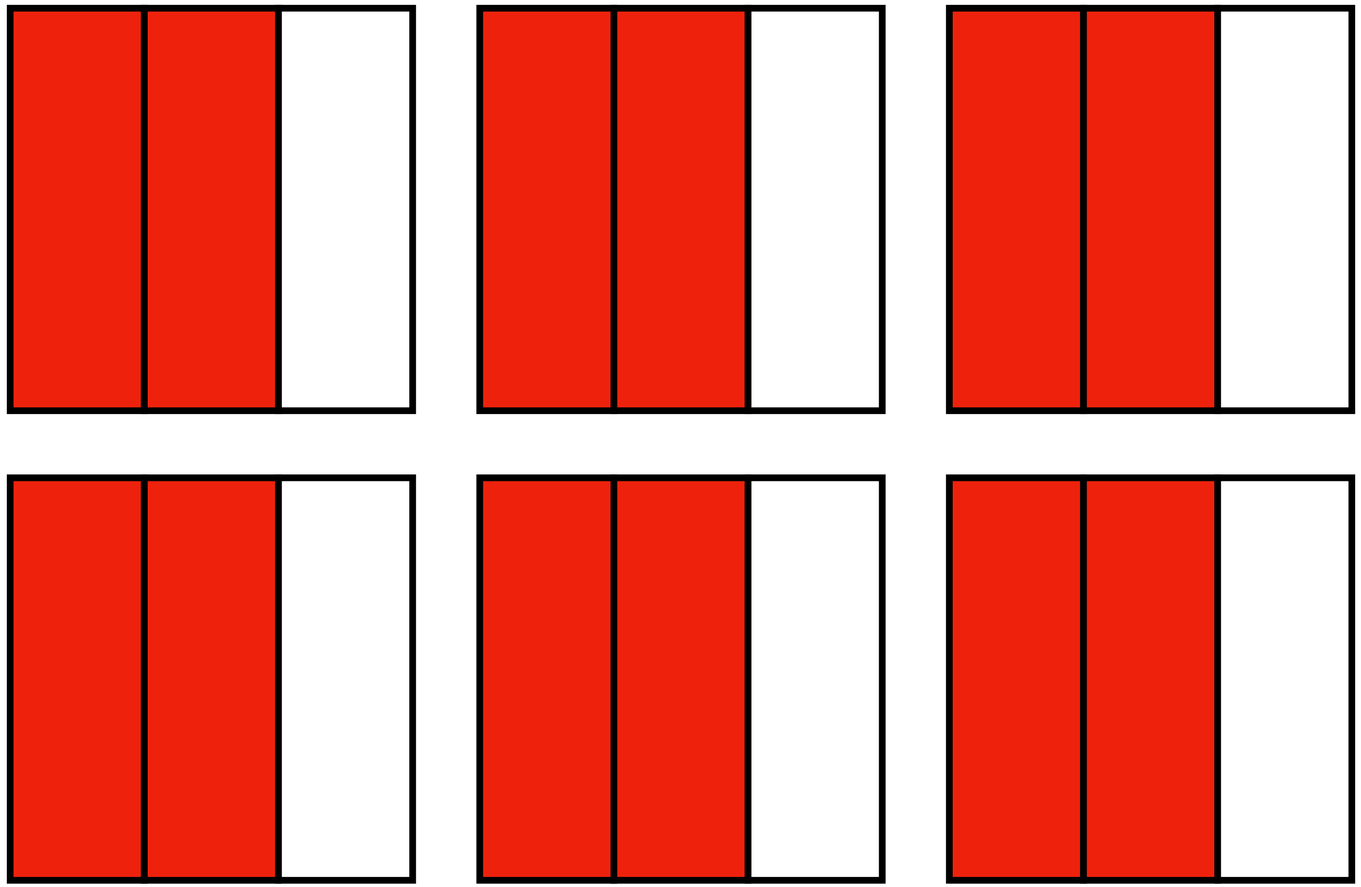
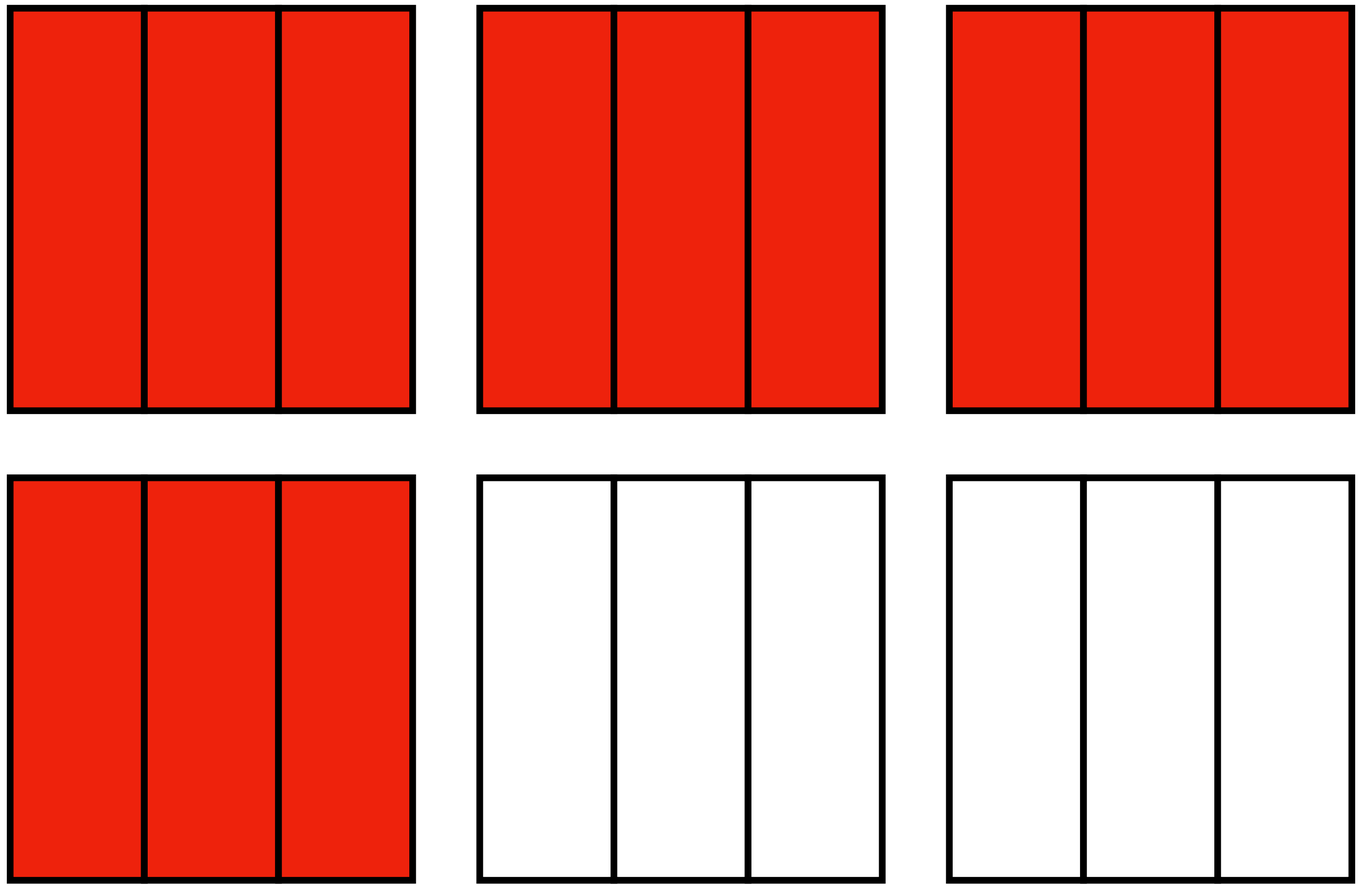
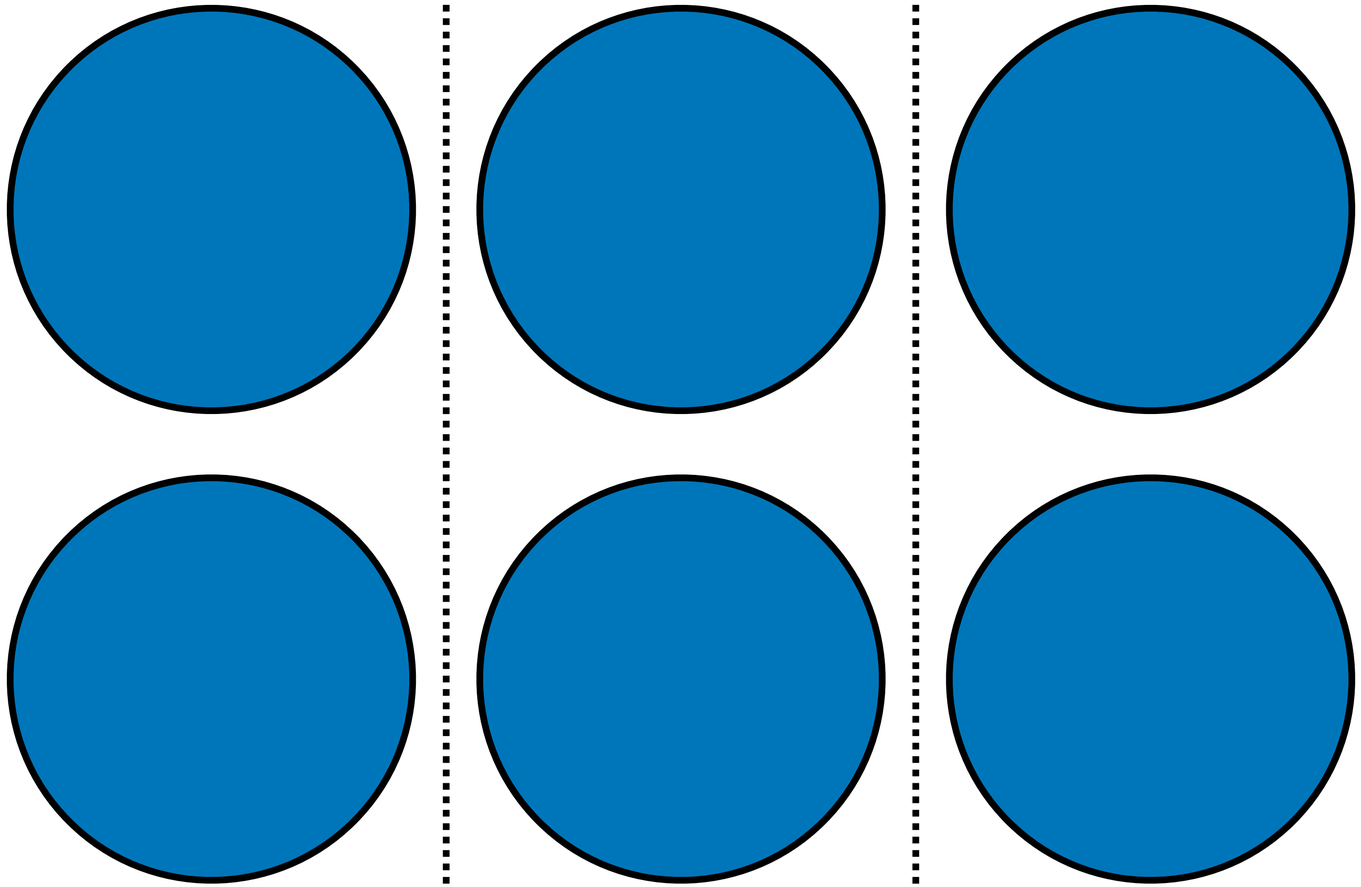
$$6 \times \frac{2}{3}$$



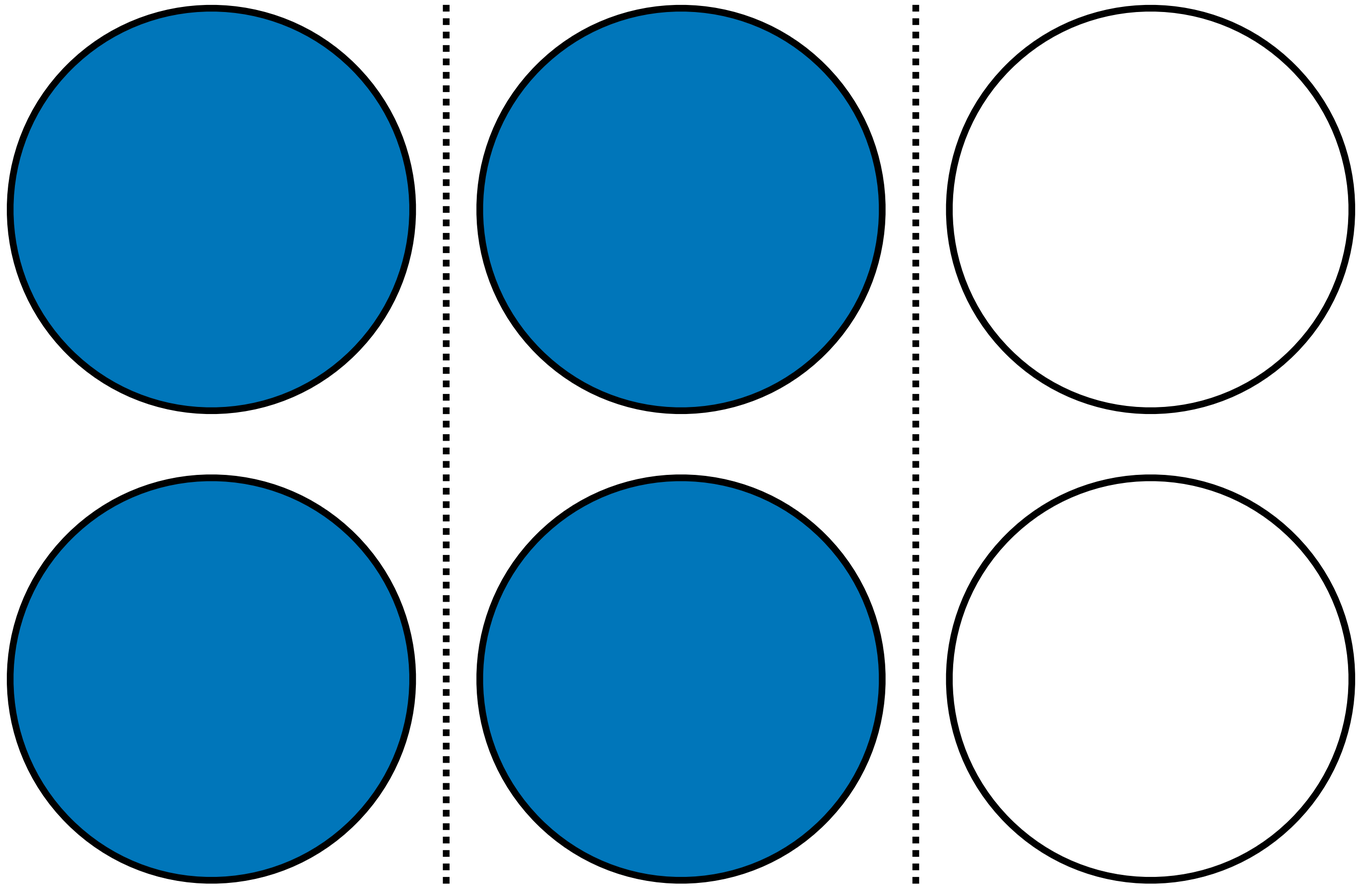
$$6 \times \frac{2}{3}$$



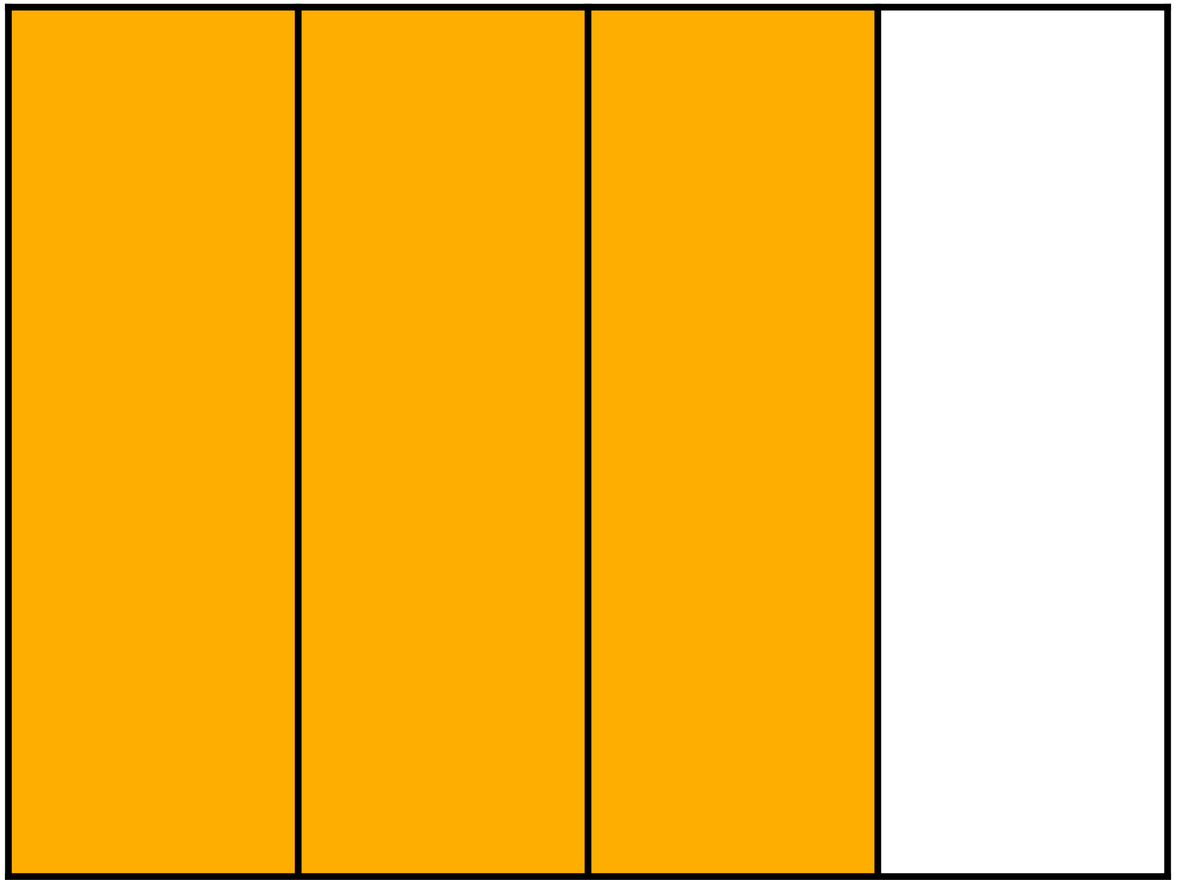
$$\frac{2}{3} \times 6$$



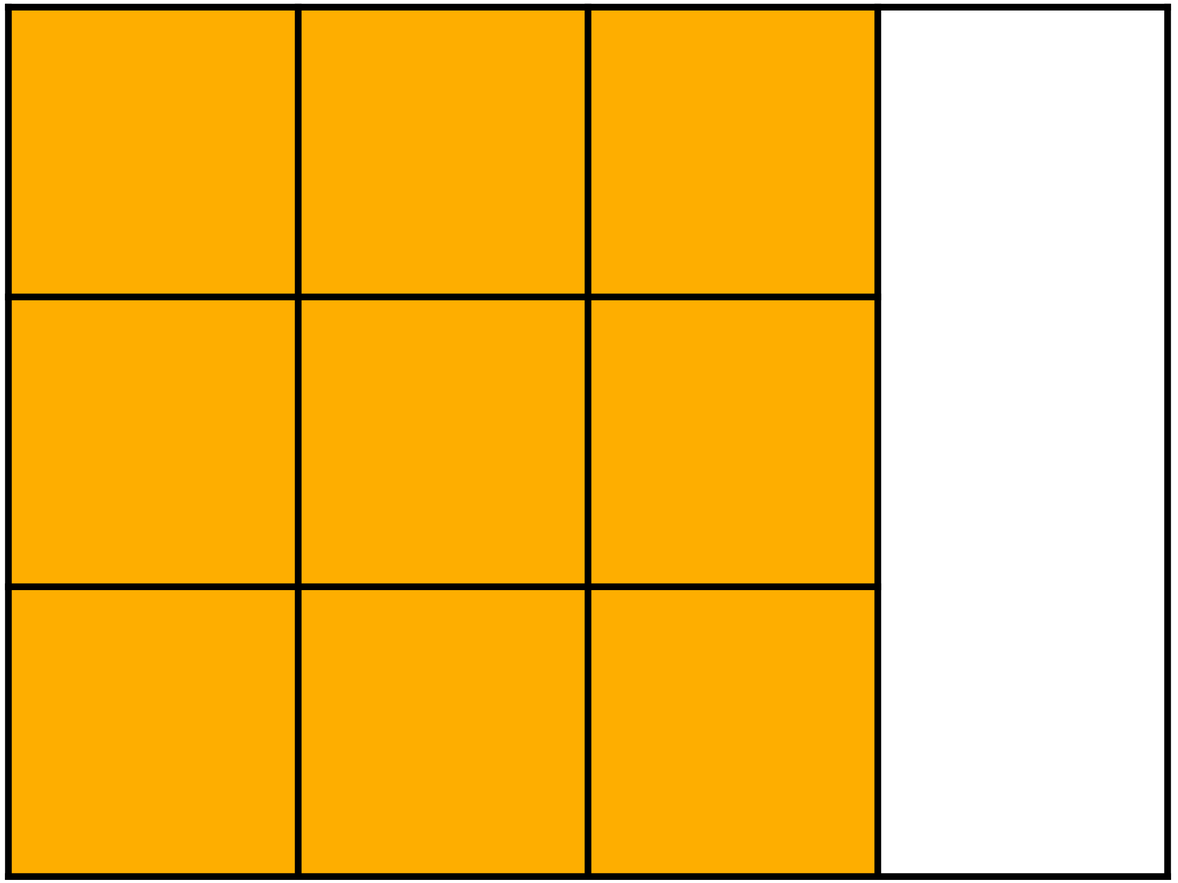
$$\frac{2}{3} \times 6$$



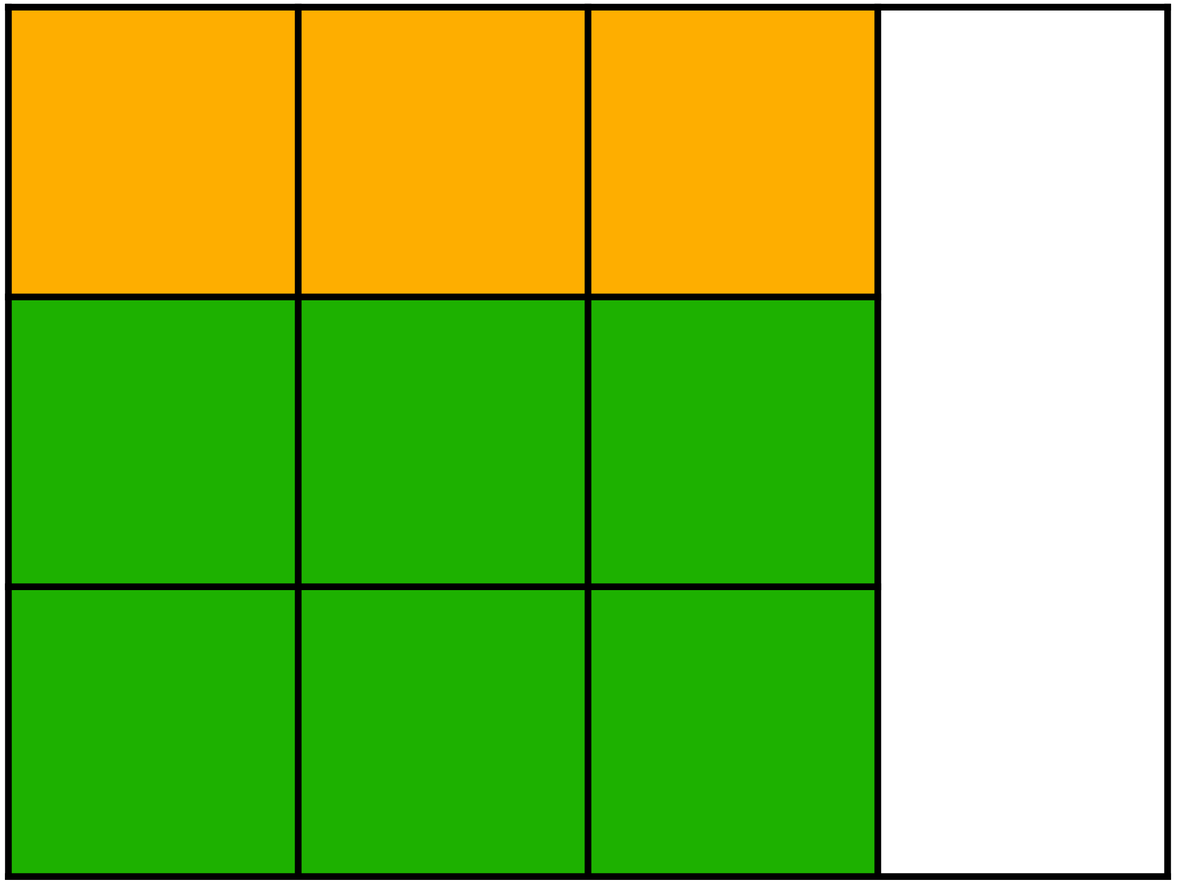
$$\frac{2}{3} \times \frac{3}{4}$$



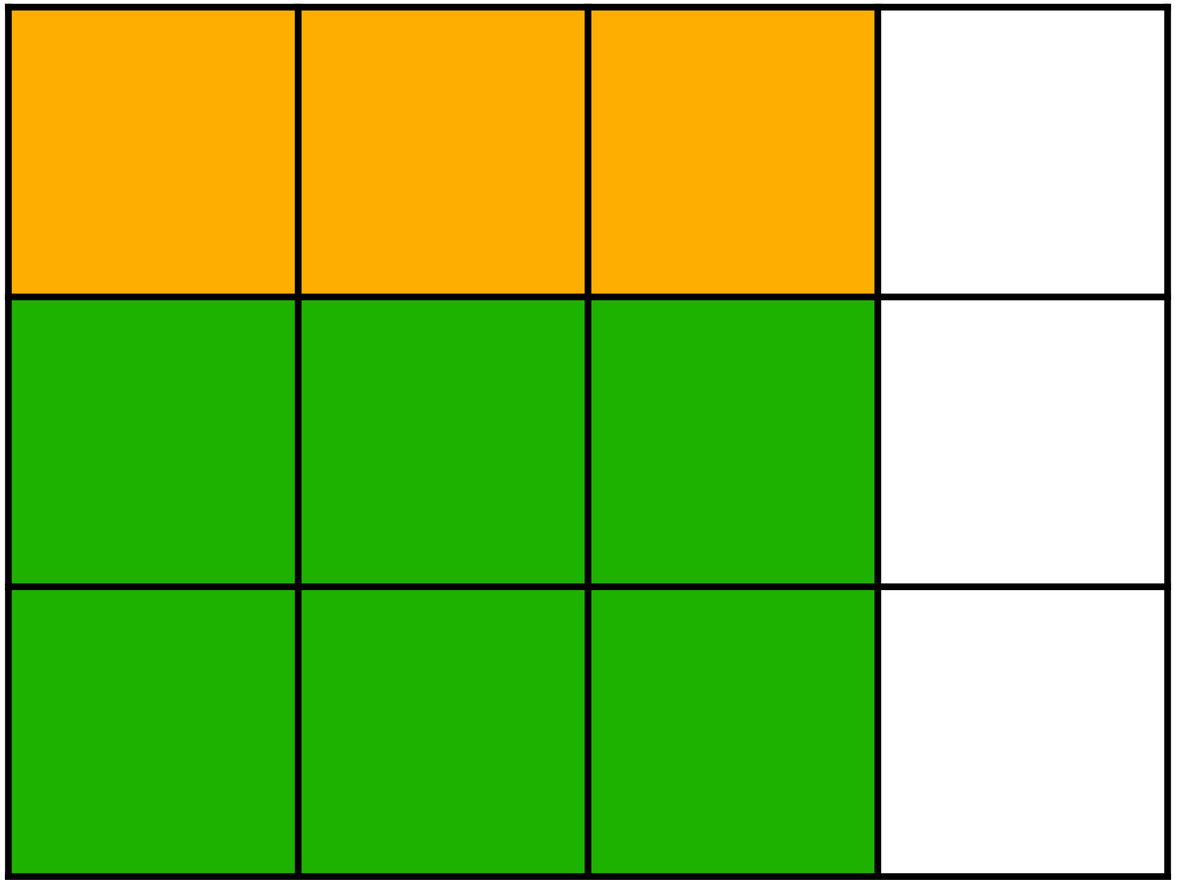
$$\frac{2}{3} \times \frac{3}{4}$$



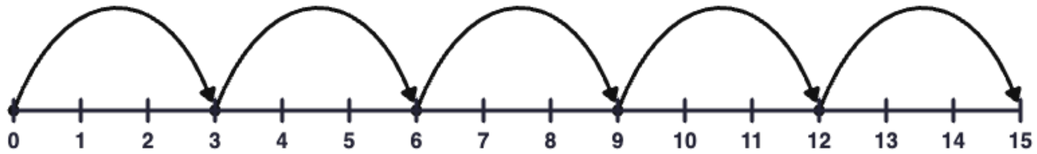
$$\frac{2}{3} \times \frac{3}{4}$$



$$\frac{2}{3} \times \frac{3}{4}$$

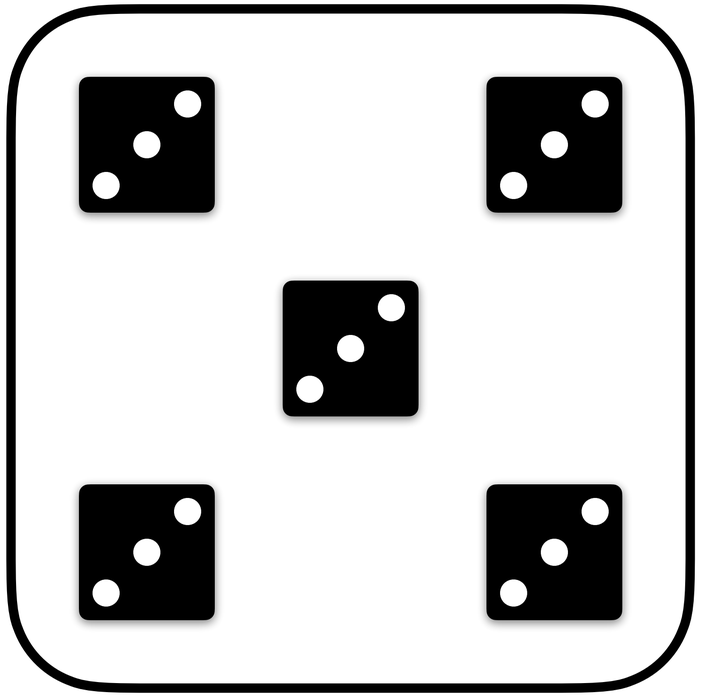


5×3

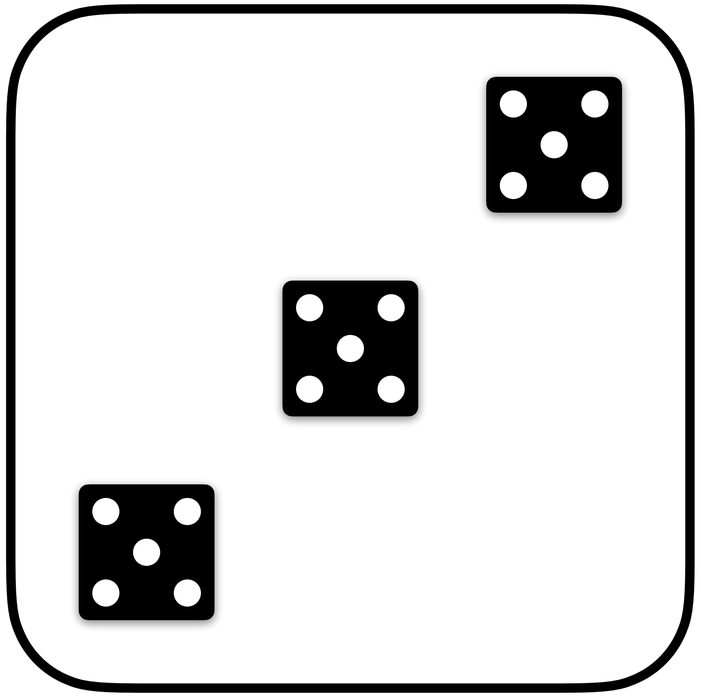


3×5

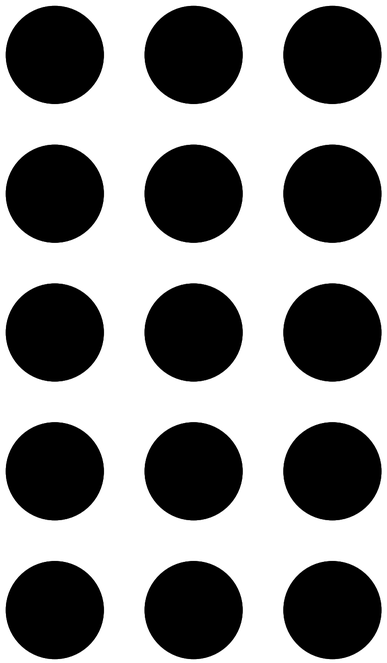
5×3



3×5

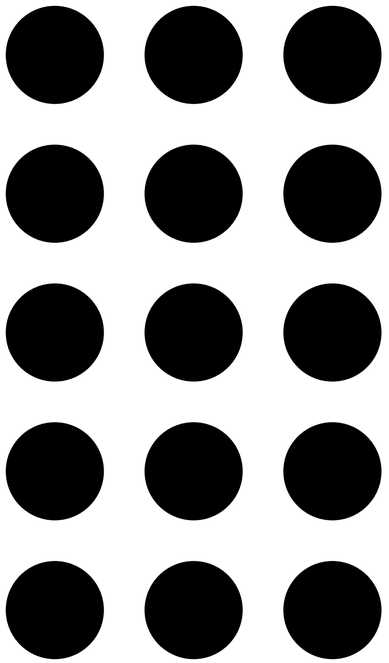


5×3

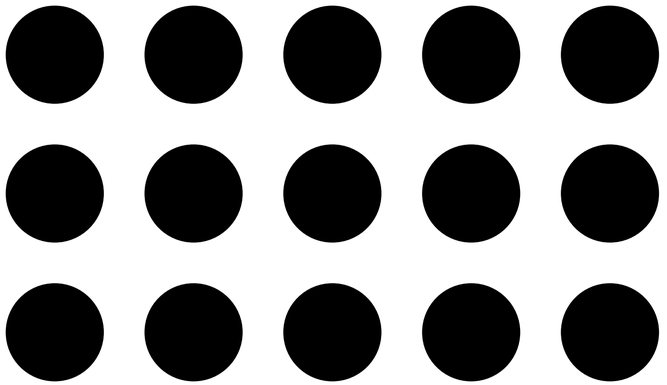


3×5

5×3

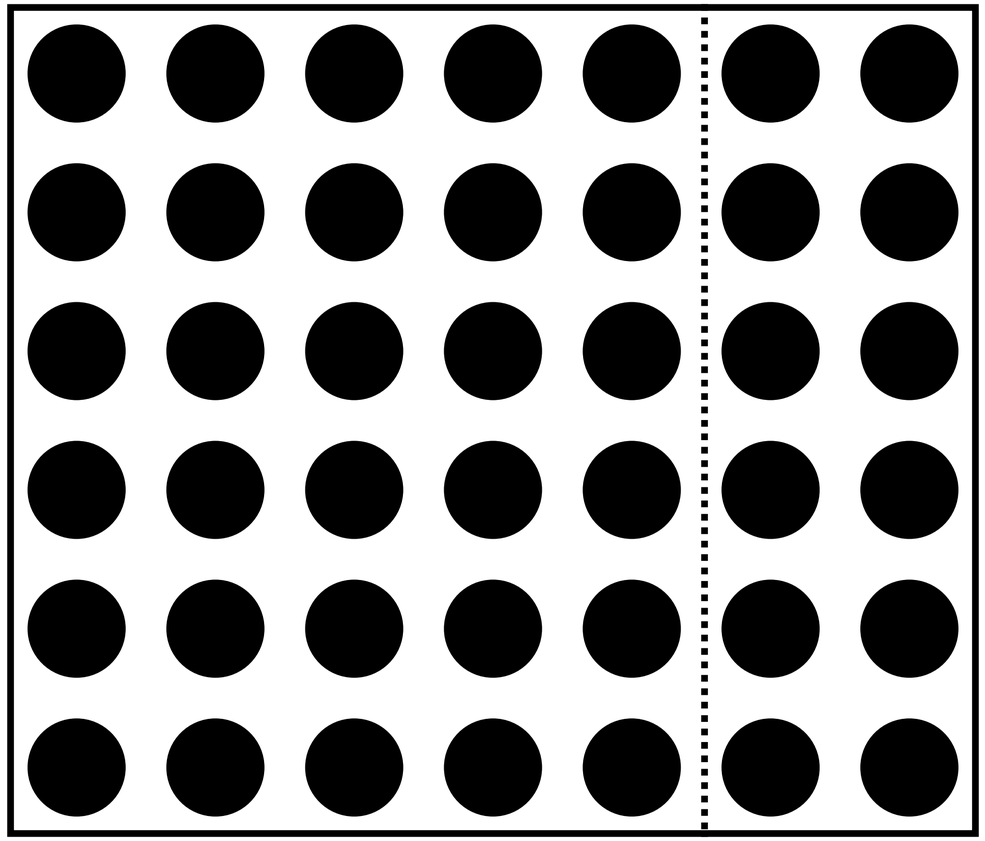


3×5



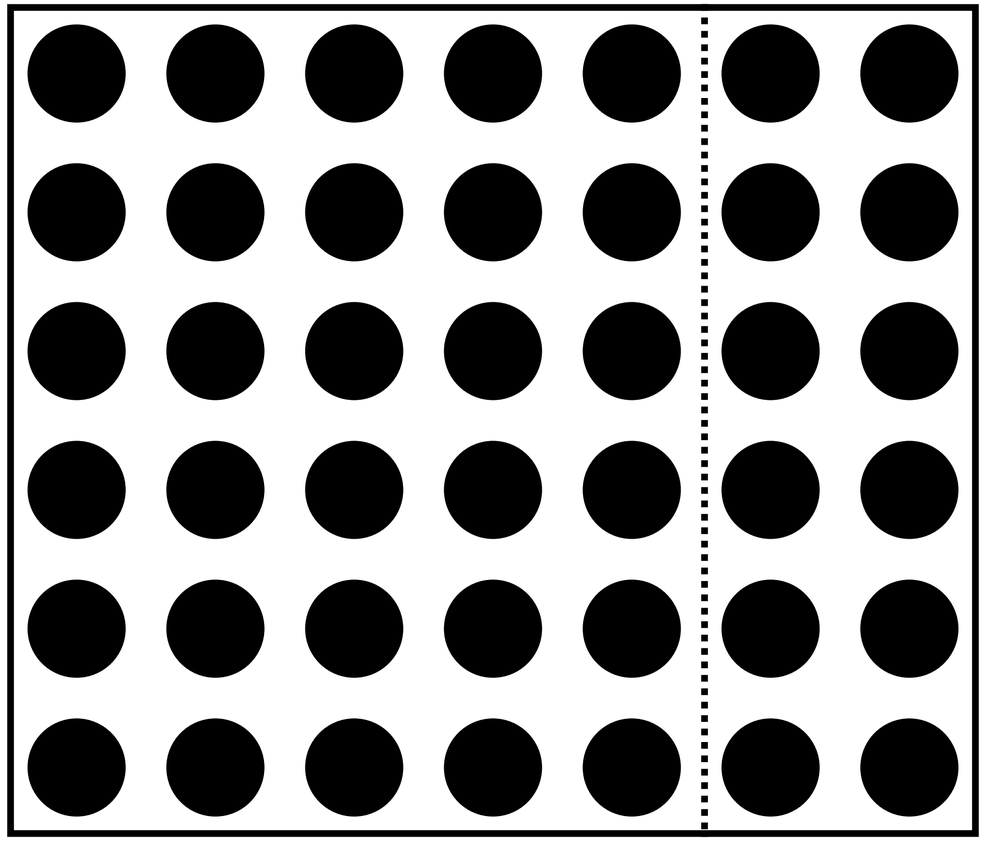
You can multiply numbers in any order.

6×7



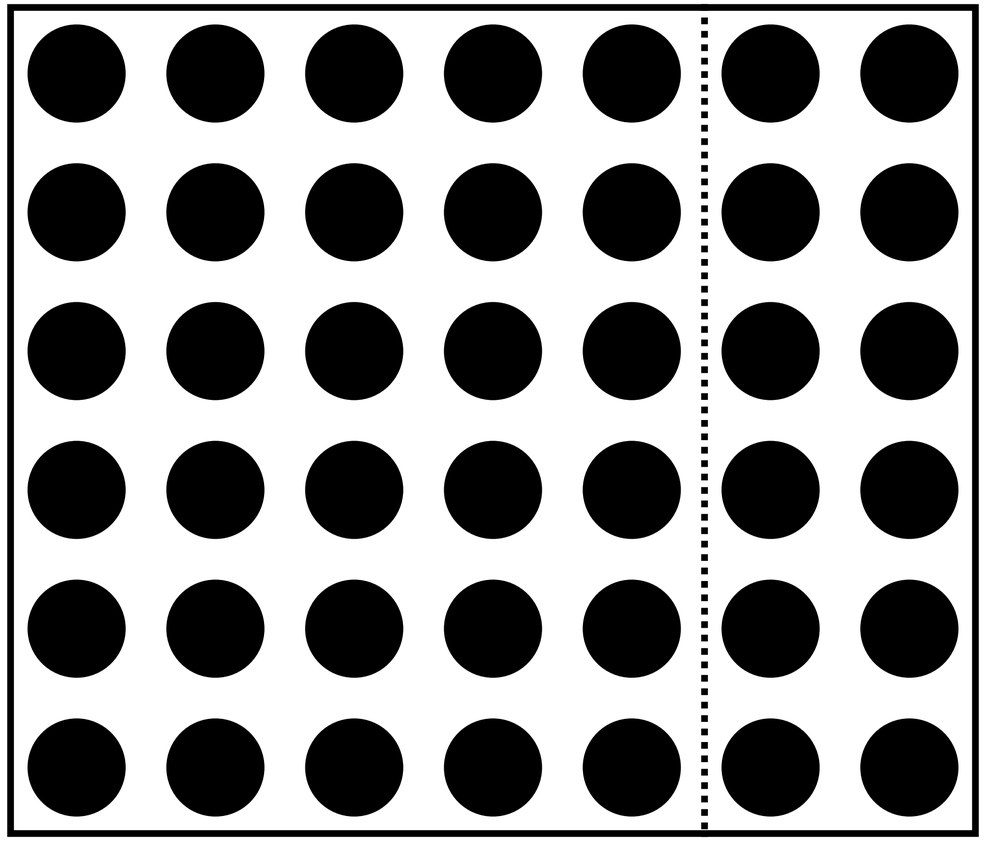
You can multiply in parts.

$$6 \times (5 + 2)$$



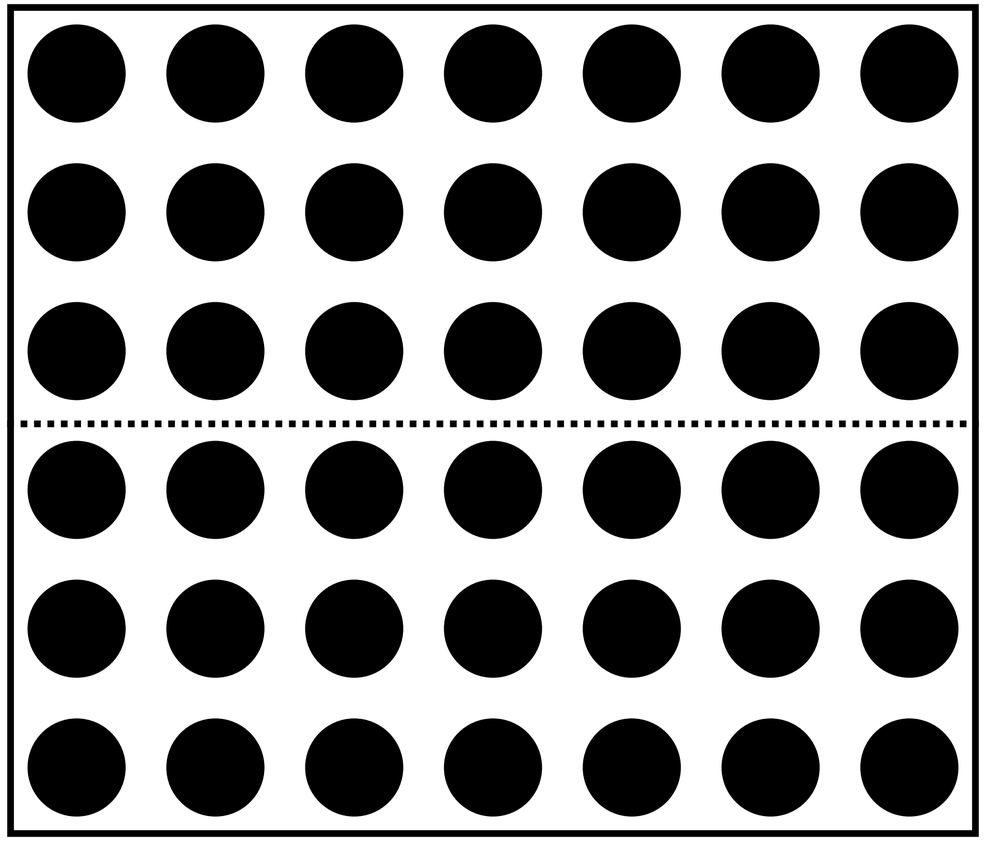
You can multiply in parts.

$$6 \times 5 + 6 \times 2$$



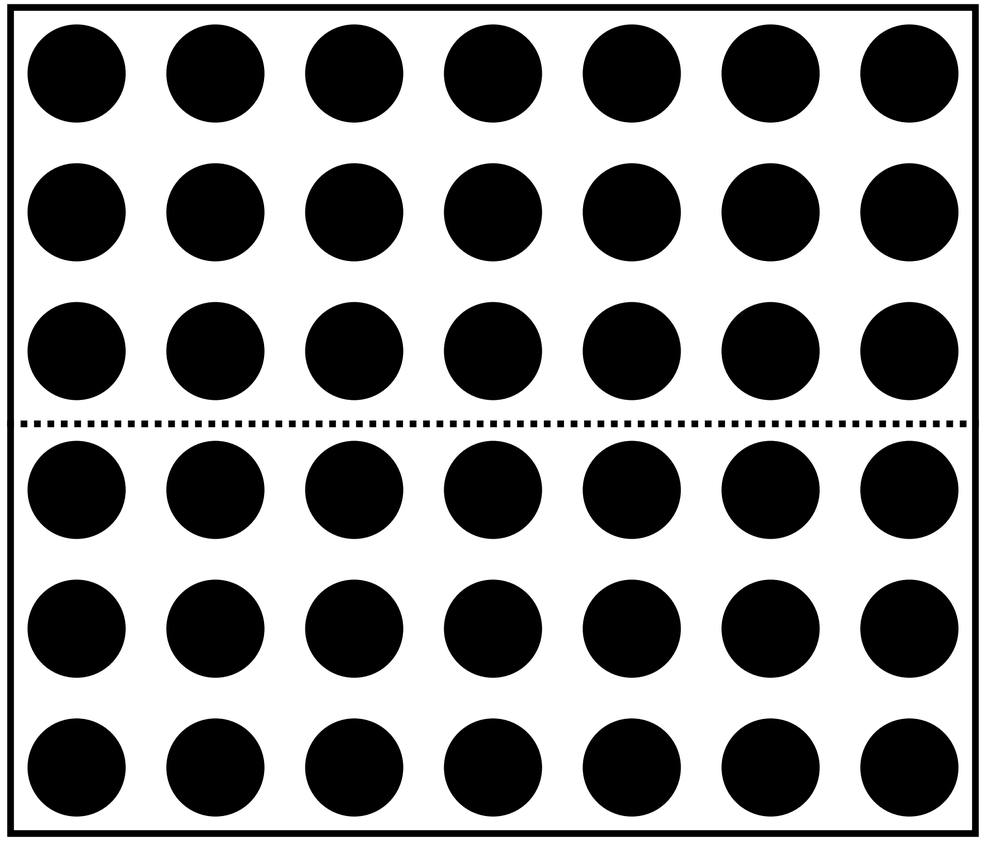
You can multiply in parts.

6×7



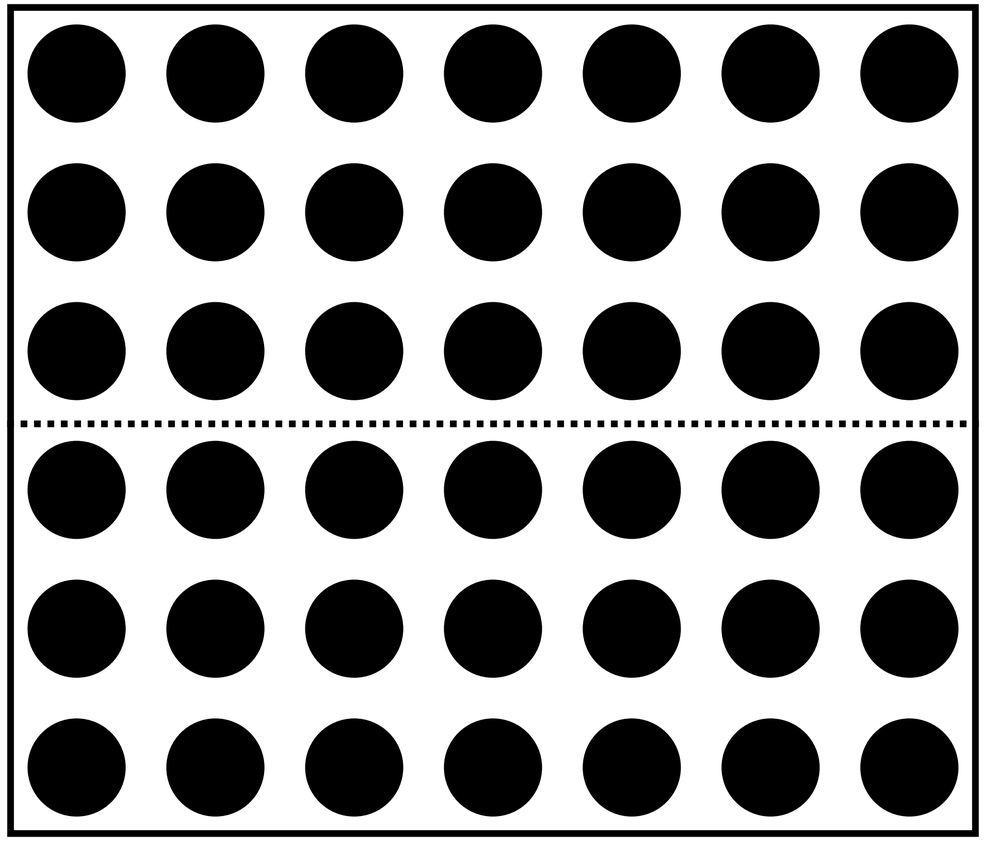
You can multiply in parts.

$$(2 \times 3) \times 7$$

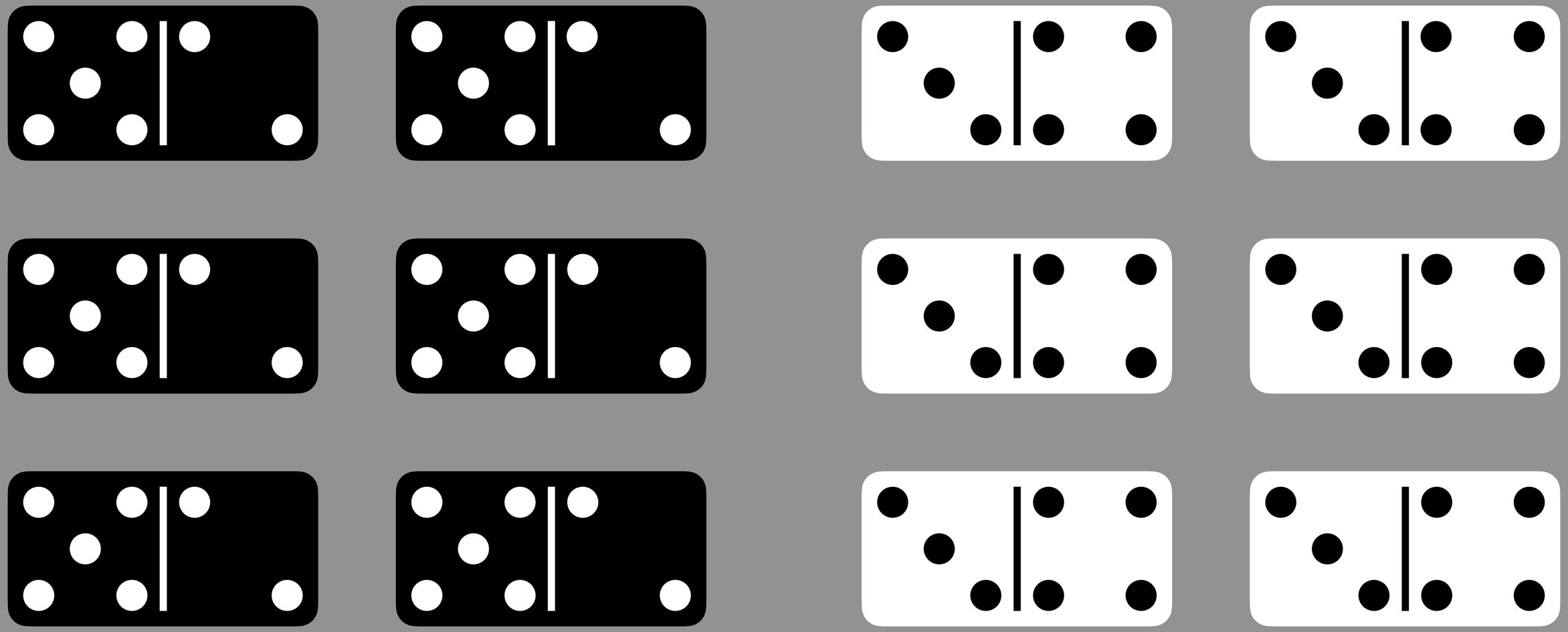


You can multiply in parts.

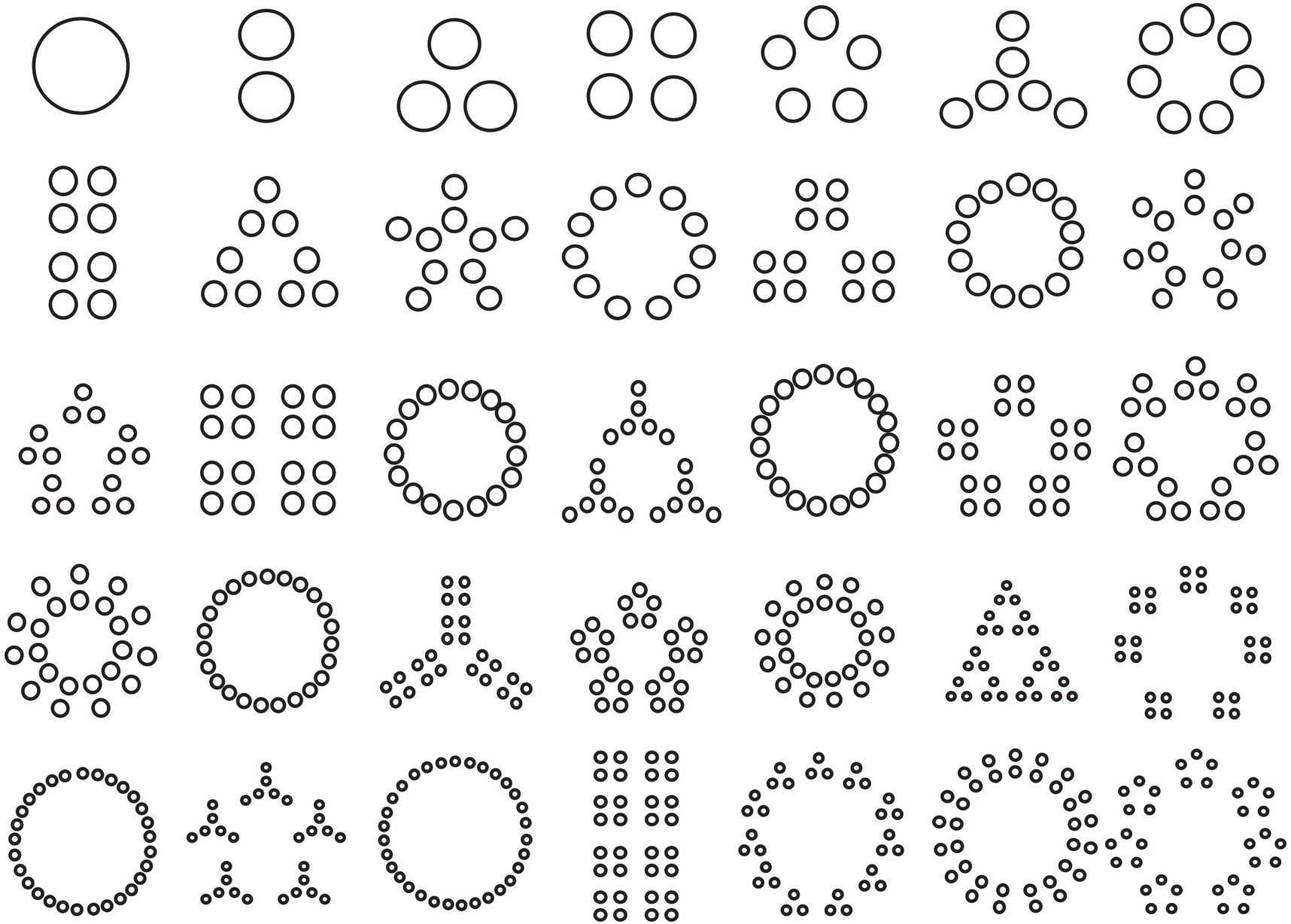
$$2 \times (3 \times 7)$$



You can multiply in parts.



Christopher Danielson



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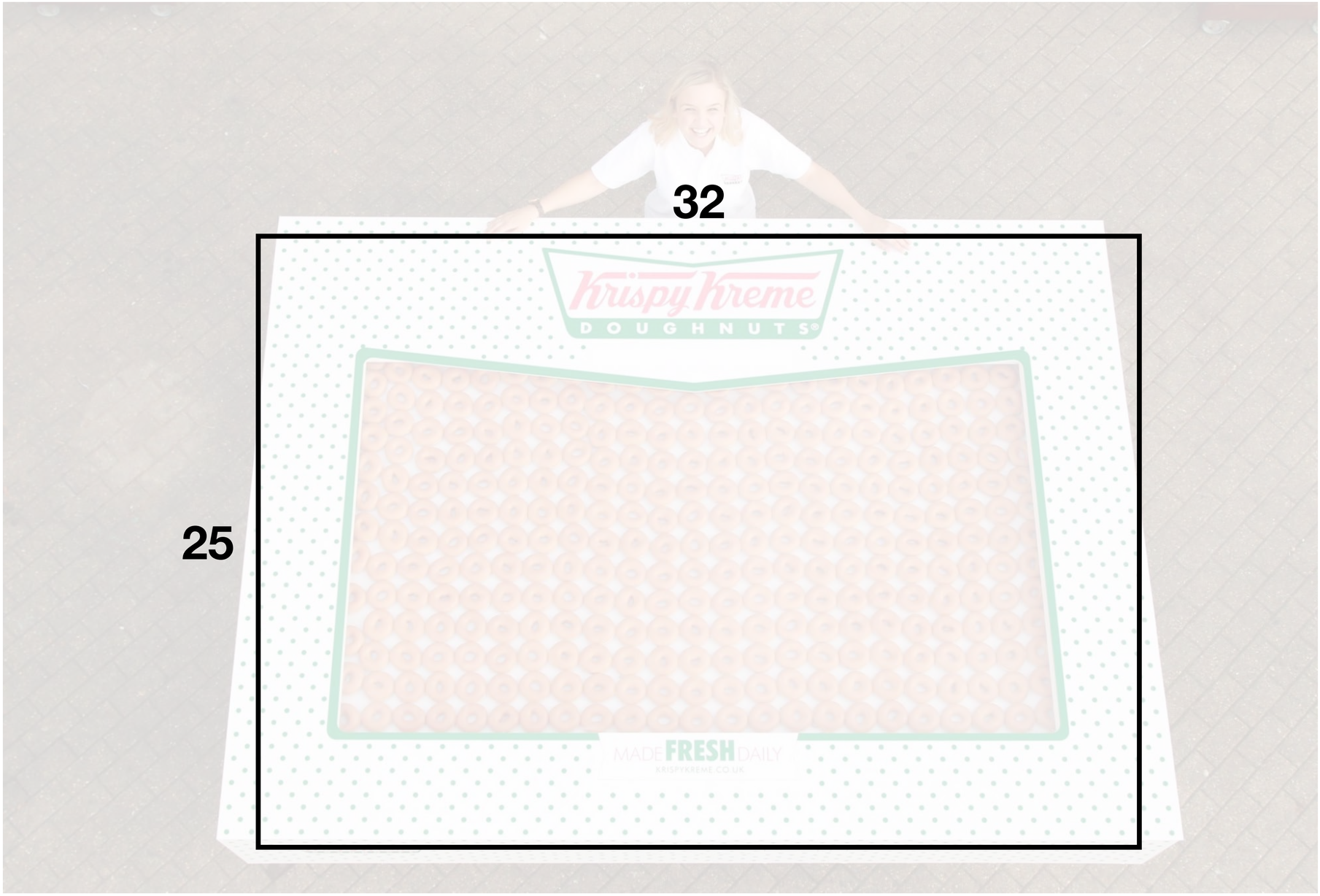
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25
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by The Math Learning Center

Help

About

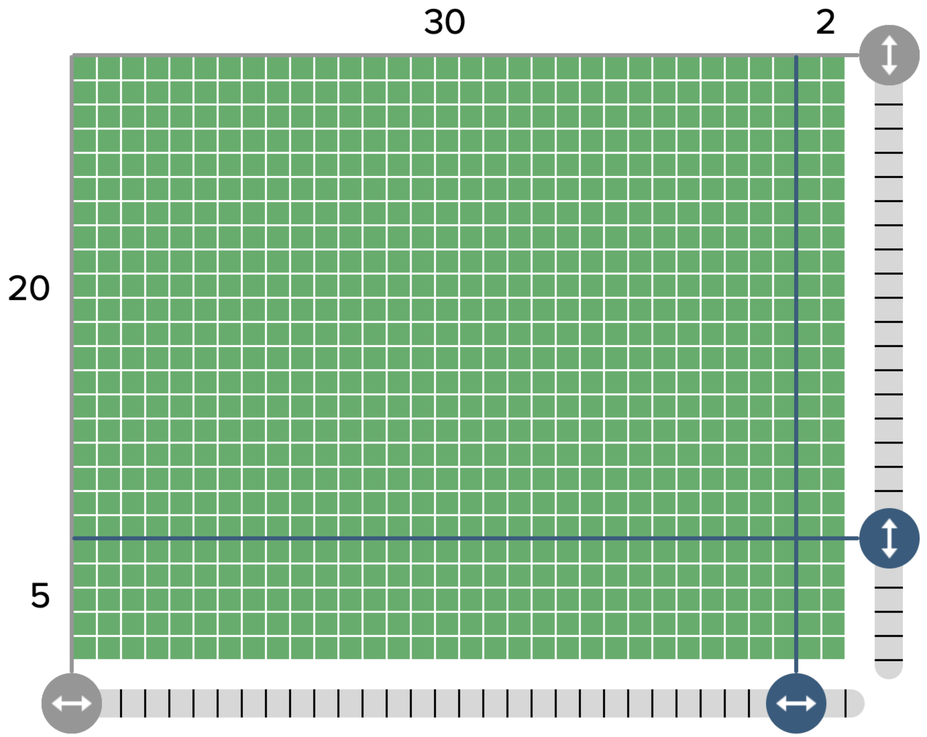
25 × 32

MAKE

Maximum size: 50 x 50

ANSWER

Go back to beta version



(20 × 30) + (20 × 2) + (5 × 30) + (5 × 2) = **(25 x 32)**

600 + 40 + 150 + 10 = **800**

Student #1

25
+25
+25
+25 → 100

+25
+25
+25
+25 → 100

+25
+25
+25
+25 → 100

+25
+25
+25
+25 → 100

+25
+25
+25
+25 → 100

+25
+25
+25
+25 → 100

+25
+25
+25
+25 → 100

800

Student #2

32 x 25

30 2

x 25 x 25

750 + 50 = 800

Student #3

32
x 25
160
+ 640
800

Student #4

	30	2
20	600	40
5	150	10

600 + 150 + 40 + 10 = 750 + 50 = 800

Student #5

32
+32
64
+32
96
+32
128
+32
160
+32
192
+32
224
+32
256
+32
288
+32
320
+32
352
+32
384
+32
416
+32
448
+32
480
+32
512
+32
544
+32
576
+32
608
+32
640
+32
672
+32
704
+32
736
+32
768
+32
800

Student #6

32 x 25
= 16 x 50
= 8 x 100
= 800

Student #7

	10	10	10	2
25	250	250	250	50

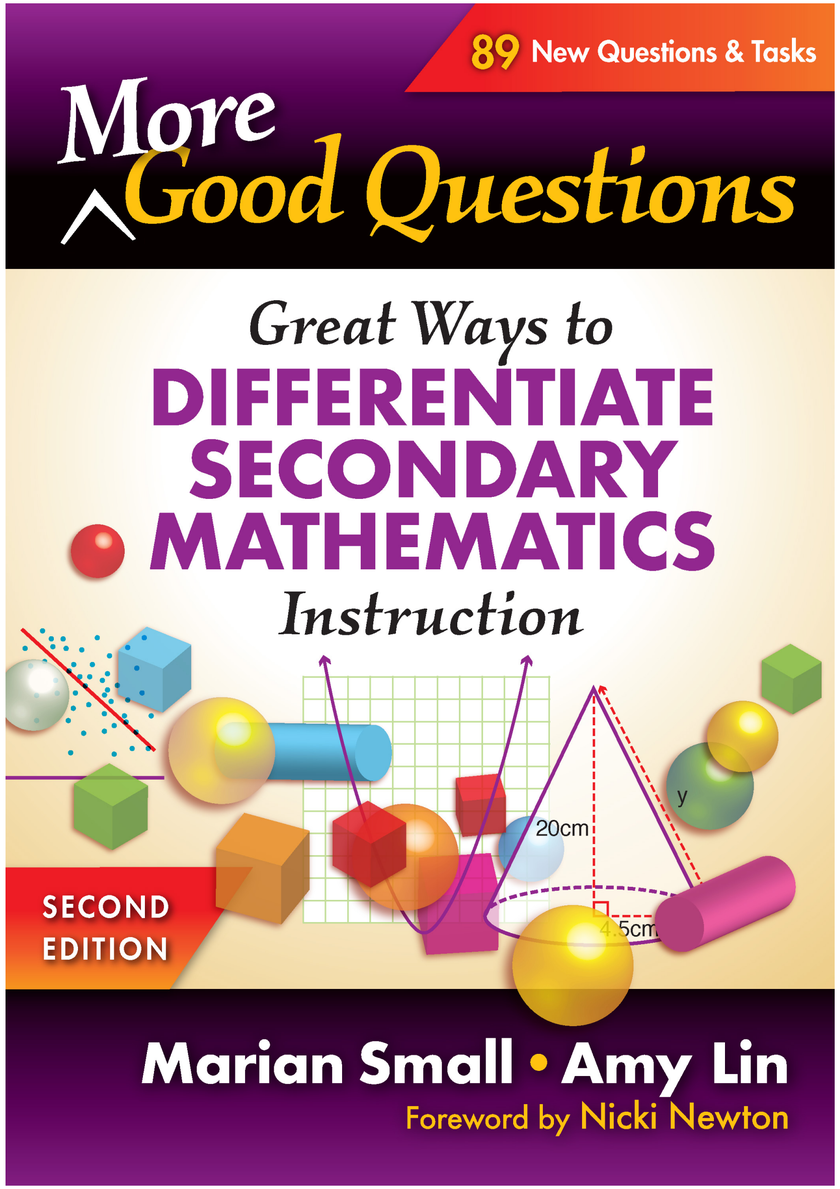
250 + 250 + 250 + 50 = 500 + 300 = 800

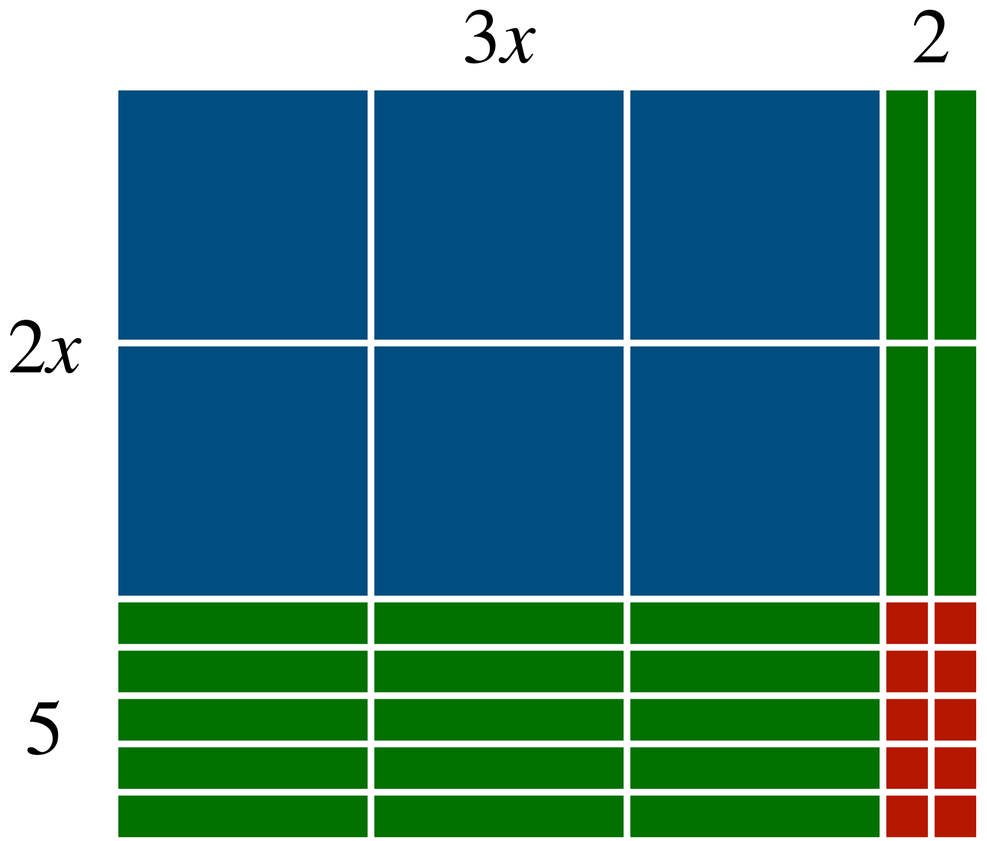
Student #8

	4	4	8	16
25	100	100	200	400

200 + 200 + 400 = 400 + 400 = 800

*"The operations of addition, subtraction, multiplication, and division hold the **same fundamental meanings** no matter the domain in which they are applied."*





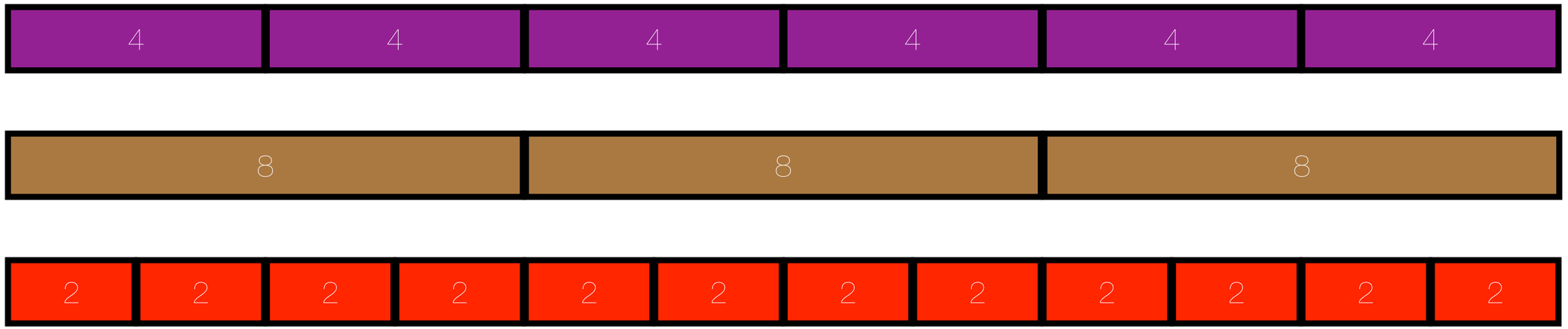
$$(2x)(3x) + (2x)(2) + (5)(3x) + (5)(2) = (2x + 5)(3x + 2)$$

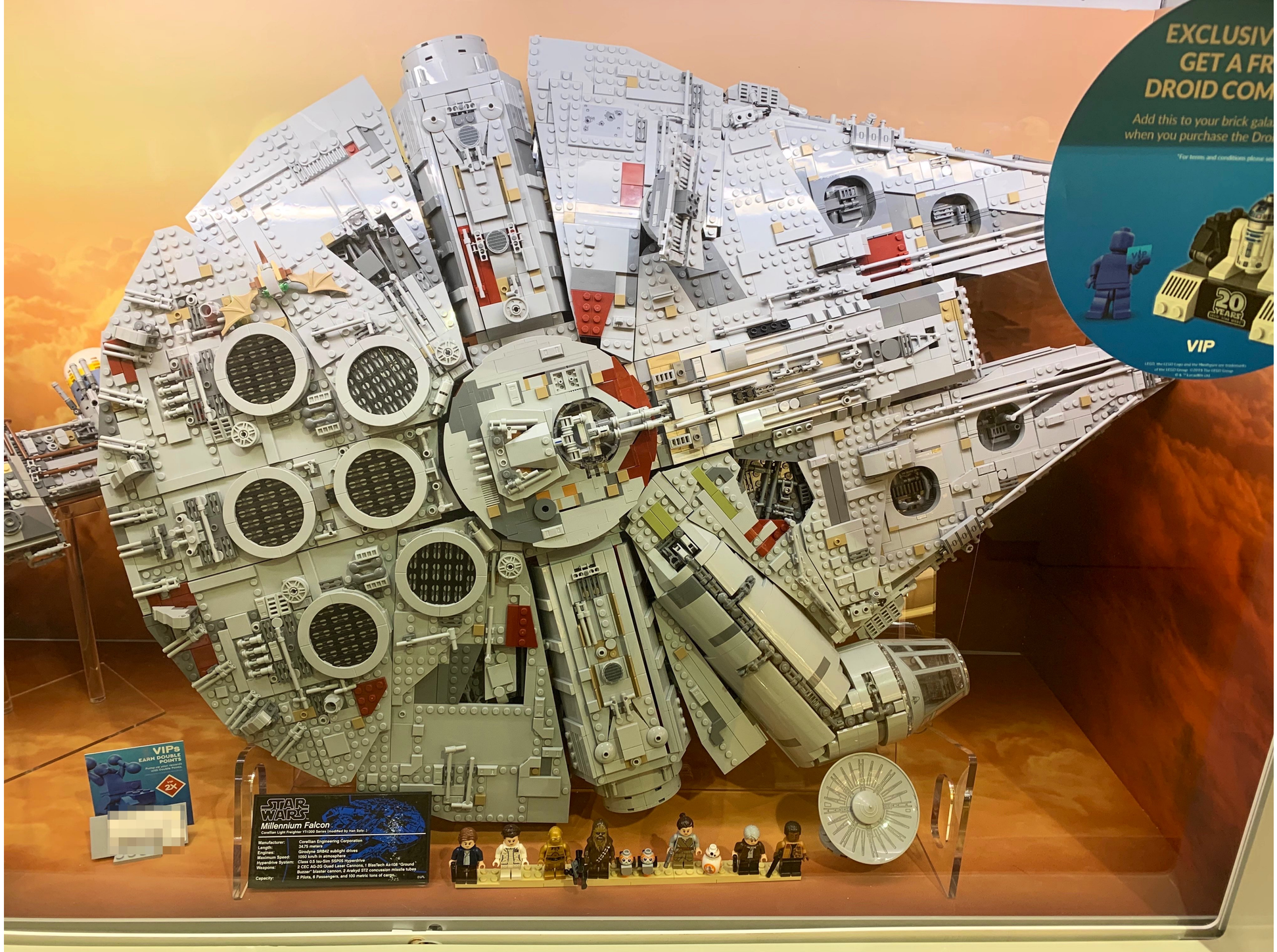
$$6x^2 + 6x + 15x + 10 = 6x^2 + 21x + 10$$

Cathy Marks Krpan

12

24

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STAR WARS
Millennium Falcon
Corellian Light Freighter YT-1300 Series (modified by Han Solo)
Manufacturer: Corellian Engineering Corporation
Length: 34.75 meters
Engines: Girodyne SRB42 sublight drives
Maximum Speed: 1050 km/h in atmosphere
Hyperdrive System: Class 0.5 Isu-Sim SSP05 Hyperdrive
Weapons: 2 CEC AG-2G Quad Laser Cannons, 1 BlasTech As108 "Ground Buzzer" blaster cannon, 2 Arakyd ST2 concussion missile tubes
Capacity: 2 Pilots, 6 Passengers, and 100 metric tons of cargo.
EXCLUSIV
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VIP
20 YEARS

3 Act Math Tasks

- What is the first question that comes to you mind?
- Write down an estimate that is:
  - too low
  - too high
  - just right
- What information would be helpful to have here?

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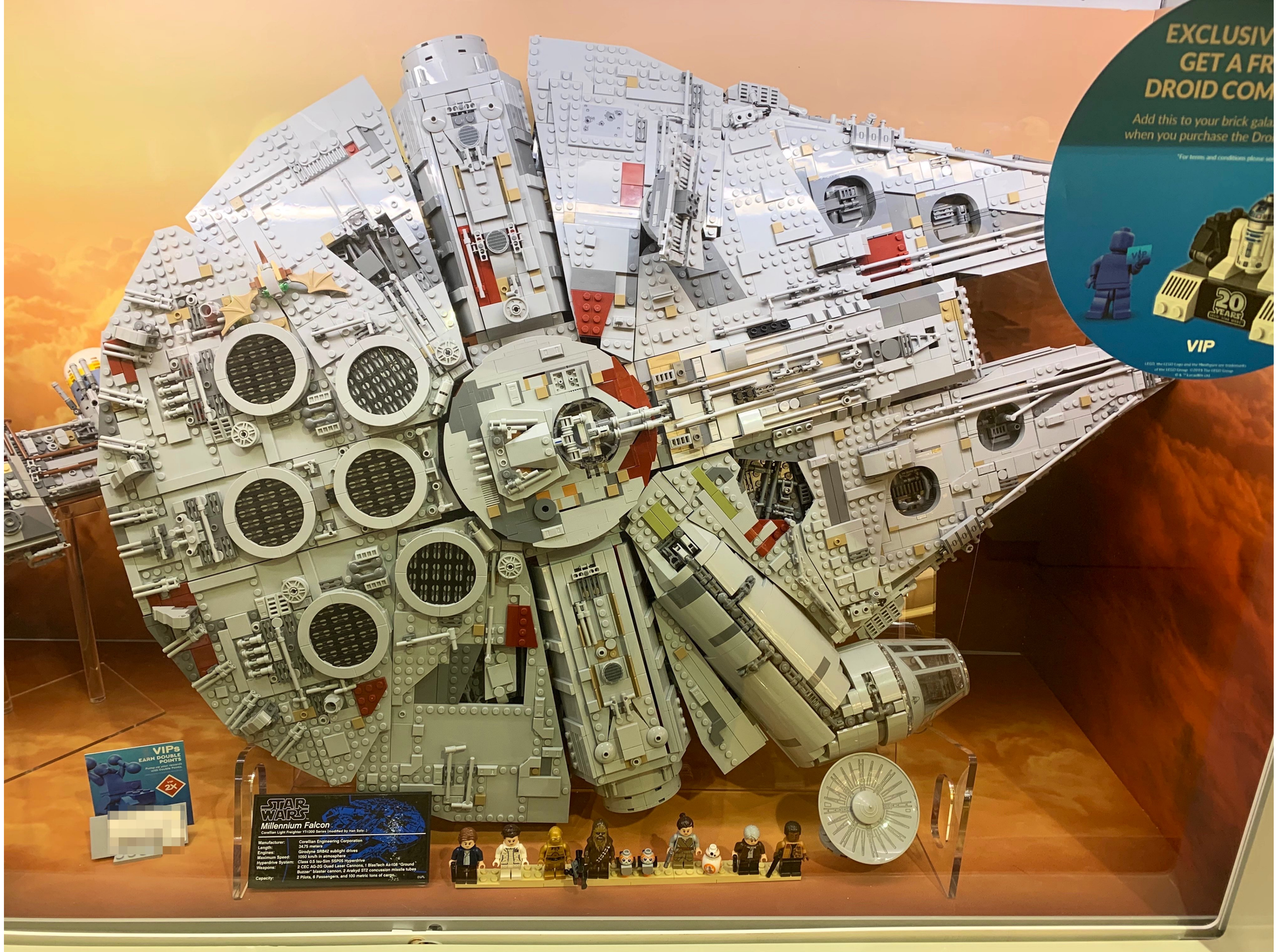
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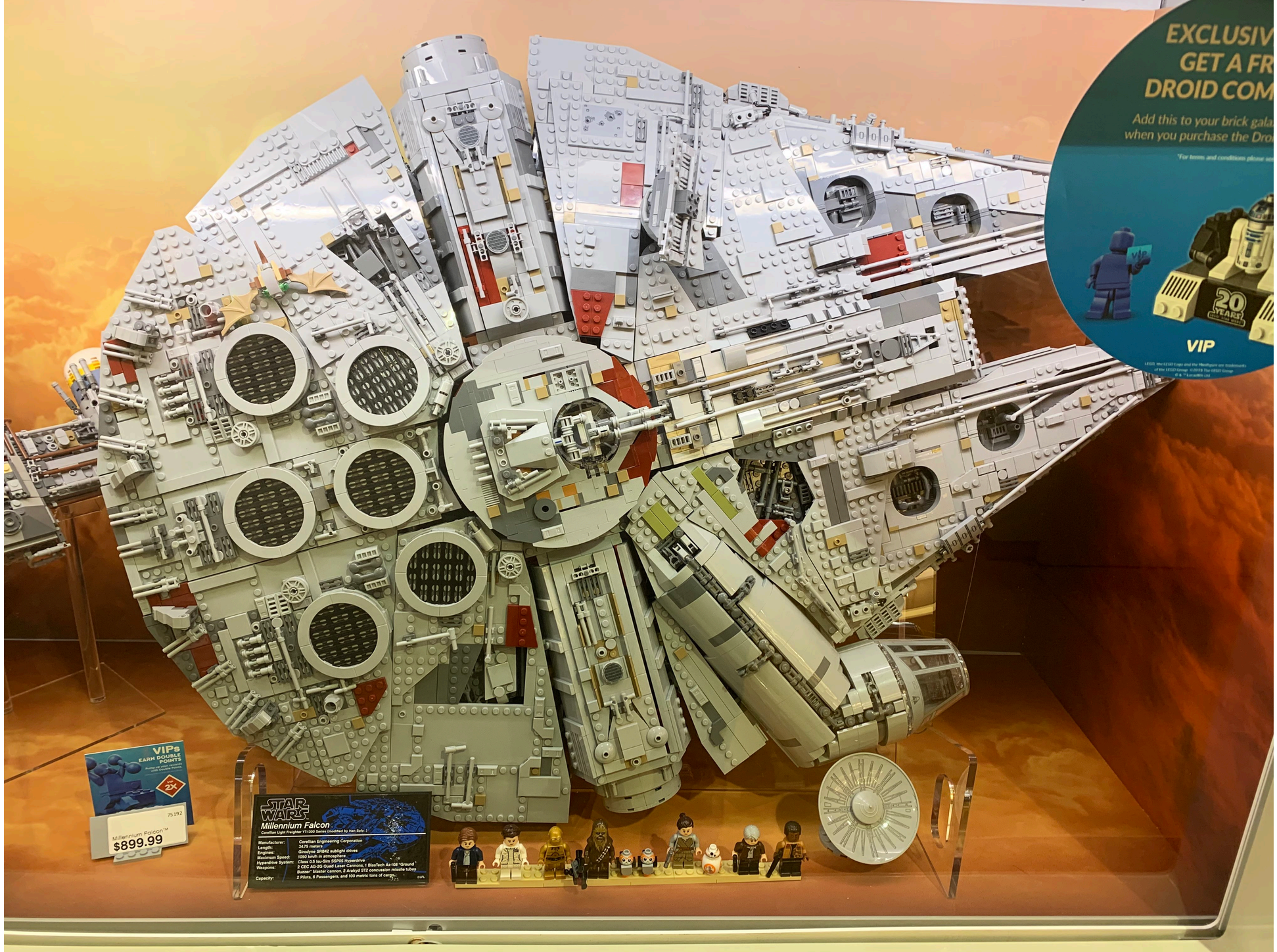
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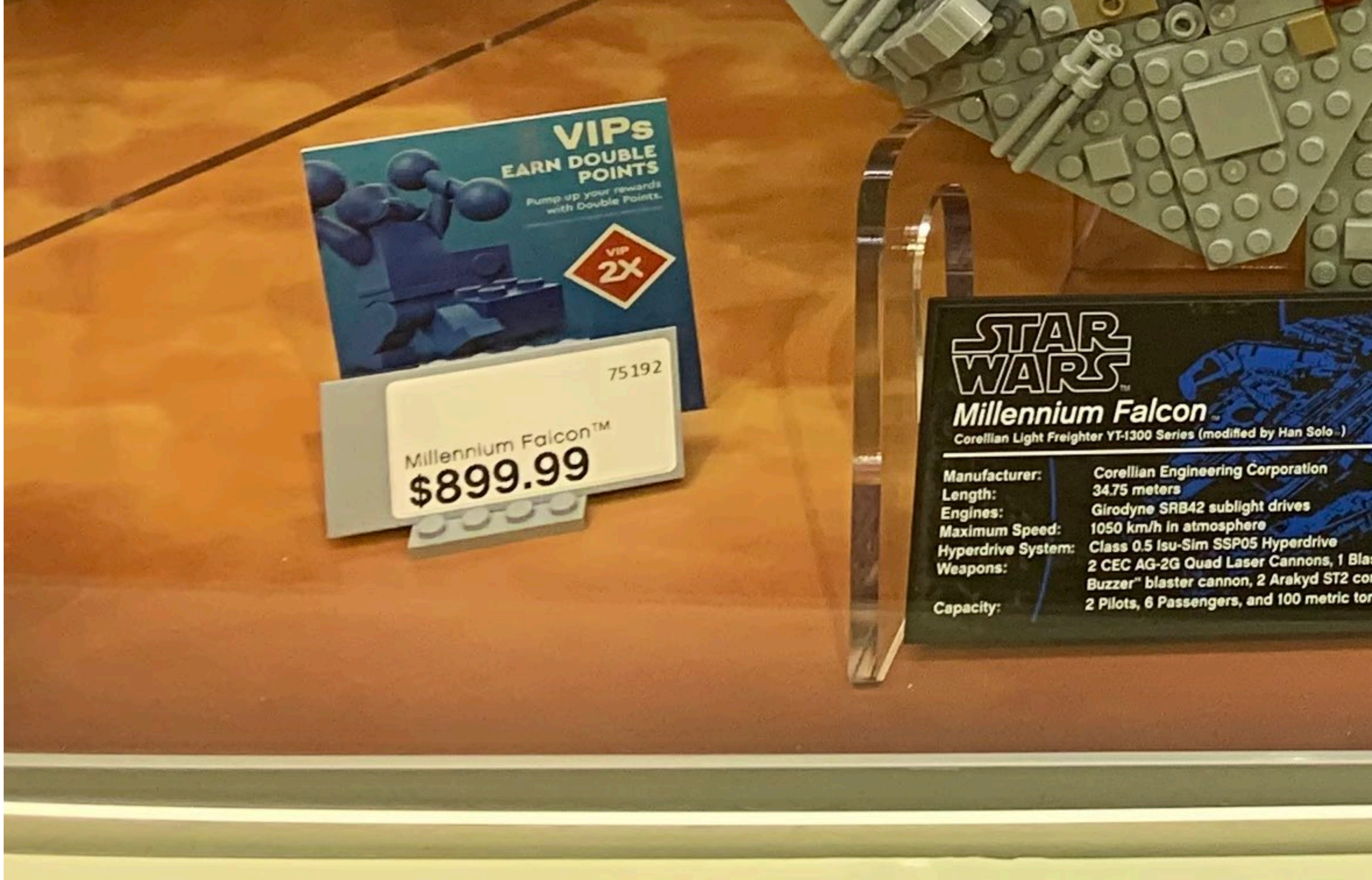
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Weapons: 2 CEC AG-2G Quad Laser Cannons, 1 Bla
Buzzer" blaster cannon, 2 Arakyd ST2 co
Capacity: 2 Pilots, 6 Passengers, and 100 metric to

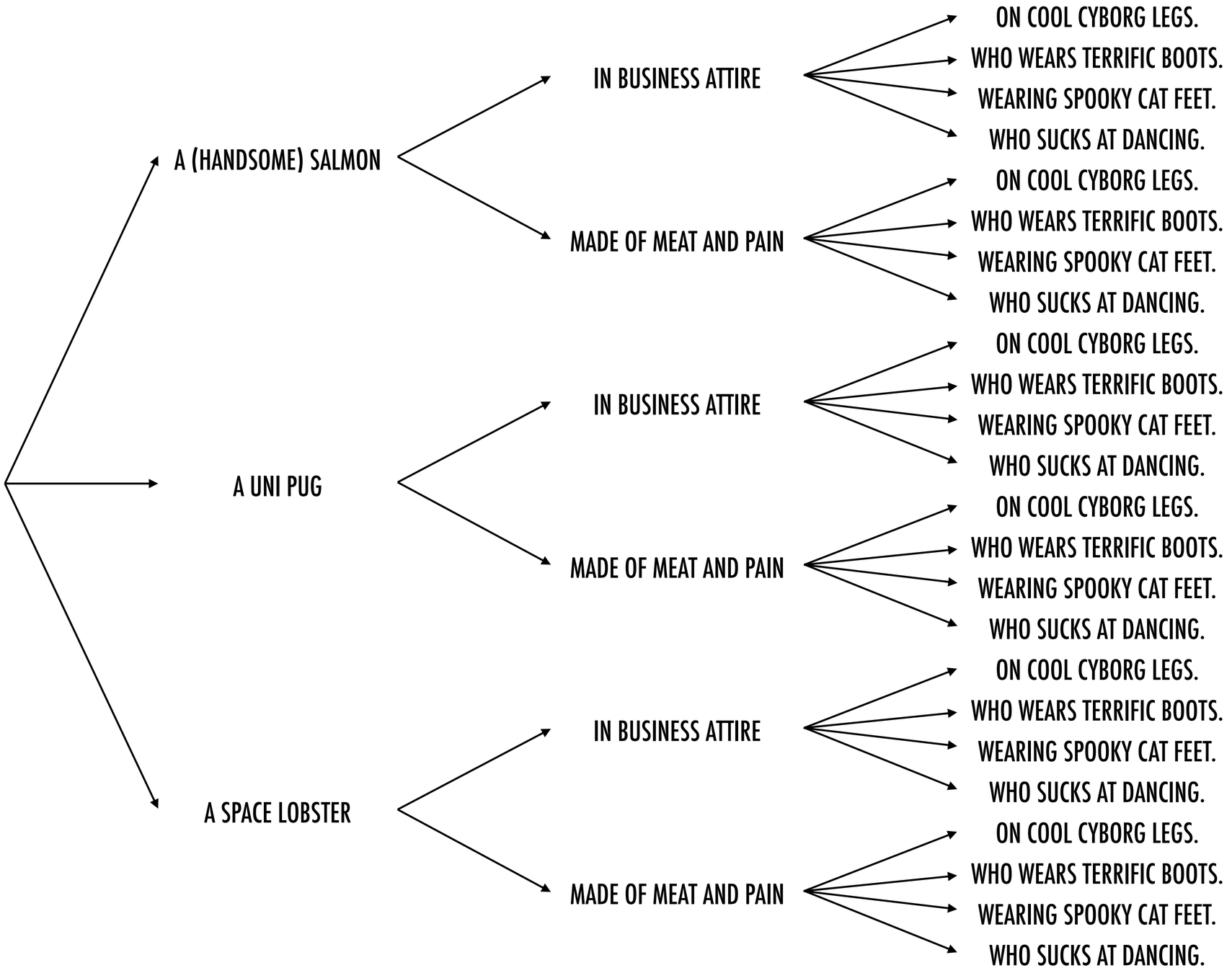

chrishunter.ca

A (HANDSOME) SALMON
A UNI PUG
A SPACE LOBSTER

IN BUSINESS ATTIRE
MADE OF MEAT AND PAIN

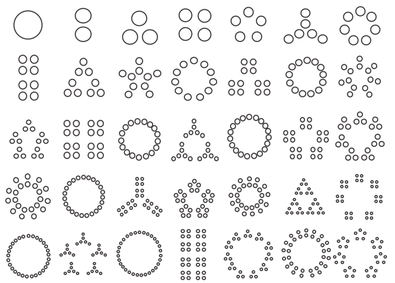
ON COOL CYBORG LEGS.
WHO WEARS TERRIFIC BOOTS.
WEARING SPOOKY CAT FEET.
WHO SUCKS AT DANCING.


A SALMON
A UNI PUG
A SPACE LOBSTER
IN BUSINESS ATTIRE
MADE OF MEAT AND PAIN
WHO WEARS TERRIFIC BOOTS
WEARING SPOOKY CAT FEET


A (HANDSOME) SALMON
A UNI PUG
A SPACE LOBSTER
IN BUSINESS ATTIRE
MADE OF MEAT AND PAIN
IN BUSINESS ATTIRE
MADE OF MEAT AND PAIN
IN BUSINESS ATTIRE
MADE OF MEAT AND PAIN
ON COOL CYBORG LEGS.
WHO WEARS TERRIFIC BOOTS.
WEARING SPOOKY CAT FEET.
WHO SUCKS AT DANCING.

Meanings

- Repeated Addition
- Equal Groups or Sets
- Array
- Area of a Rectangle
- Comparison
- Rate
- Combinations



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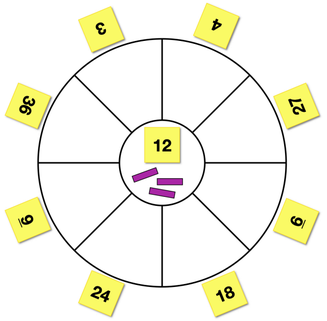
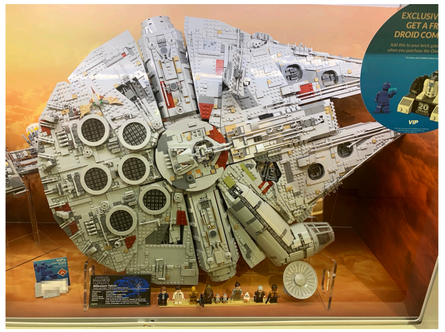




More Than Times Tables

Effective Mathematics Teaching Practices

- Establish mathematics goals to focus learning
- Implement tasks that promote reasoning and problem solving
- Use and connect mathematical representations
- Facilitate meaningful discourse
- Pose purposeful questions
- **Build procedural fluency *from* conceptual understanding**
- Support productive struggle in learning mathematics
- Elicit and use evidence of student thinking

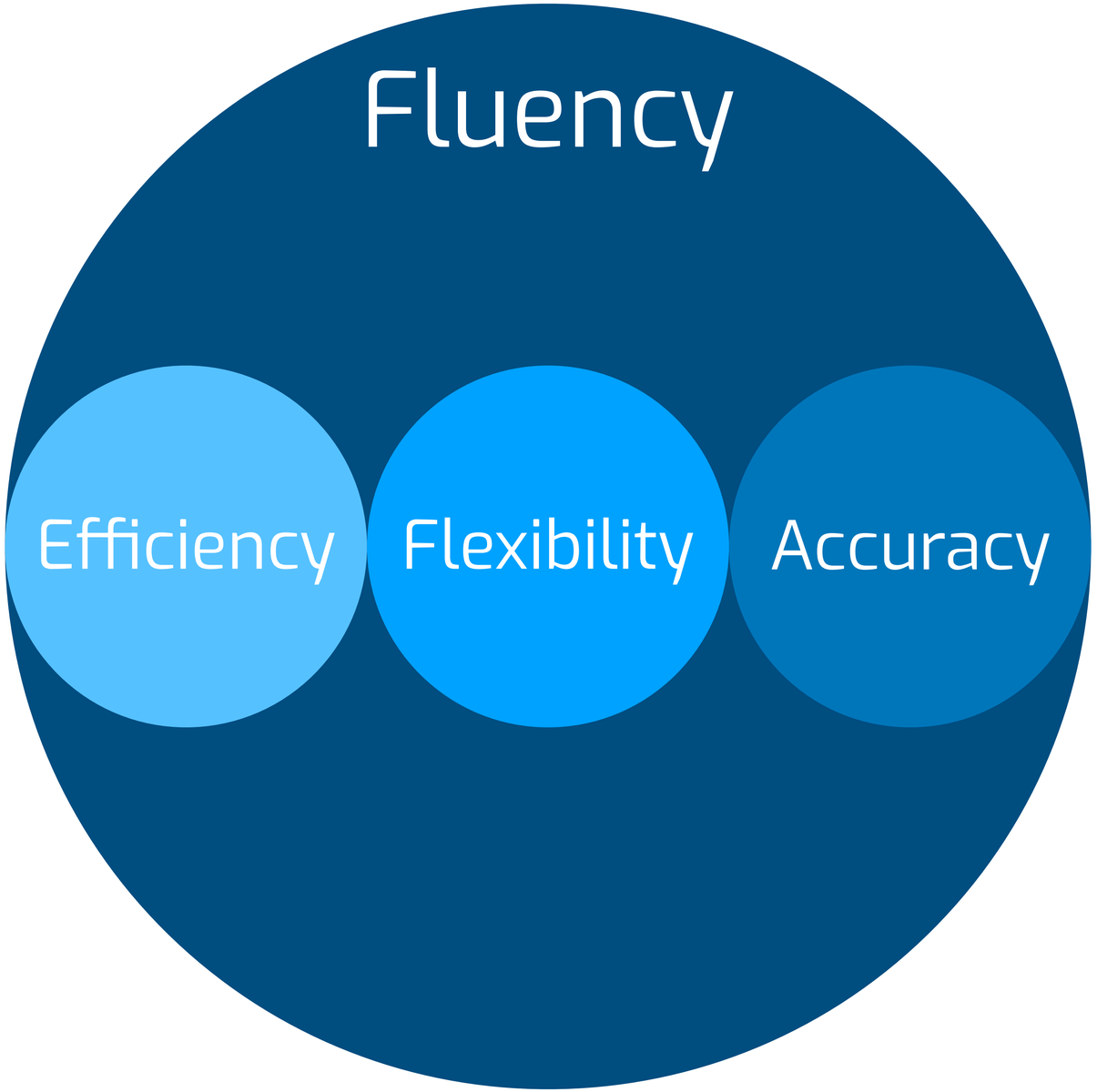
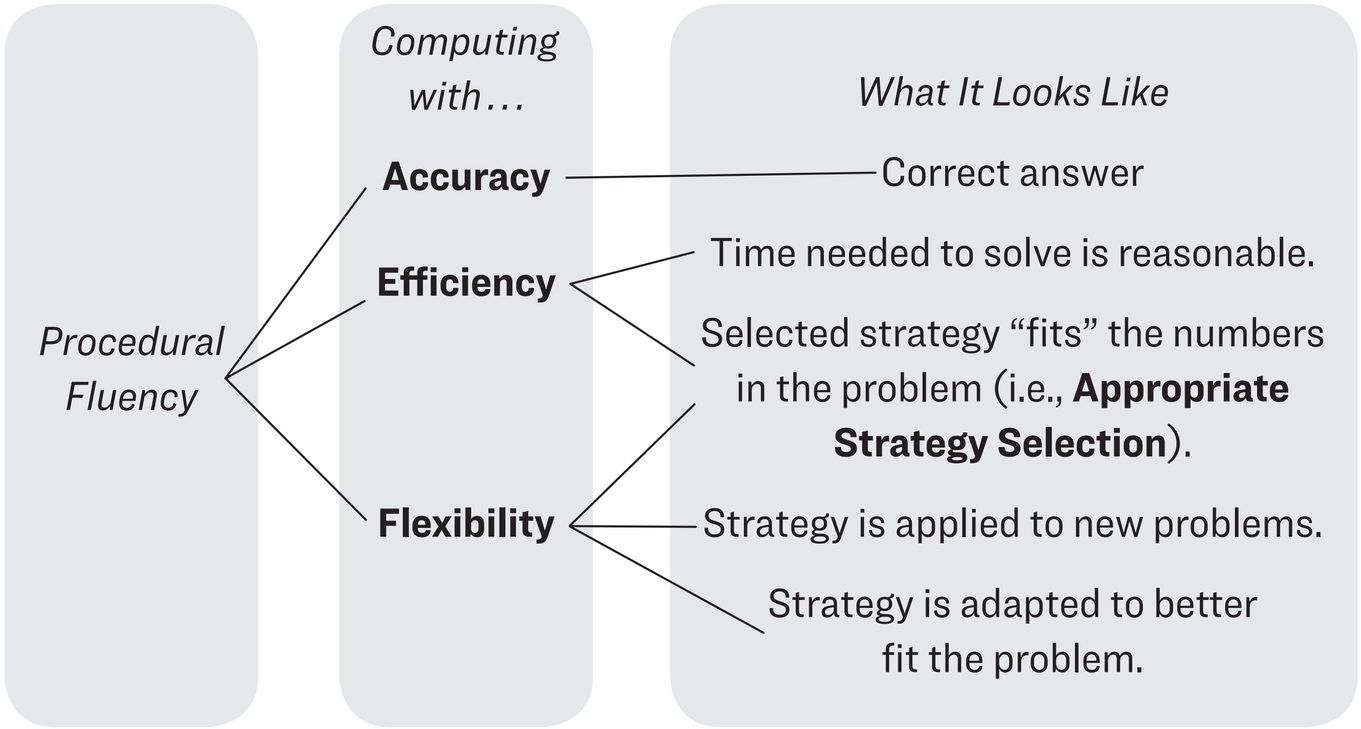
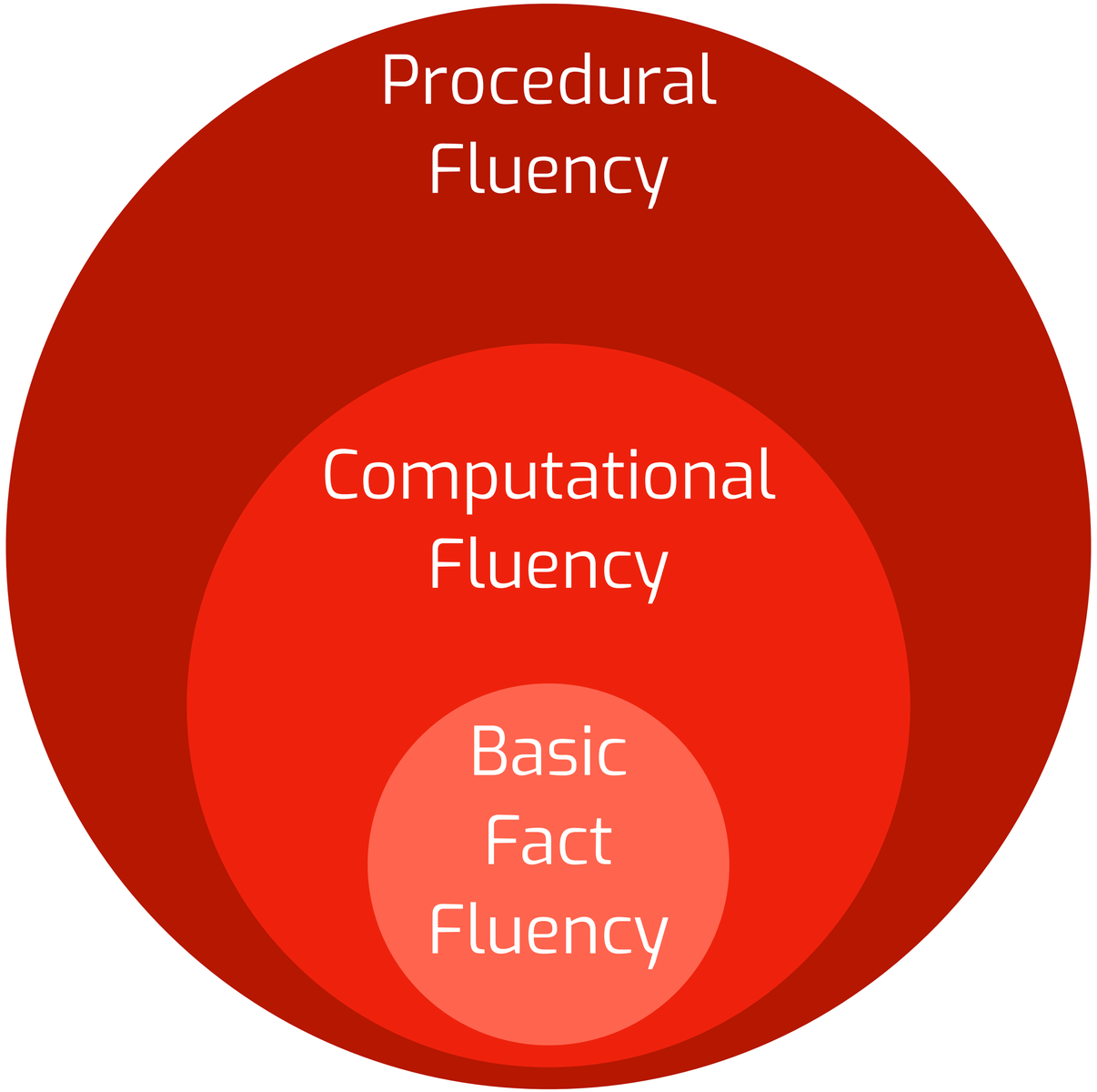

Fluency
Efficiency
Flexibility
Accuracy

FIGURE 1.1 **What Procedural Fluency Is and What It Looks Like**



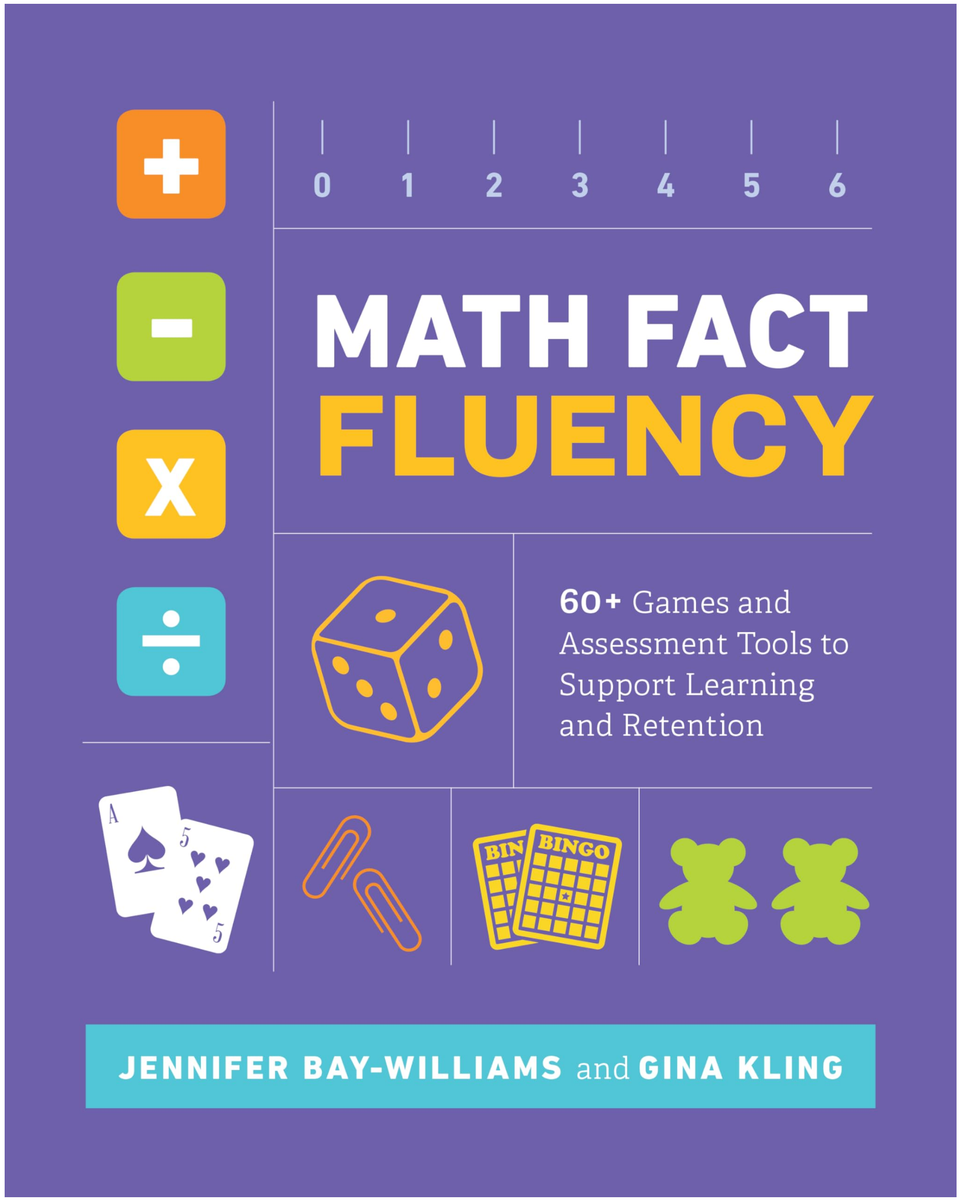
The four components (**bolded**) are interrelated. Appropriate strategy selection is required for efficiency and flexibility.

Bay-Williams, J. & Kling, G. (2019). *Math fact fluency: 60+ games and assessment tools to support learning and retention*. National Council of Teachers of Mathematics.


Procedural
Fluency
Computational
Fluency
Basic
Fact
Fluency

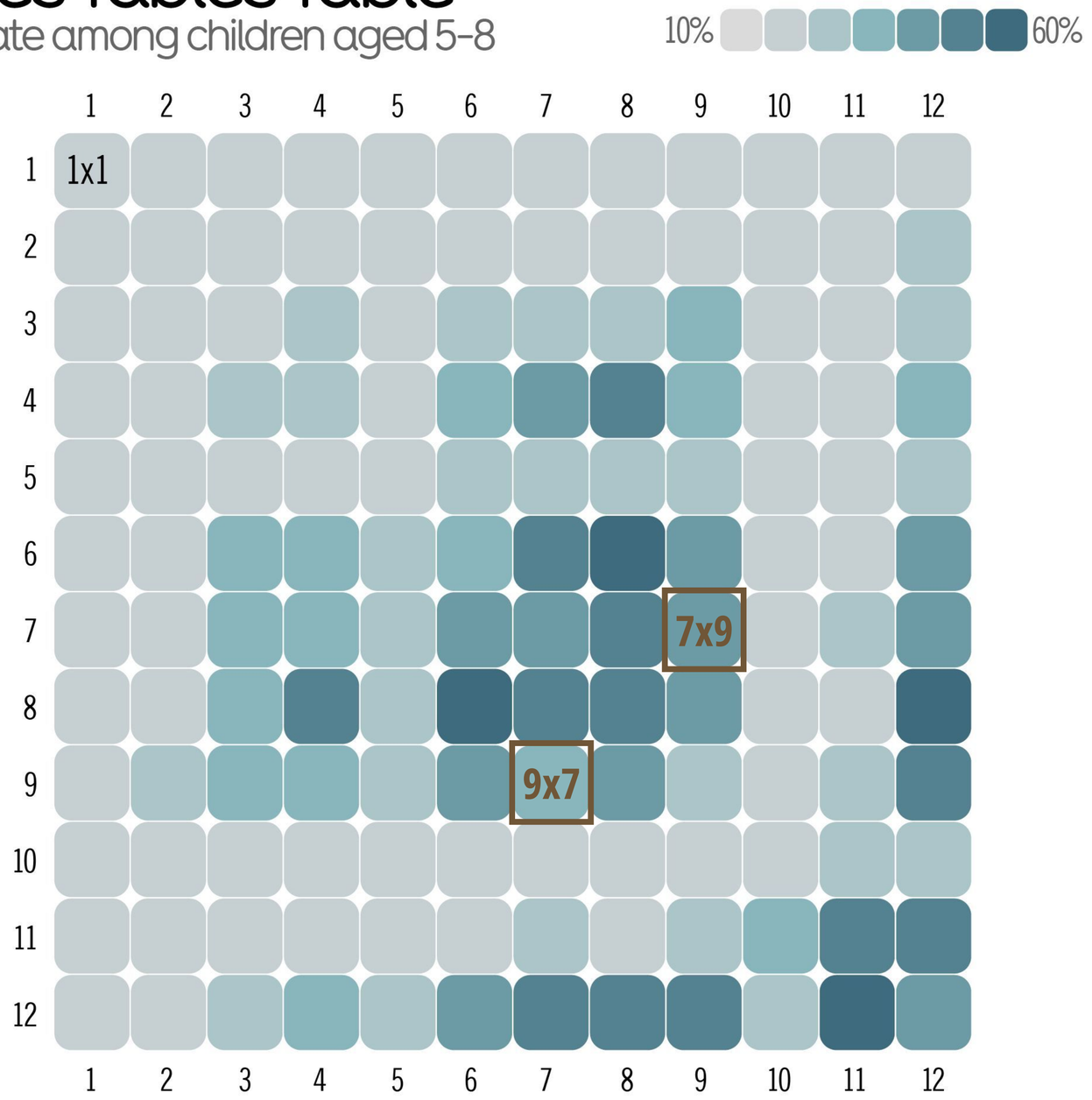
The Five Fundamentals

1. Mastery Must Focus on Fluency
2. Fluency Develops in Three Phases
3. Foundational Facts Must Precede Derived Facts
4. Timed Tests Do Not Assess Fluency
5. Students Need Substantial and Enjoyable Practice



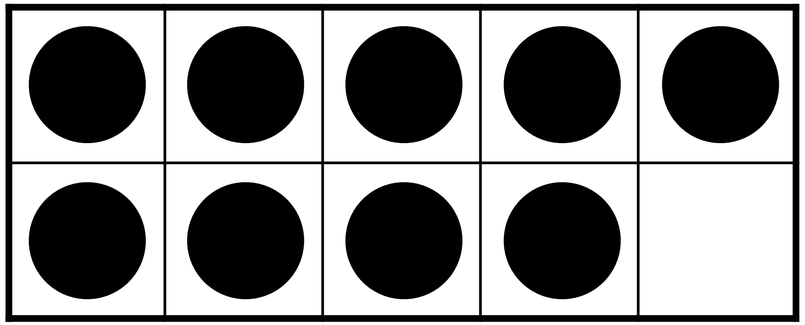
Times Tables Table

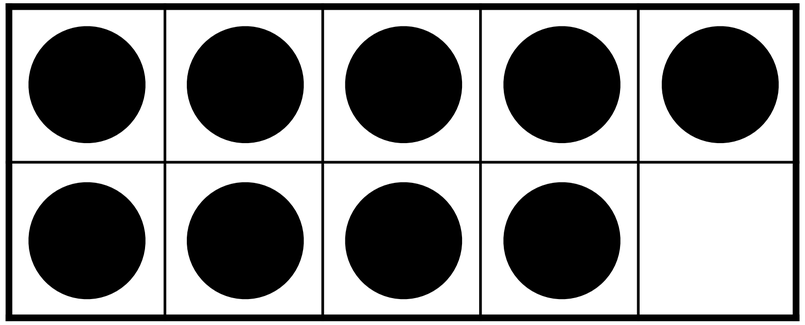
Error rate among children aged 5-8

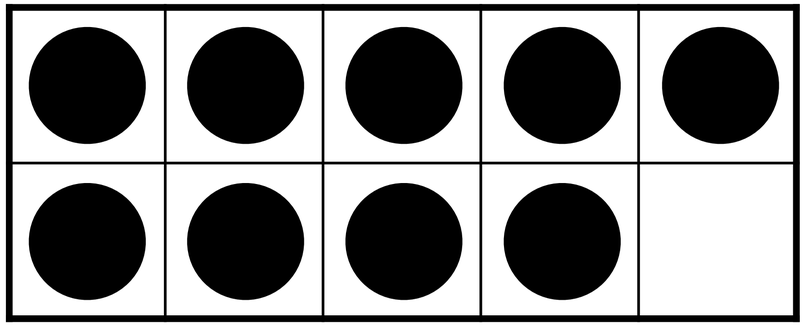


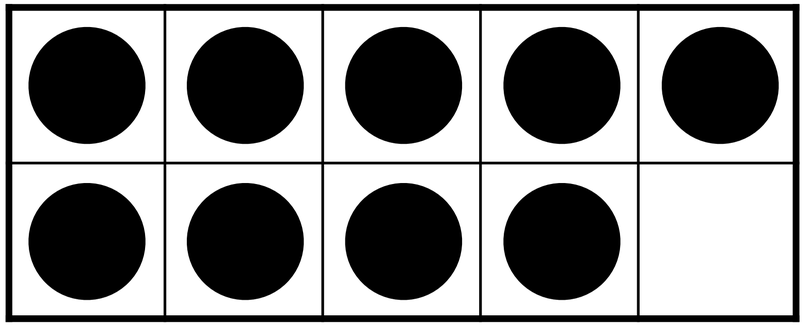
Information is Beautiful

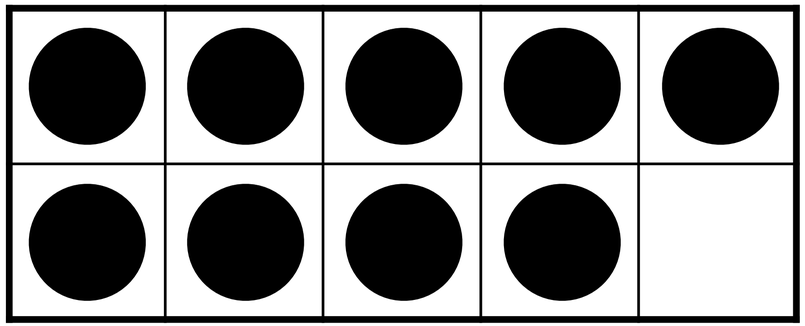
source: private study by Caddington School of 232 children

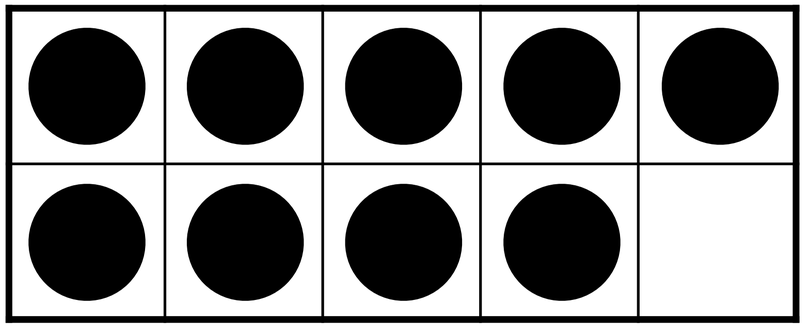


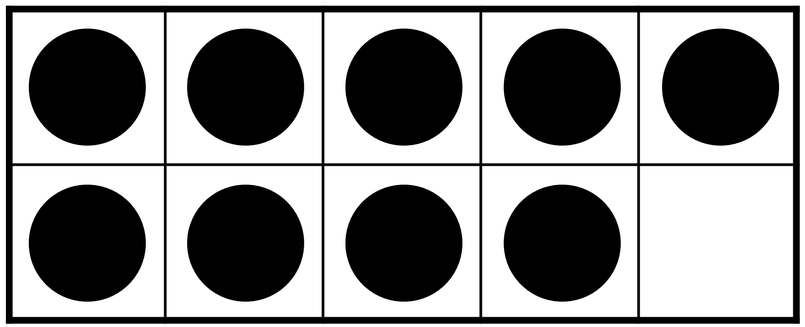


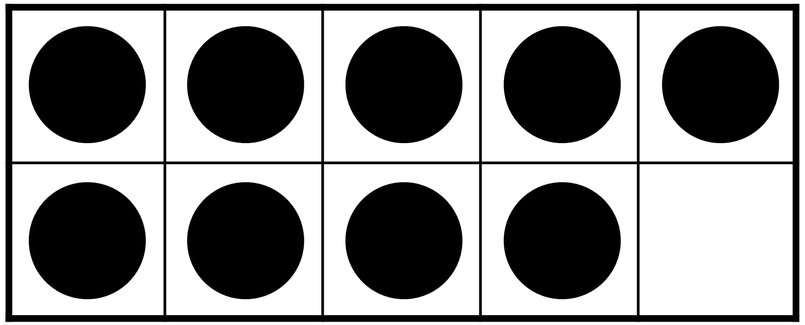


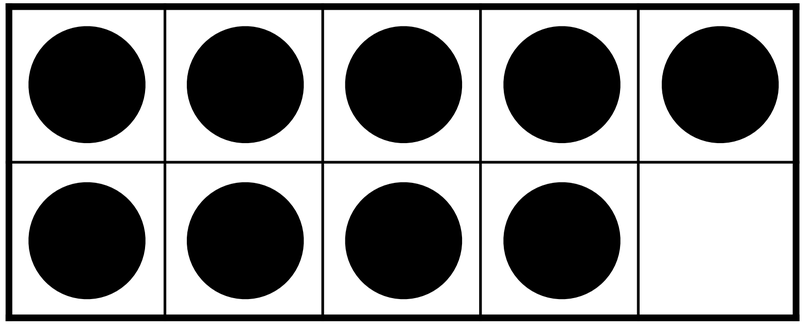


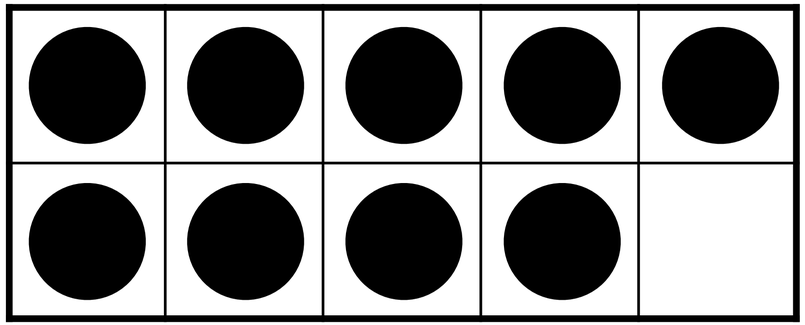


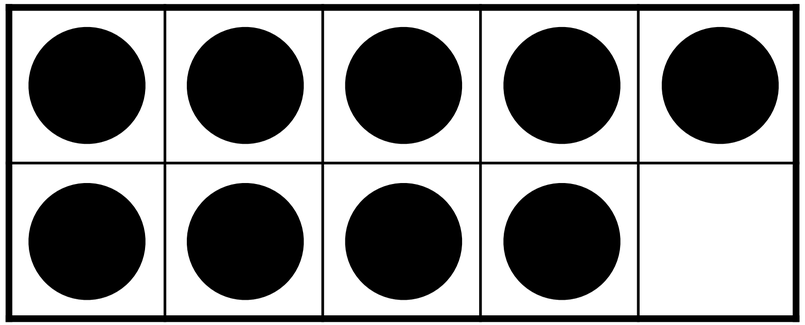


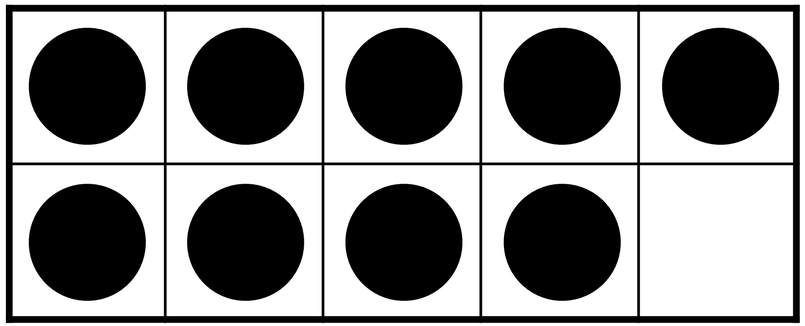


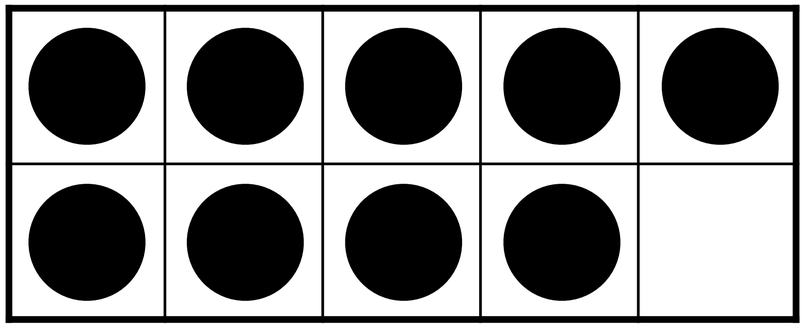


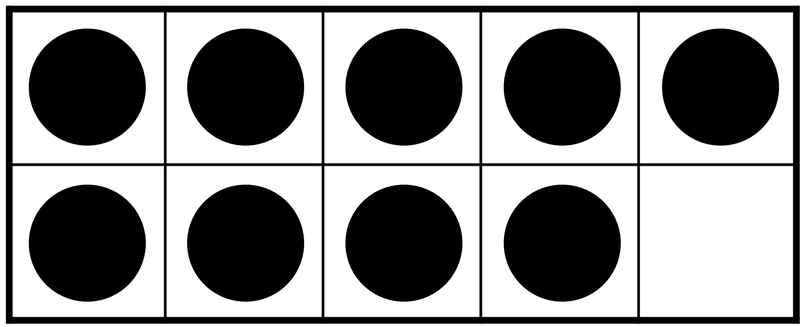


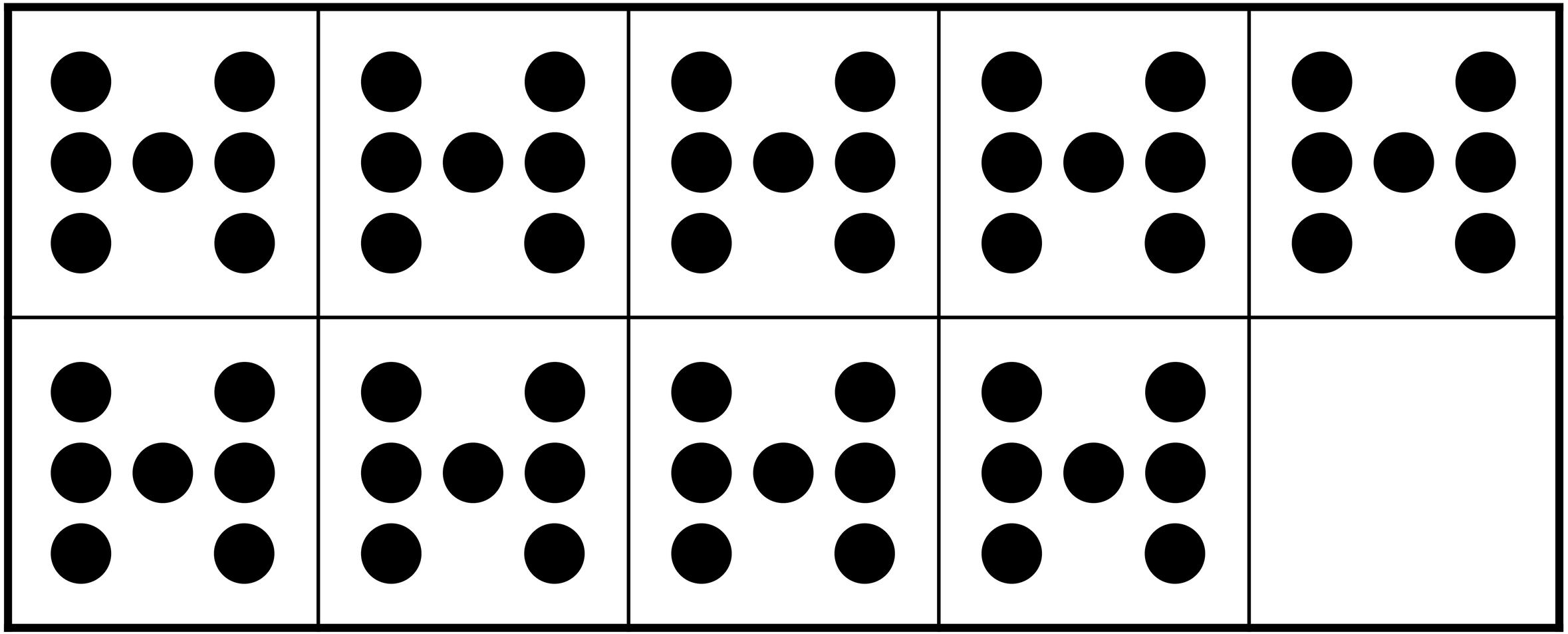


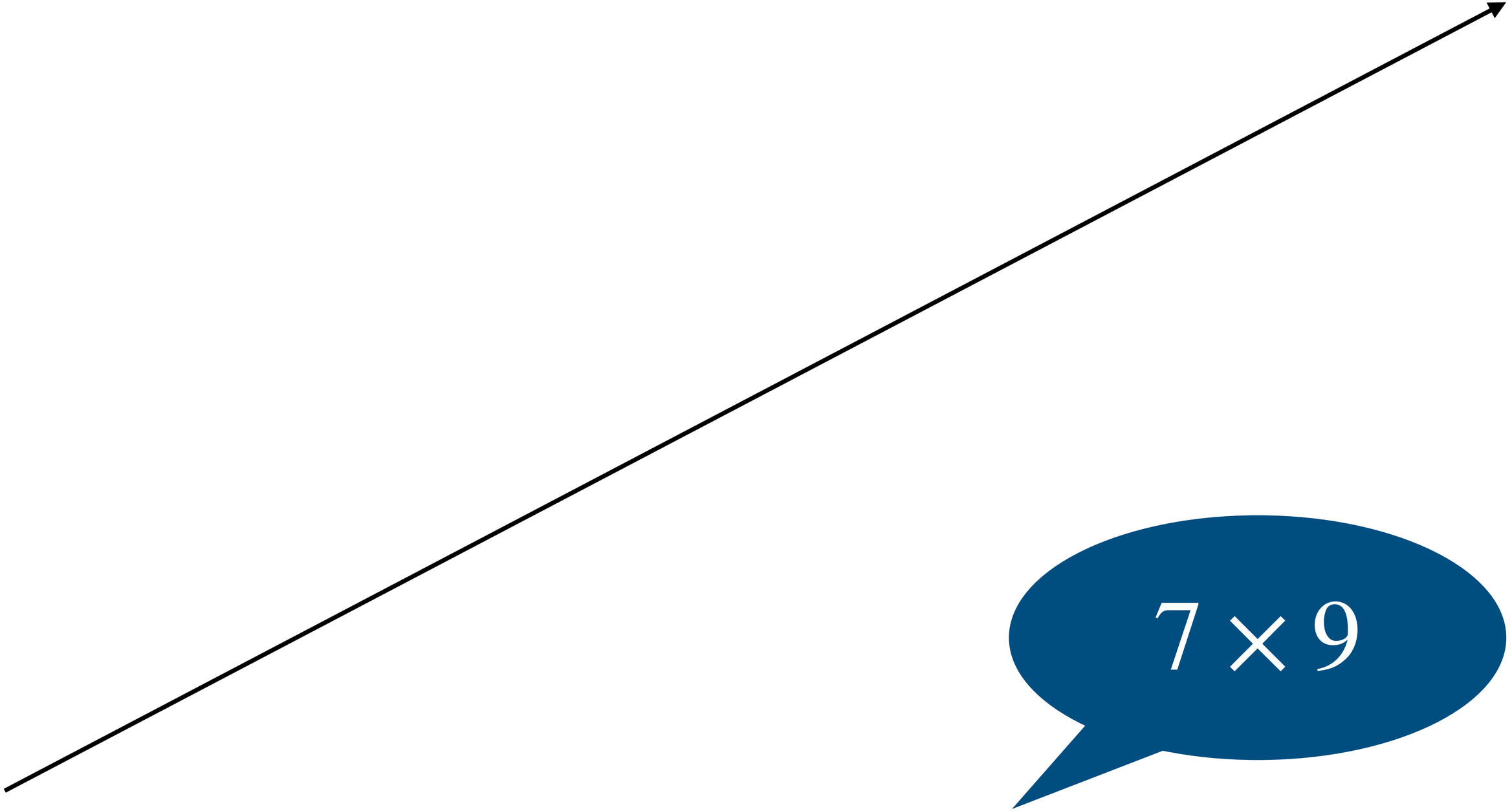


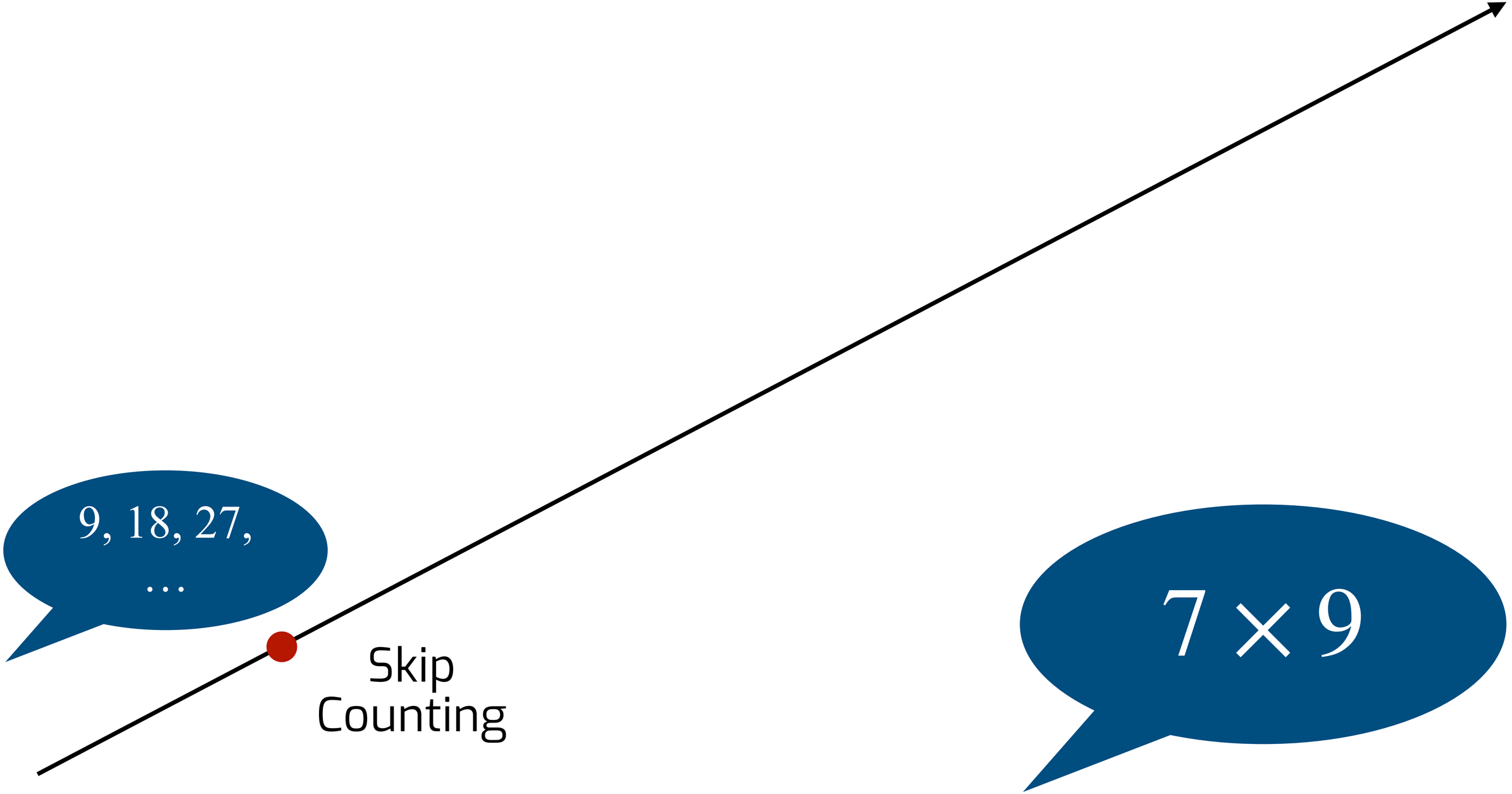


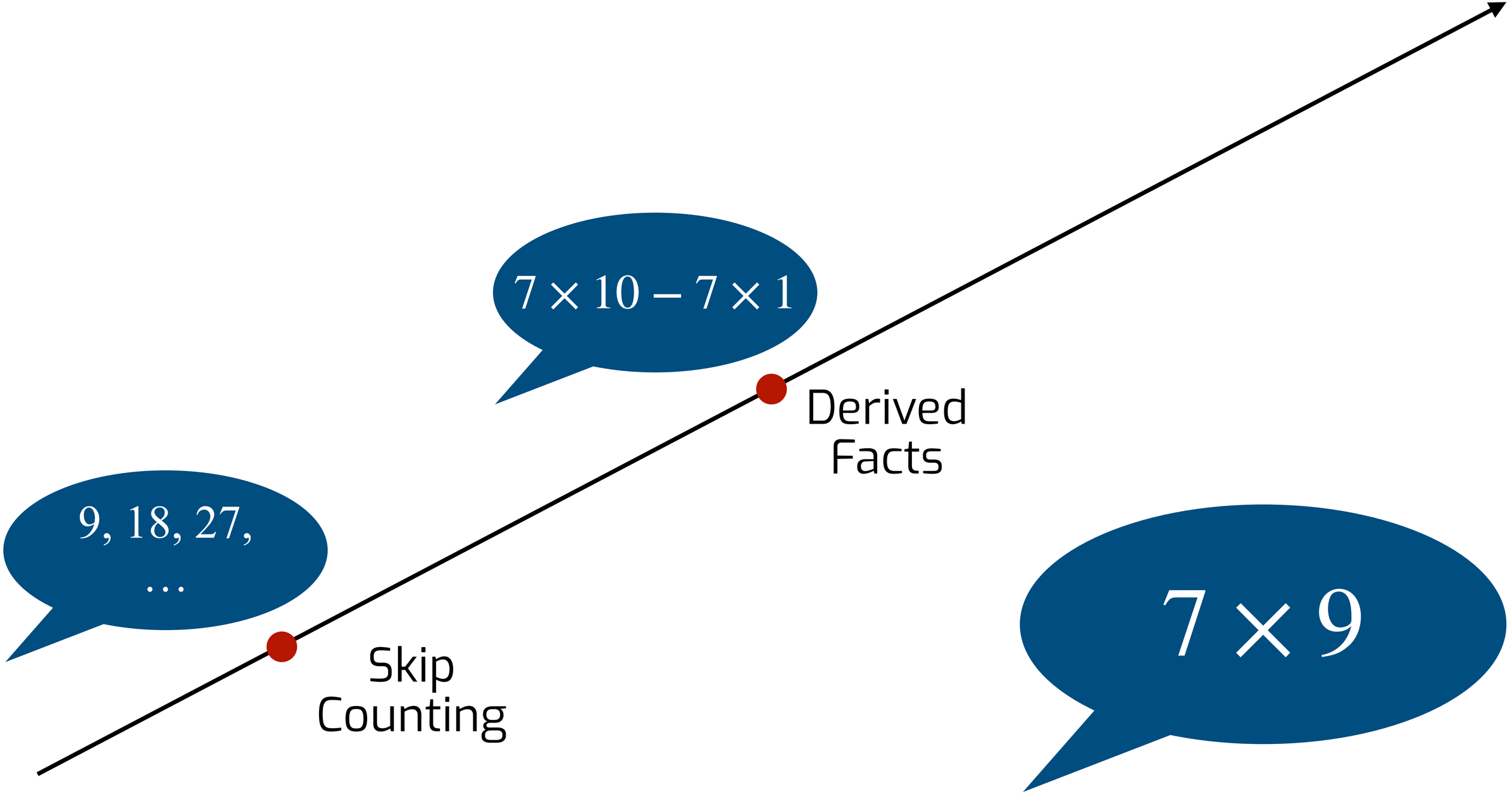


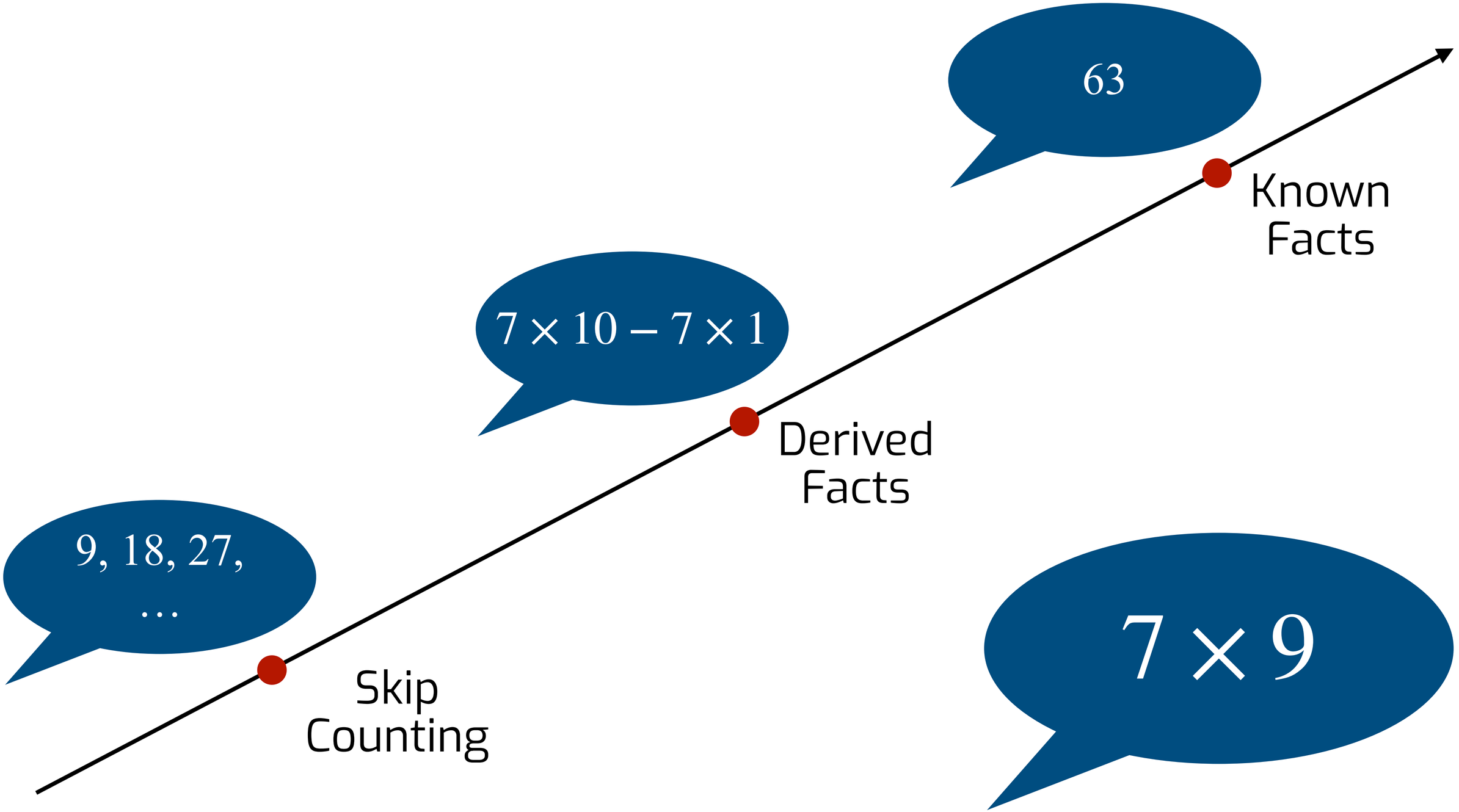


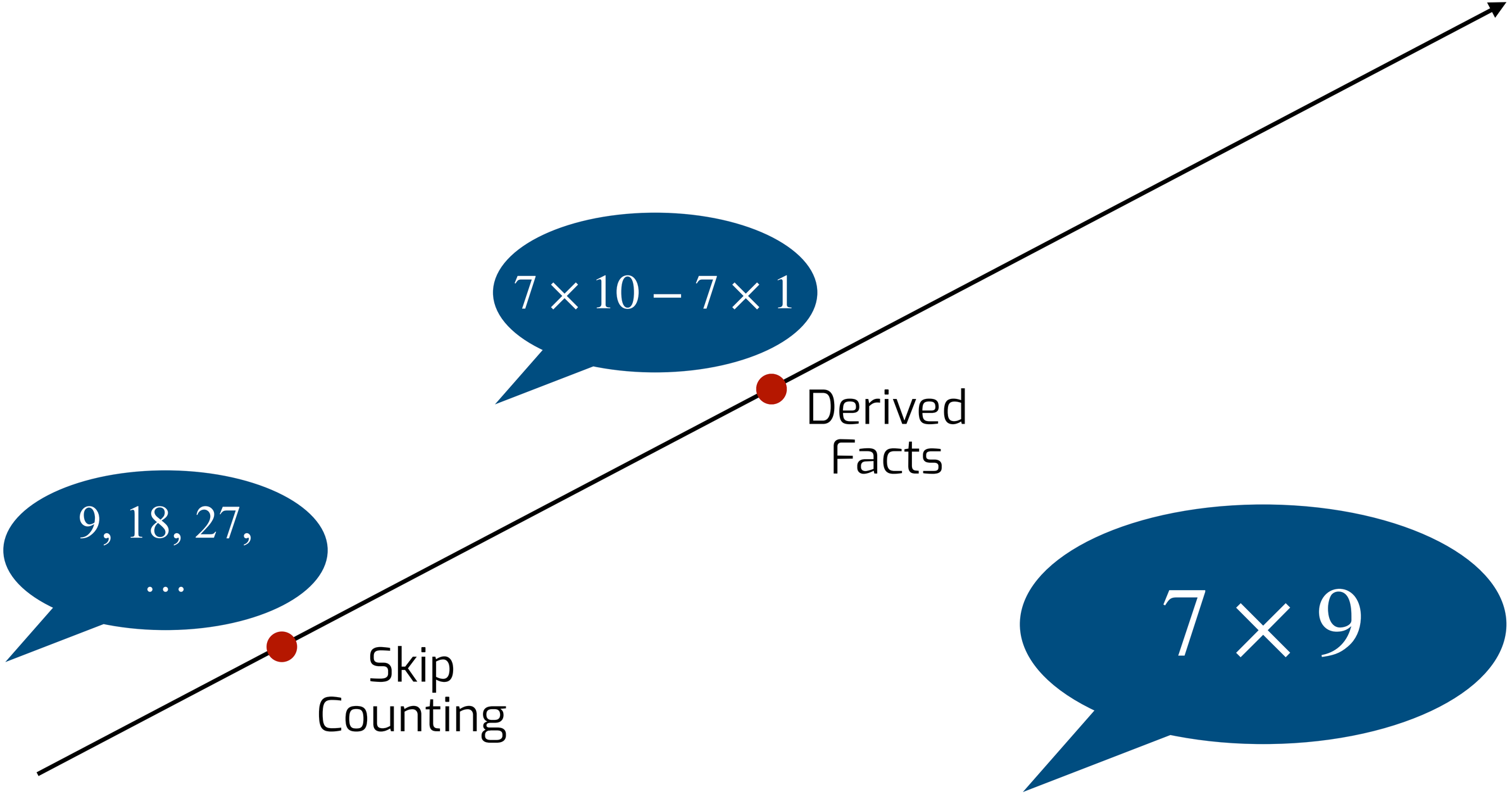


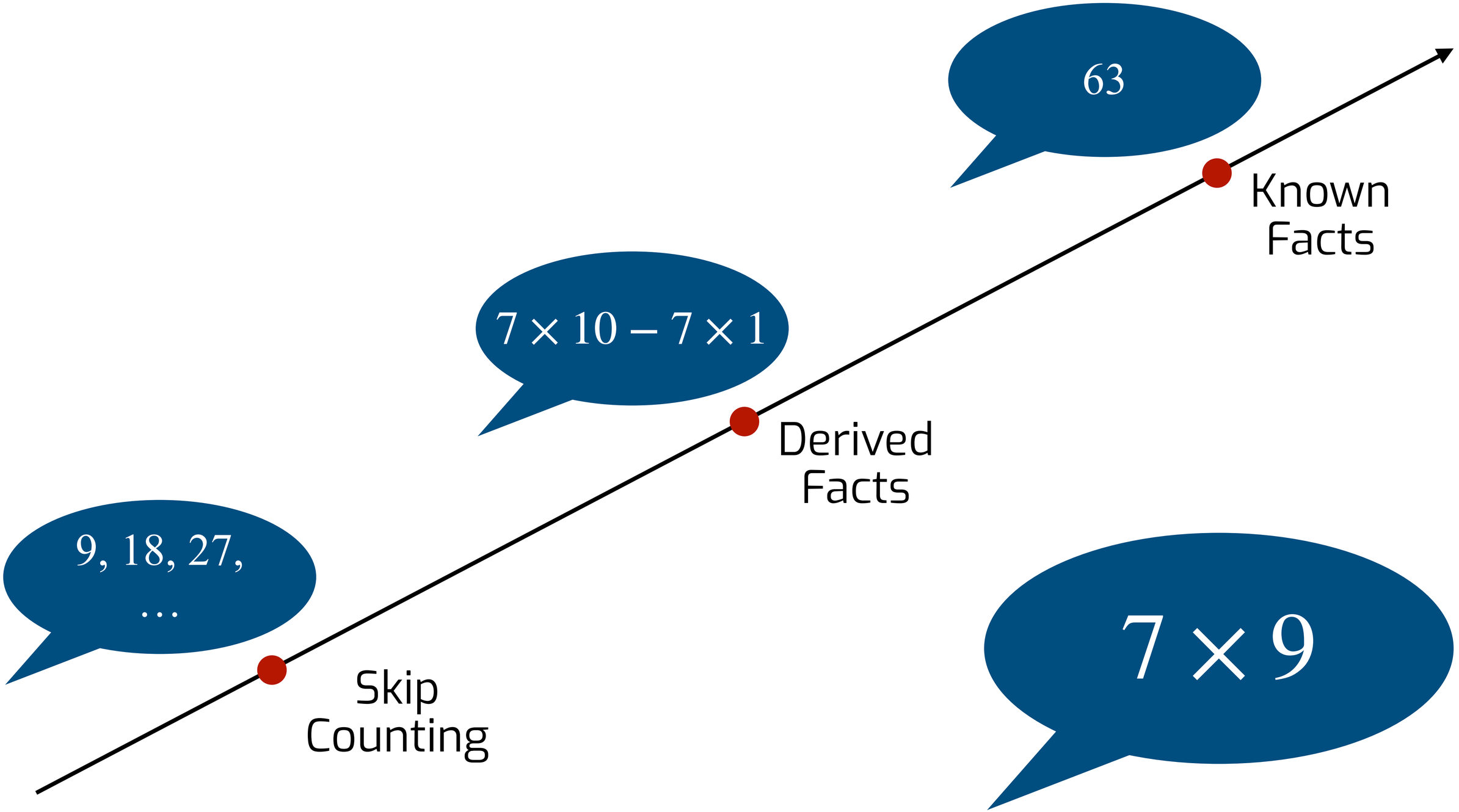

7×9

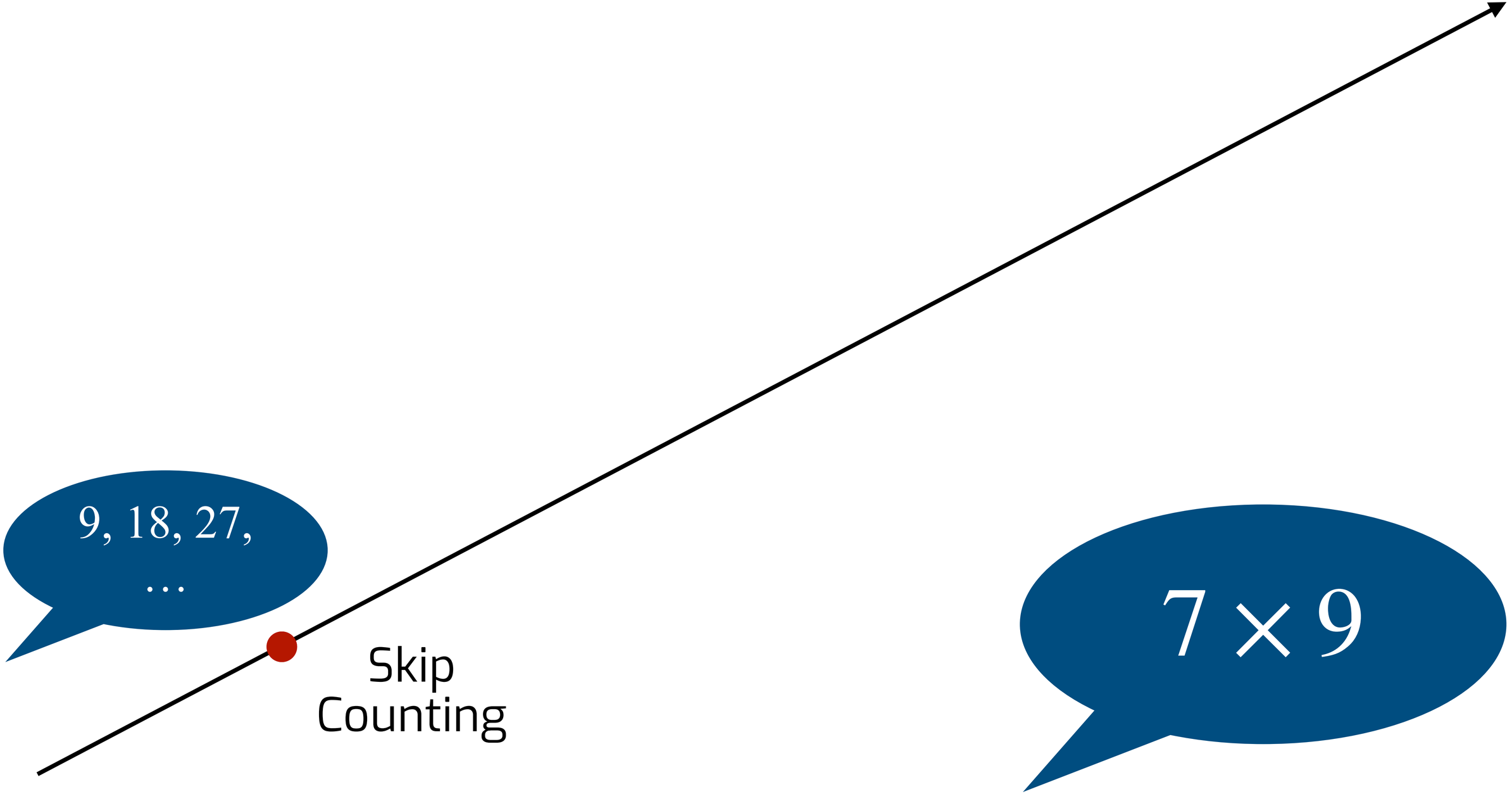

9, 18, 27,
…
Skip
Counting
7 × 9

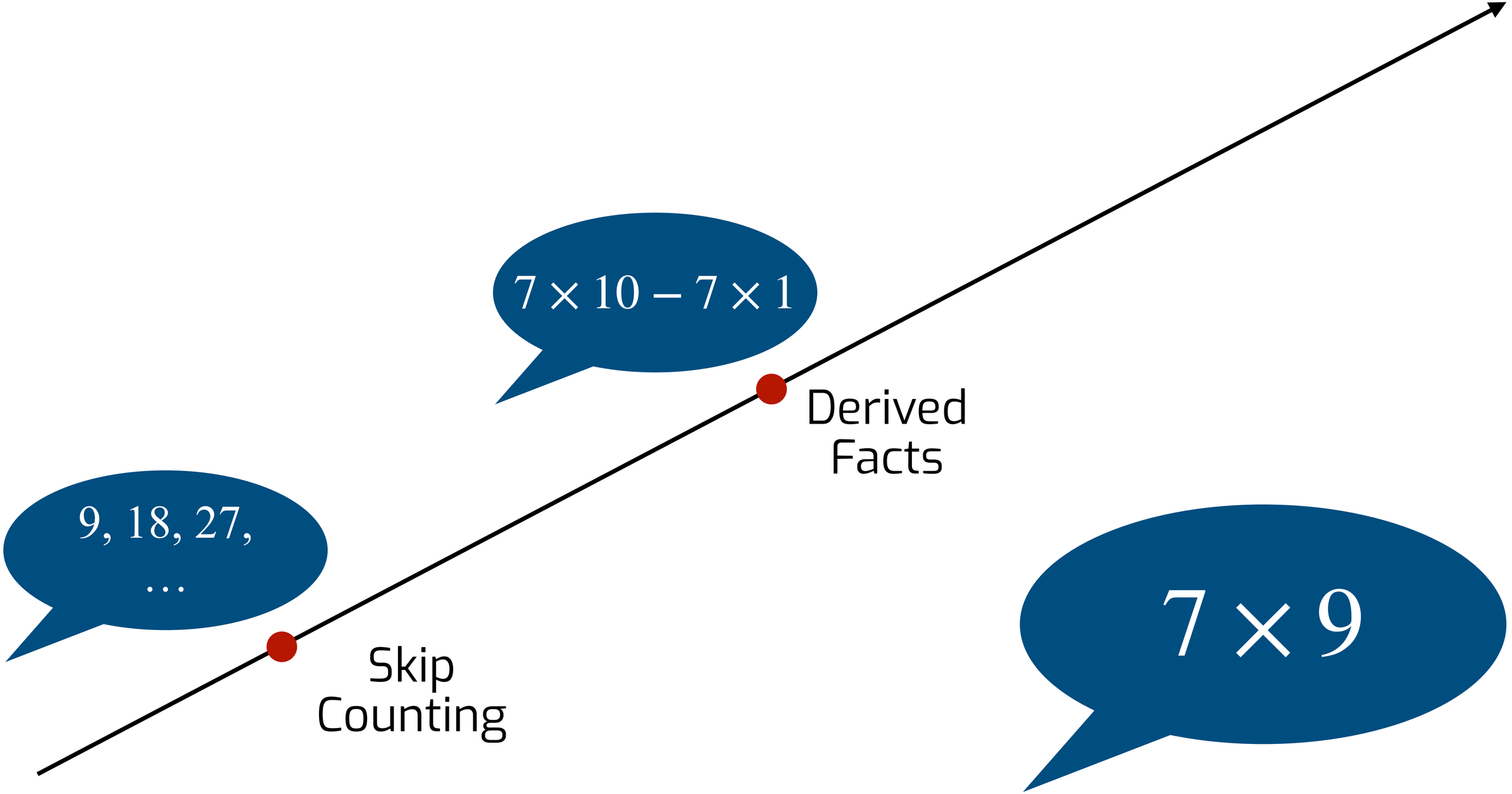

9, 18, 27, …
Skip Counting
$7 \times 10 - 7 \times 1$
Derived Facts
7×9

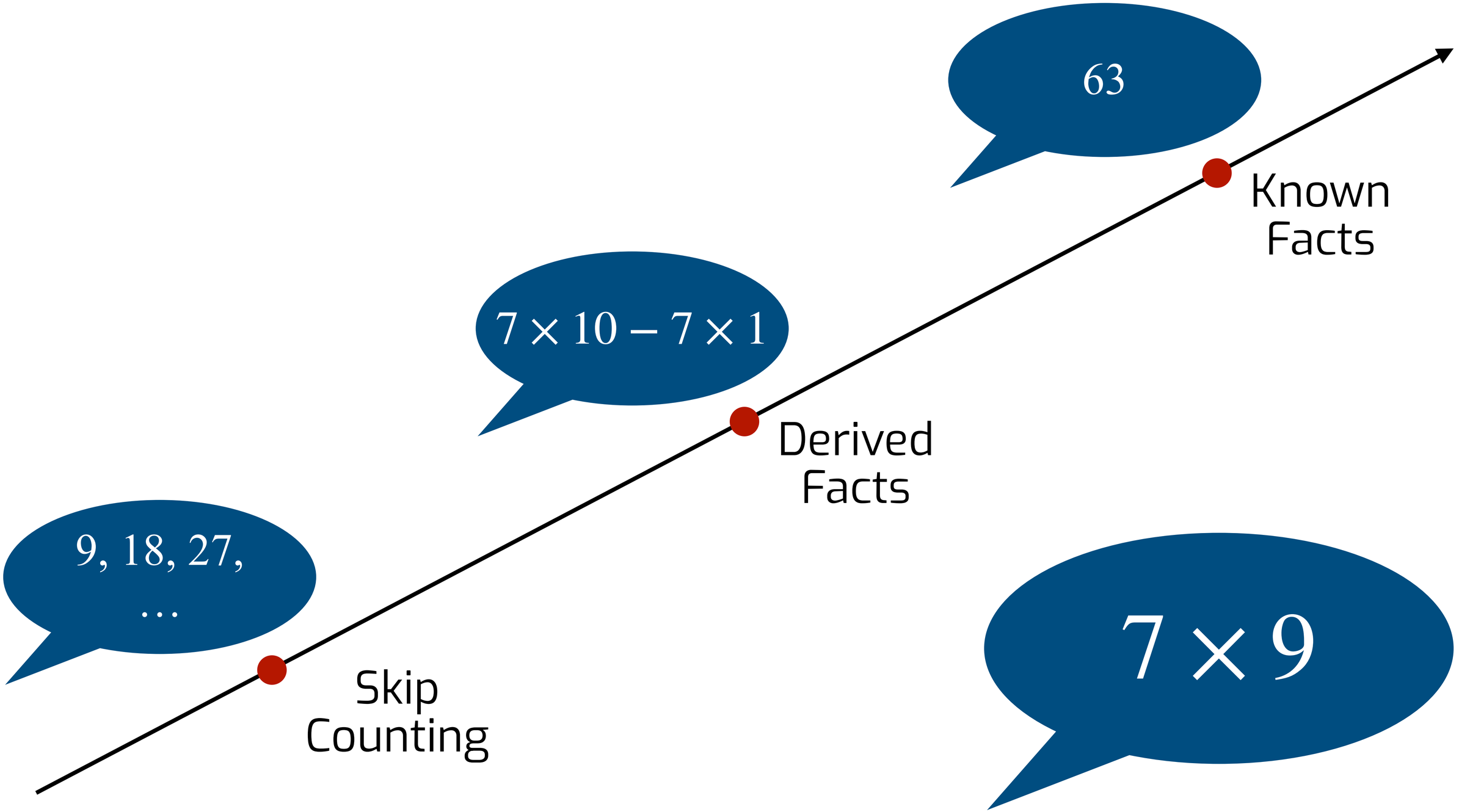

63
Known Facts
$7 \times 10 - 7 \times 1$
Derived Facts
9, 18, 27, …
Skip Counting
7×9


9, 18, 27, …
Skip Counting
$7 \times 10 - 7 \times 1$
Derived Facts
7×9


63
Known Facts
$7 \times 10 - 7 \times 1$
Derived Facts
9, 18, 27, …
Skip Counting
7×9

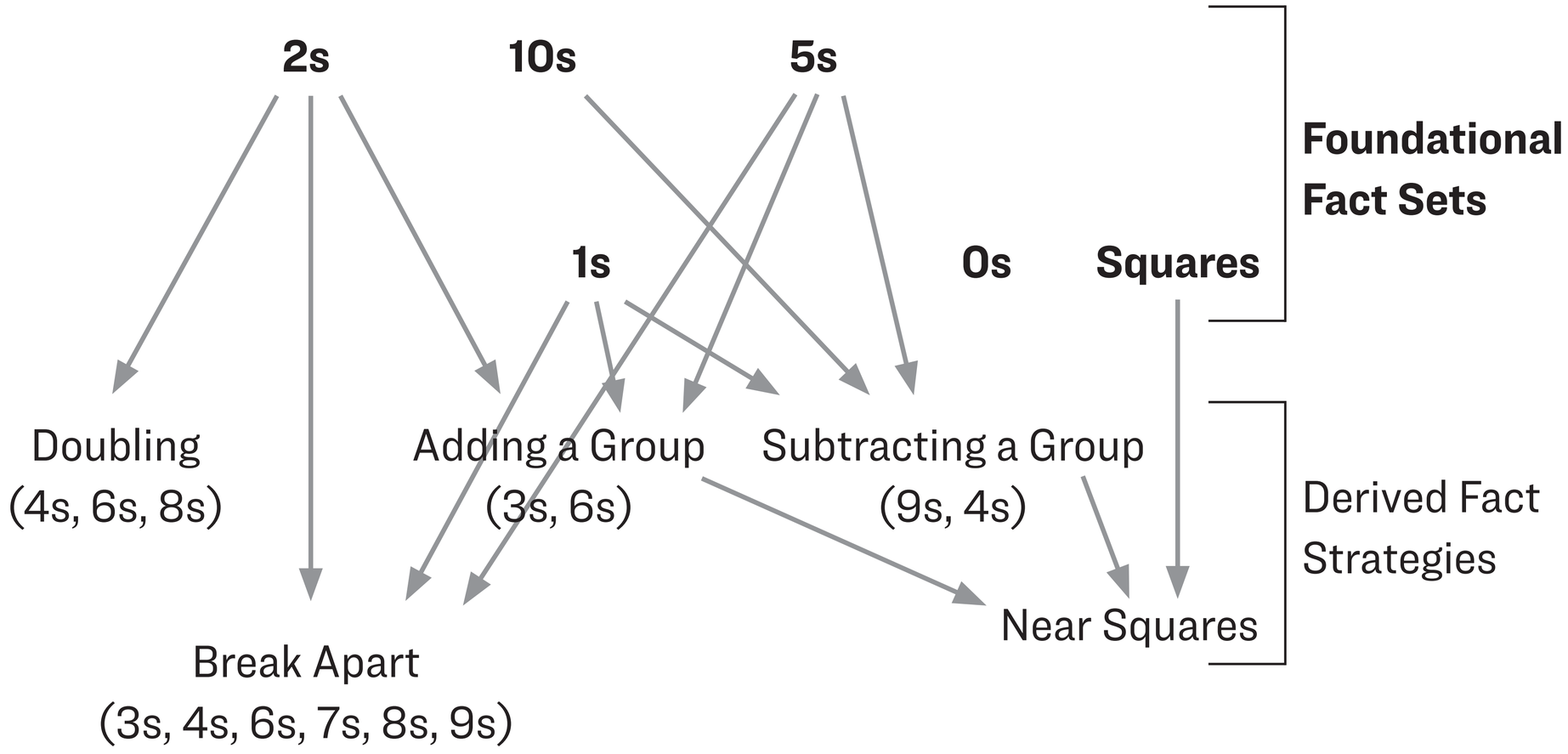

9, 18, 27,
…
Skip
Counting
7 × 9


9, 18, 27, …
Skip Counting
$7 \times 10 - 7 \times 1$
Derived Facts
7×9

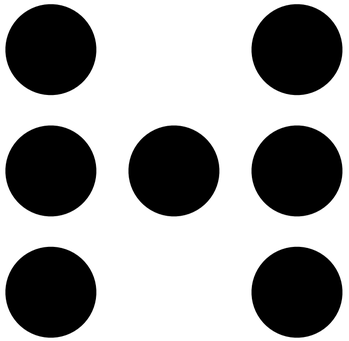

63
Known Facts
$7 \times 10 - 7 \times 1$
Derived Facts
9, 18, 27, …
Skip Counting
7×9

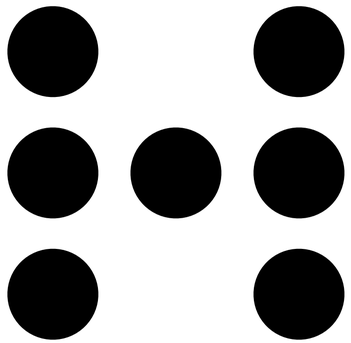
recall (from memory) ≠ memorize

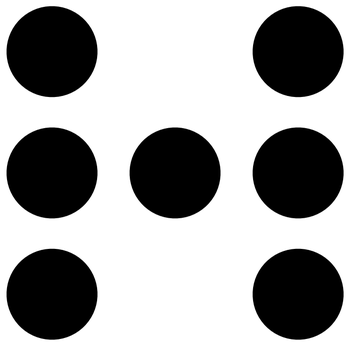
FIGURE 1.3 **Multiplication Fact Fluency Flexible Learning Progression**

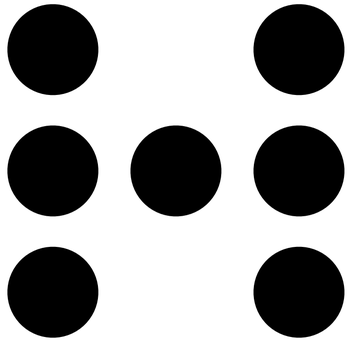


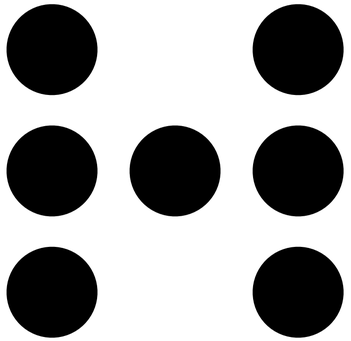
Bay-Williams, J. & Kling, G. (2019). *Math fact fluency: 60+ games and assessment tools to support learning and retention*. National Council of Teachers of Mathematics.

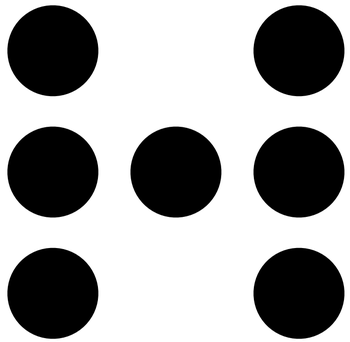


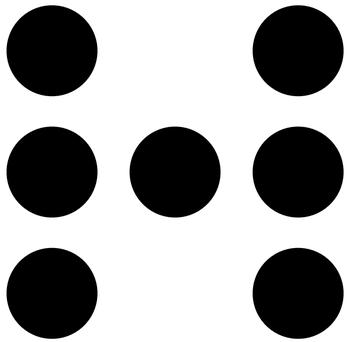




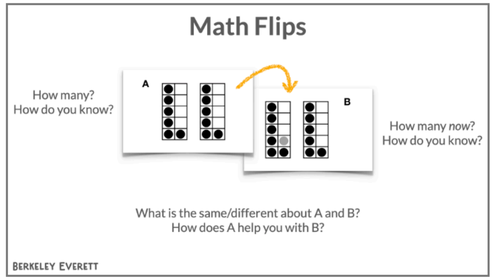




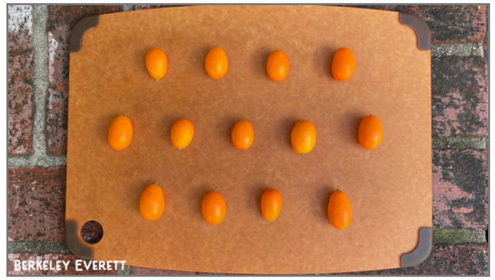




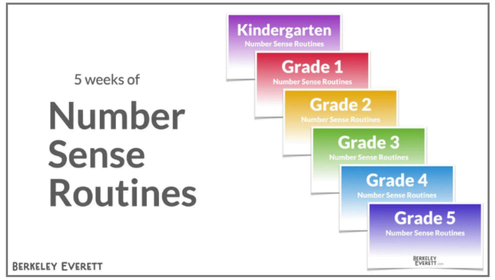
	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100



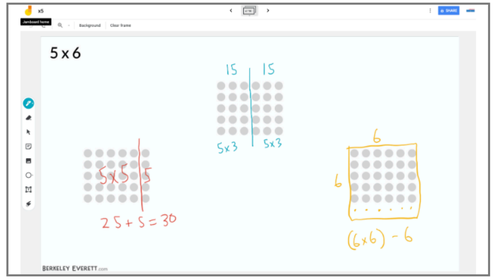
Math Flips: A new kind of flashcard



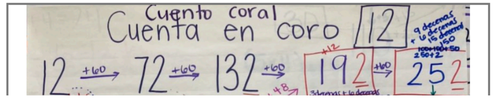
Images: Noticing, Wondering, and How Many?

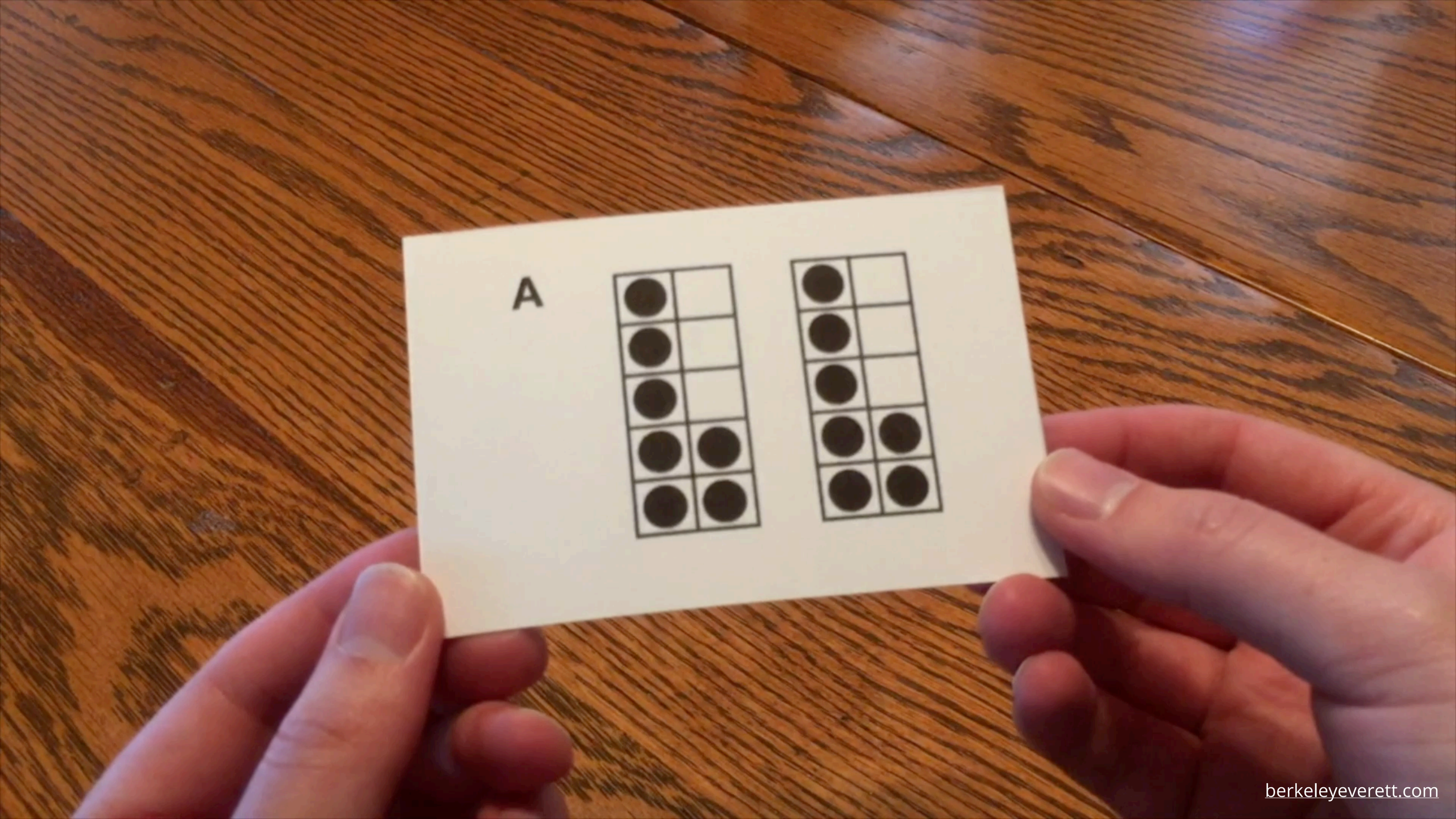


Number Sense Routines: 5 weeks for every grade

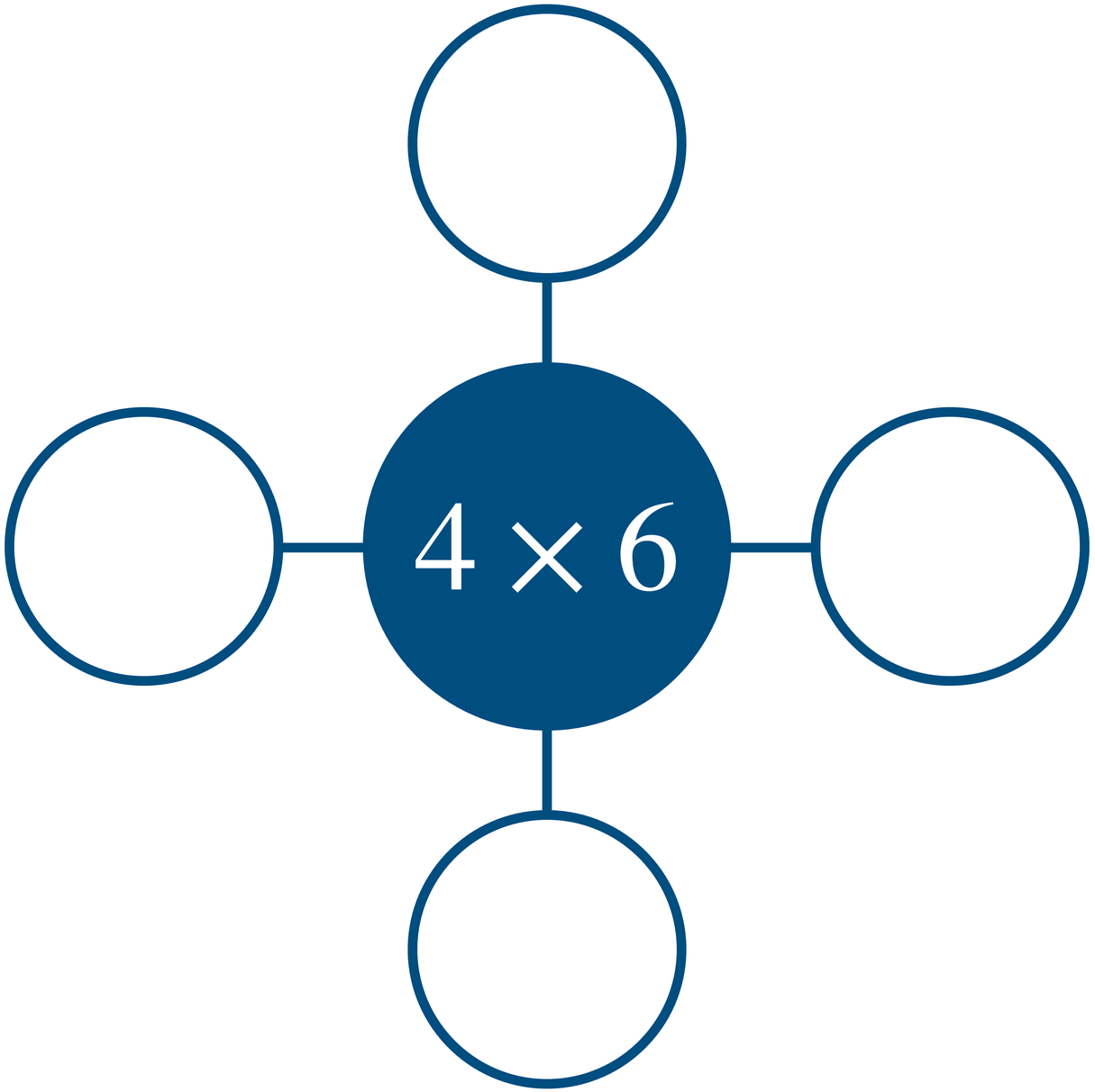


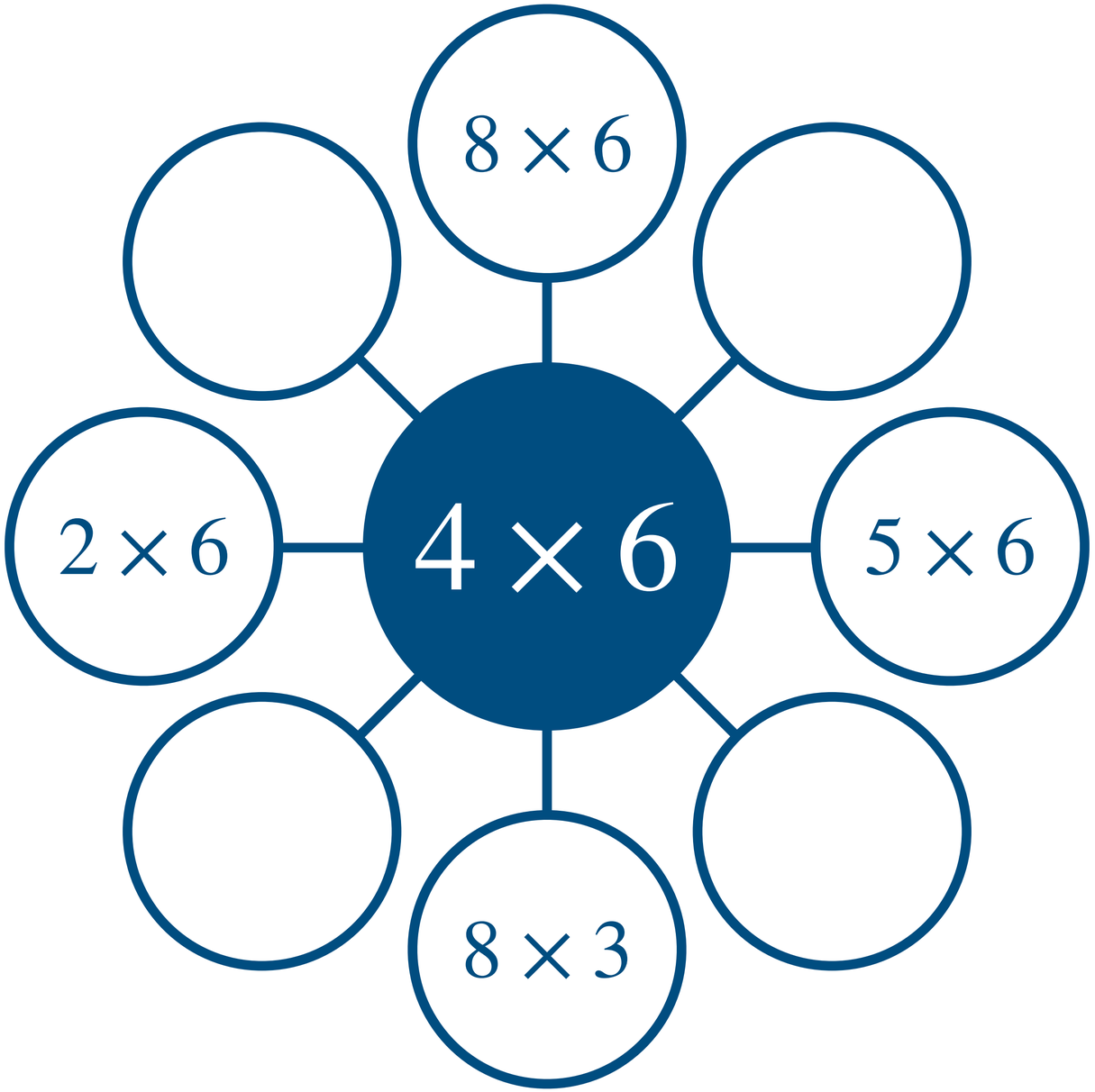
Arrays: All single-digit multiples as slides and PDFs

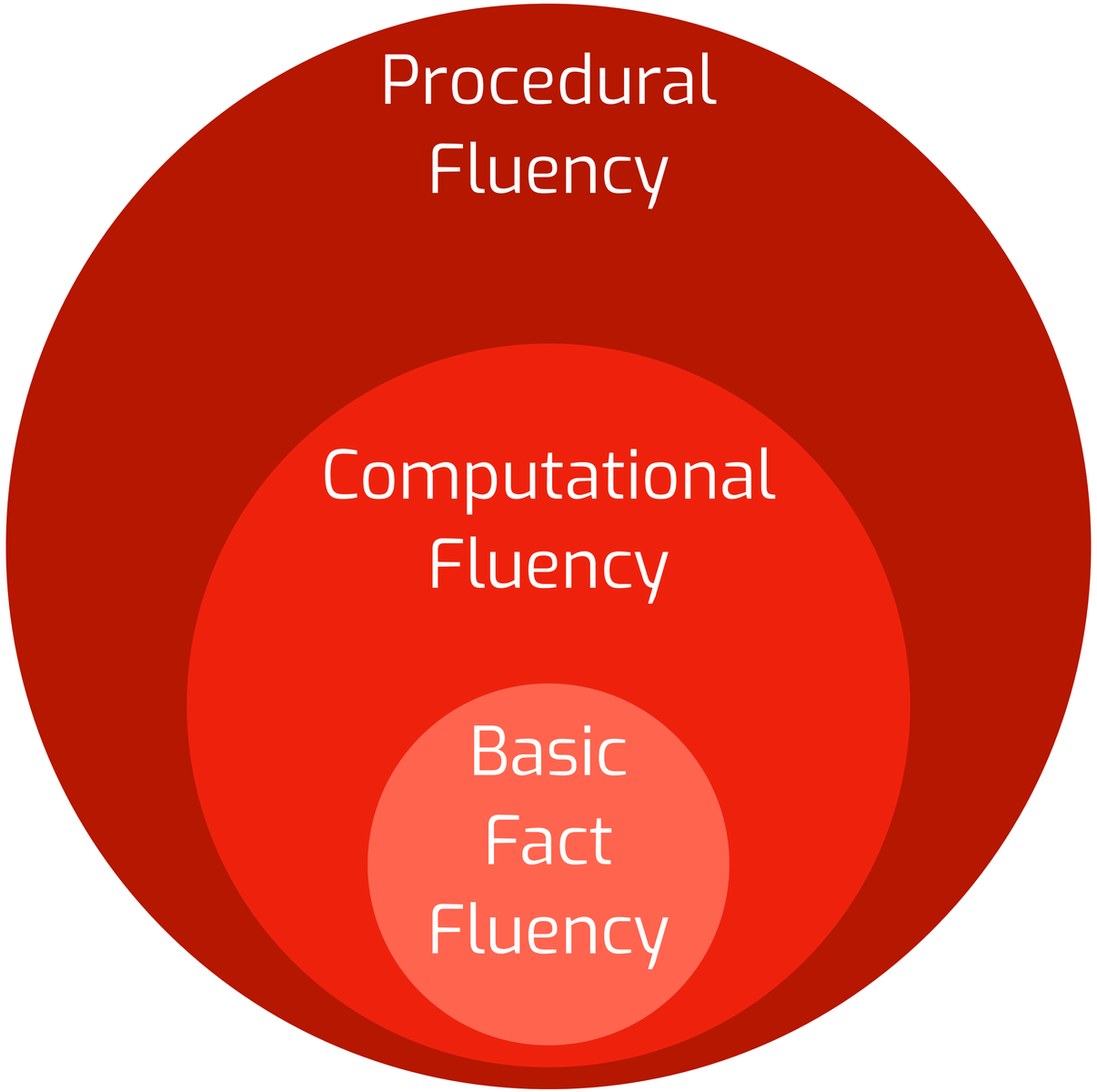




Suppose you know that $4 \times 6 = 24$.
What ***other*** *multiplication facts could you figure out based on that?*


4 × 6


8 × 6
2 × 6
4 × 6
5 × 6
8 × 3


Procedural
Fluency
Computational
Fluency
Basic
Fact
Fluency

$$7 \times 9$$

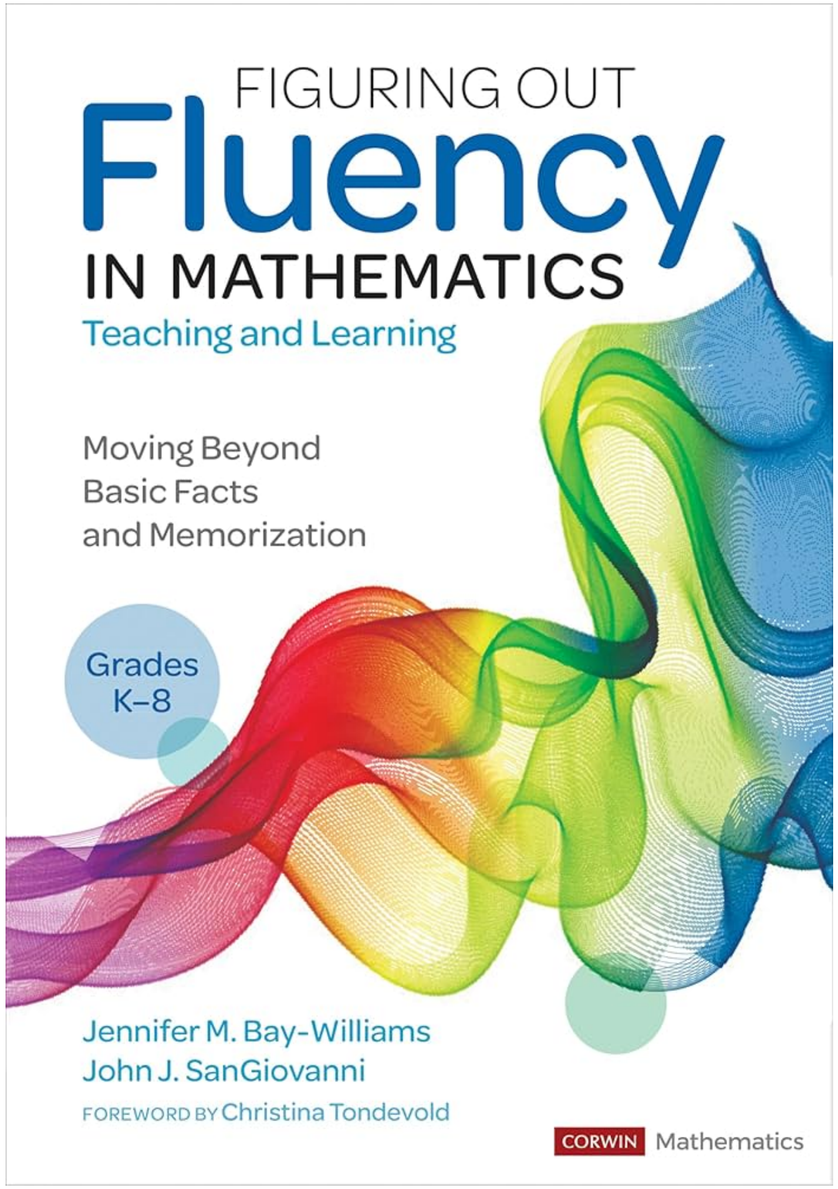
$$7 \times 19$$

$$7 \times 49$$

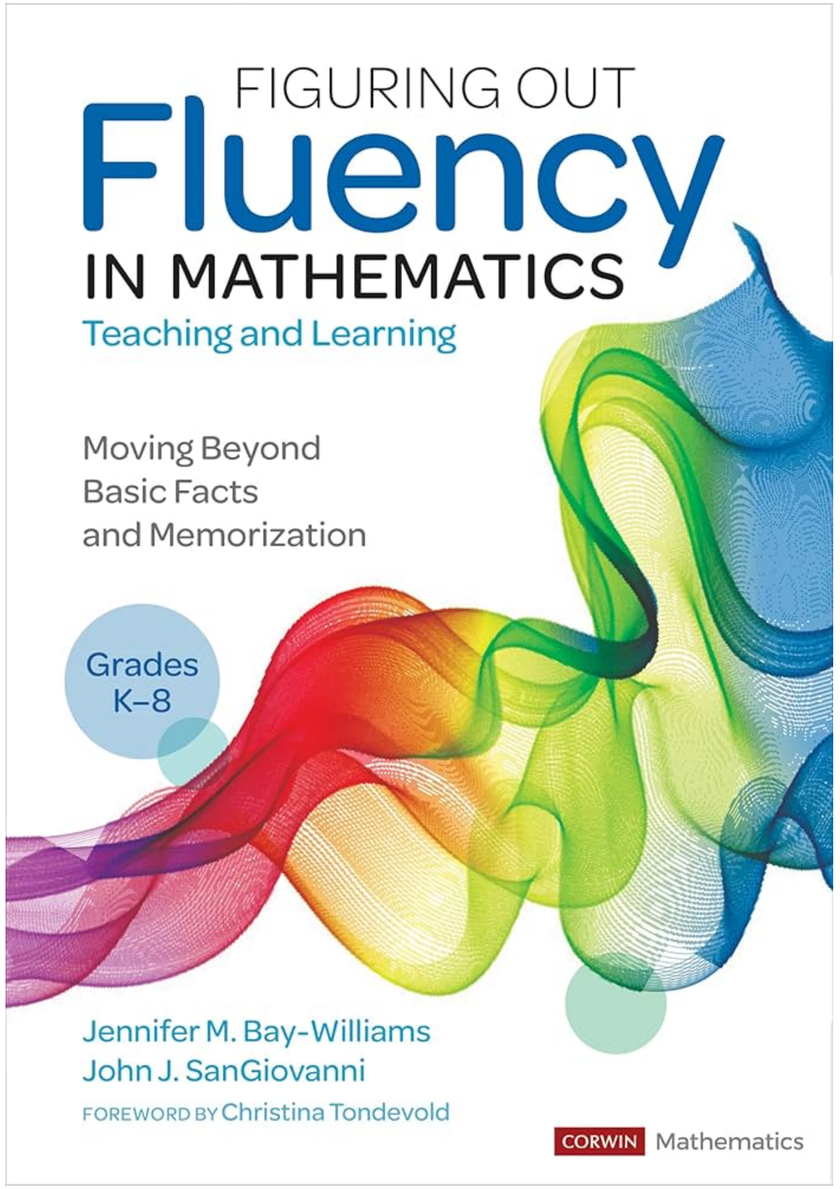
$$7 \times 99$$

$$7 \times 199$$

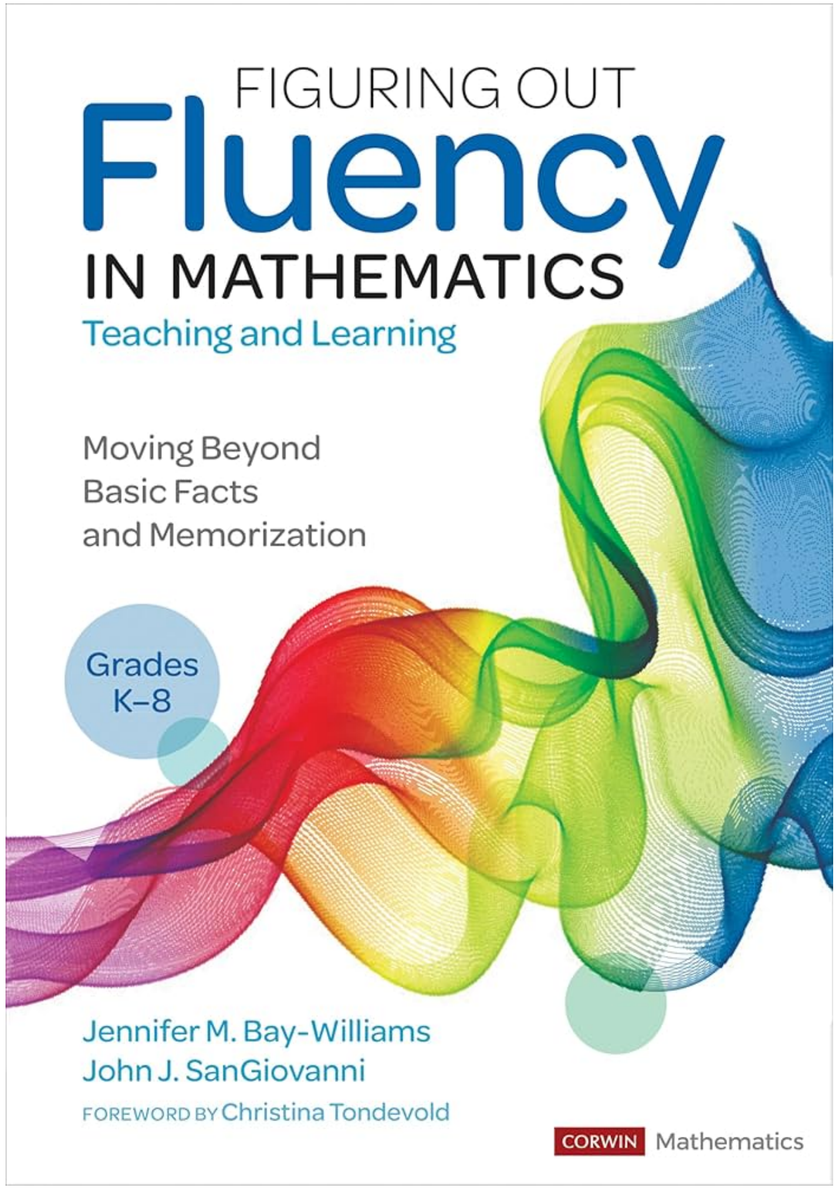
Fallacy: Once learned, the standard algorithm is the best choice.



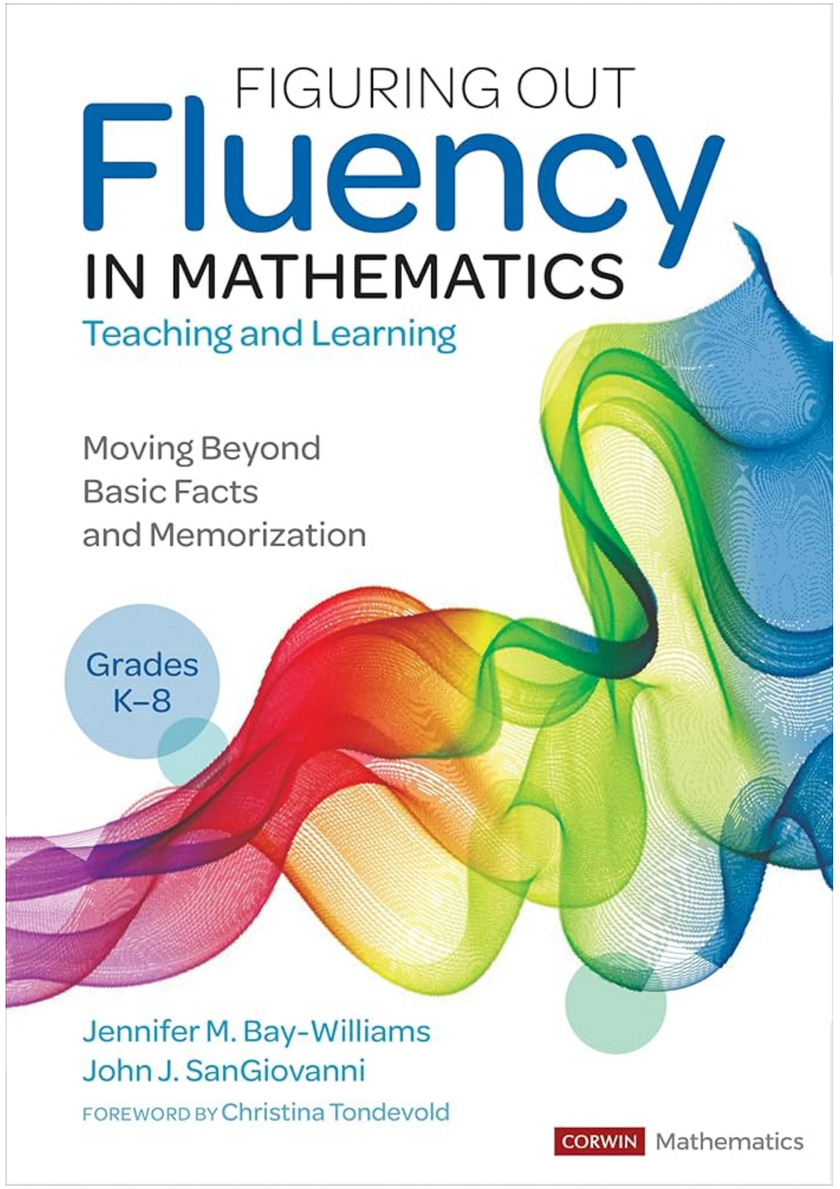
Truth: Standard algorithms are sometimes the best choice, sometimes not; standard algorithms are added to a repertoire of strategies, but they are not a replacement for them.



Fallacy: Some students are better off with knowing just one way.



Truth: Each and every student is better off knowing a set of useful strategies and learning when each is useful (and when they are not).



FOCUS ISSUE: the value of "Why?"

Number Talks Build Numerical Reasoning

Strengthen accuracy, efficiency, and flexibility with these mental math and computation strategies.

By Sherry D. Parrish

ARTISTICCO, LLC/ISTOCKPHOTO.COM

Mary, a third grader, solves twelve minus five on her paper by crossing out the twelve and recording a zero above the ten and a twelve above the two. When asked to share why she solved the problem this way, Mary quickly replies, "Because you have to do it that way when the bottom number is bigger than the top number."

We would like to believe that this is a unique situation; however, our classrooms are filled with students like Mary who view mathematics as a collection of rules and procedures to memorize instead of a system of relationships to investigate and understand (NRC 2001).

The math Process Standards highlighted in *Principles and Standards for School Mathematics* (NCTM 2000) and the National Research Council's Strands of Mathematical Proficiency discussed in *Adding It Up* have encouraged mathematics instruction to move beyond rote procedural knowledge, but these instructional shifts have yet to be consistently embraced or reflected in student performance nationally or internationally (NRC 2001). The recently drafted Common Core State Standards (CCSS) continue to build on these processes and proficiencies with eight Mathematical Practices and calls for instruction grounded in conceptual understanding and mathematical reasoning (CCSSI 2010).

How can educators make shifts in their instructional practices that foster sense making in mathematics and move forward in developing mathematical dispositions as outlined in each of these documents? *Classroom number talks*, five- to fifteen-minute conversations around purposefully crafted computation problems, are a productive tool that can be incorporated into classroom instruction to combine the essential processes and habits of mind of doing math. During number talks, students are asked to communicate their thinking when presenting and justifying solutions to problems they solve mentally. These exchanges lead to the development of more accurate, efficient, and flexible strategies. What does it mean to compute accurately, efficiently, and flexibly? *Accuracy* denotes the ability to produce an accurate answer; *efficiency* denotes the ability to choose an appropriate, expedient strategy

“Simply defined, number talks are five- to fifteen-minute ***classroom conversations*** *around* ***purposely crafted computation problems*** *that are solved mentally.”*

$$8 \times 6$$

$$8 \times 60$$

$$8 \times 59$$

$$8 \times 0.1$$

$$8 \times 6.1$$

$$8 \times 5.9$$

# of Packs	# of Sticks
1	12
2	24
4	48
8	96
10	120
9	108

FACILITATION NOTES

Packs of Gum	Sticks
1	12
2	___
4	___
8	___
10	___
9	___

I've got a pack of gum that has 12 sticks in it. Is that a big pack, small, medium? Do you like gum?
If one pack has 12 sticks in it, how many sticks are in two packs? Model doubling in tandem.
Repeat. *Did anyone use the problem before to help you?*
Repeat. *How could you use the problem before to help you?*
Repeat. *Did anyone use the eight packs? Did anyone not use the 8 packs?*
Repeat. *Did anyone use the eight packs? Did anyone use the 10 packs? Which of those do you wish your brain would think of if you randomly have to find the number of sticks in 9 packs?*

SAMPLE FINAL DISPLAY

Packs of Gum	Sticks
1	12
2	24
4	48
8	96

×2, ×2, ×2 (doubling)

48 = 40 + 8
96 = 80 + 16

Packs of Gum	Sticks
1	12
2	24
4	48
8	96
10	120

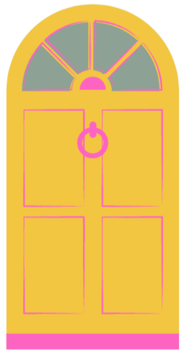
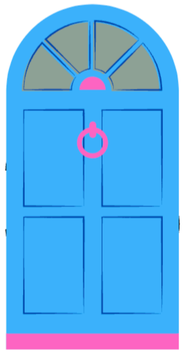
×10, 2+8, 24+96, ×10

Packs of Gum	Sticks
1	12
2	24
4	48
8	96
10	120
9	108

10–1, 1+8, 12+96, 120–12

Note: Alternate using vertical and horizontal ratio tables so students see and work with both orientations.

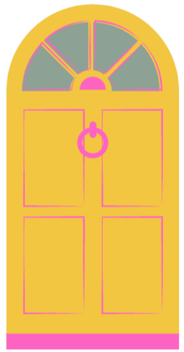
How many possible solutions can you think of?

3 x  = 6 x 

OPEN A DOOR

What must be behind the yellow door?

$$3 \times \square = 6 \times 3.5$$

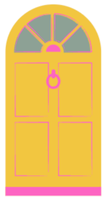


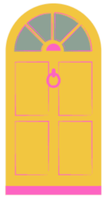
OPEN A DOOR

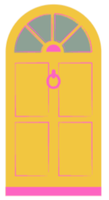
Which side would you rather solve? How might this help you with other problems?

$$3 \times 7 = 6 \times 3.5$$

OPEN PRACTICE

4 x  = 8 x 4.5

24 x 1.5 = 12 x 

 x 6 = 5.5 x 12

REVEAL

$4 \times 9 = 8 \times 4.5$

$24 \times 1.5 = 12 \times 3$

$11 \times 6 = 5.5 \times 12$

Menu

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